INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY
IJARIIT
ISSN: 2454-132X
Impact Factor: 6.078
(Volume 9, Issue 6 - V9I6-1213)
Available online at: https://www.ijariit.com

A relationship between burning number of a graph and its domination number

Hao Yu Zhang
z_aoyu@live.concordia.ca
Concordia University, Montreal, Canada

ABSTRACT

Graph burning, introduced by Elham Rashanbin in 2016(Rashanbin et al,. 2016), has gained significant research attention in recent years. This innovative concept involves iteratively selecting nodes in a given graph to 'burn' at each time step. The previously burnt nodes then ignite the adjacent nodes, and this process continues until all nodes are burnt. The minimum number of nodes required to ignite the entire graph is known as the burning number. Similarly, the domination number of a graph refers to the size of the smallest vertex subsets that ensure every vertex in the graph is either part of the subset or adjacent to a vertex within the subset. In this paper, we demonstrate that if the domination number of a graph is at least 6, then its burning number is smaller than or equal to its domination number.

Keywords *– graph theory, graph, burning number, domination number*

1. PRELIMINARIES

In this section, we will provide the essential definitions required to elucidate our main result, commencing with the fundamental definition of a graph.

Definition: A graph G = (V, E) is a mathematical structure defined as a set V of vertices and a set E of edges. The edges in E are composed of pairs of vertices from V. To denote the vertex set and edge set of graph G, V(G) and E(G) are commonly used, respectively.

Example: The graph shown in the figure below consists of the vertex set V = {0, 1, 2, 3, 4} and edge set E = {{0, 1}, {0, 2}, {0, 3}, {0,4}}.

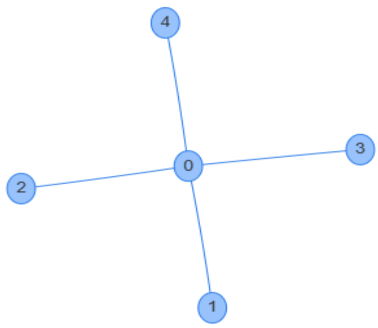

Figure 1

Definition 1: Given a graph G=(V,E), we introduce the burning process as follows: a sequence $v_1, v_2, ..., v_k$ of vertices in V is selected, and each vertex in the sequence is burned at a constant time step. As the burning propagates, the vertices adjacent to those burned in the previous time step also ignite. The burning number of a graph is thereby defined as the minimum length of a burning sequence necessary to cover all vertices.

Example 1: The vertex sequence {0,1} is considered a minimum burning sequence in the graph depicted in the previous figure. This is because, at the second time step, vertex 0 ignites all the other vertices in the graph. Therefore, the burning number of the graph in the previous figure is 2.

Definition 2: A domination set S of a graph G=(V,E) is a subset of V, satisfying the condition that for any vertex in G, it either belongs to set S or is adjacent to a vertex in set S. The domination number of a graph refers to the size of the smallest dominating set in the graph. We use ɤ(G) to denote its domination number.

Example 2: The vertex subset {0} in Figure 1 is an example of a domination set, showcasing that the graph has a domination number of 1.

2.MAIN RESULT

Theorem 1: If a graph G=(V,E) has a domination number greater than 6, then its burning number is smaller than or equal to its domination number.

Proof: Suppose S = {v_1, v_2, ..., v_k} is a domination set of G. In this case, the vertex set V(G) can be expressed as the union of the closed neighborhoods N[v_1], N[v_2], ..., N[v_k], where $k \geq 6$. Here, N[v_i] denotes the ball of radius 1 centered at v_i.

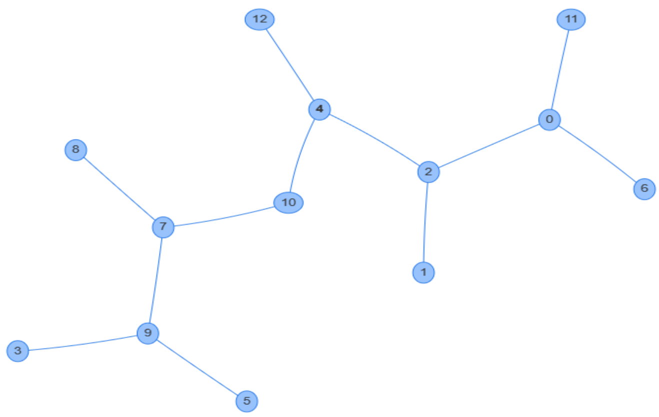

Figure 2

For example, the graph in figure 2 could have as domination set {9,7,4,2,0} and N[9]={3,9,7,5}, N[7]={8,7,9,10},N[4]={12,4,10,2},N[2] = {4,2,1,0},N[0]={2,0,11,6}. We begin by constructing a graph T, where the vertices of T represent the balls N[v_i]. In T, two balls N[v_i] and N[v_j] are connected if they share a common vertex or are directly connected by an edge in G. As a result, T becomes a connected graph with a minimum of 6 vertices. Consequently, it is guaranteed to contain a path of length 2. Without loss of generality (W.L.O.G), let us denote this path as N[v_1]- N[v_2]- N[v_3].

For example, N[9]-N[7]-N[4] in the example in figure 2 form a path of length 2 in the corresponding constructed graph T. The path N[v_1]- N[v_2]- N[v_3] in T induces a subgraph that contains a path of length at most 9 in G. Without loss of generality (W.L.O.G), let u_1,u_2,…,u_9 be such a path in the induced subgraph G[N[v_1] U N[v_2] U N[v_3]] (The argument remains valid even if the path has a length less than 9).

For example, G[N[9] U N[7] U N[4]] has the subpath 3-9-7-10-4-2-1 in G. We will construct a burning sequence for graph G by setting fire to ɤ(G) vertices, namely {u_5, v_4, v_5, v_6, ..., u_1, u_2, u_3}. Here, u_1, u_2, and u_3 are arbitrary vertices in G. Our goal is to demonstrate that this constructed sequence does indeed form a burning sequence for G.

For example a constructed burning sequence for graph in figure 2 could be {4,2,0,0,0}. To begin, we will group the vertices in G based on whether they can be lit on fire by propagating it from a manually ignited vertex. The resulting groups are: $N_{ɤ(G)-1}$ [u_5], $N_{ɤ(G)-2}$[v_4], ..., N_2[u_1], N_1[u_2], and N_0[u_3].

For example for graph in figure 2, we get the following grouping N_4[4]={0,1,2,3,4,5,6,7,8,9,10,11,12}, N_3[2]={7,10,12,4,2,1,0,11,6}, N_2[0]={4,1,2,0,6,11}, N_1[0]={2,0,6,11},N_0[0]={0}

We can verify that these grouped vertices successfully burn all vertices in G. For instance, $N_{ɤ(G)-1}$ [u_5] will have incinerated all vertices in subgraph G[N[v_1] U N[v_2] U N[v_3]] .

For example, N_4[4]={0,1,2,3,4,5,6,7,8,9,10,11,12} coves all vertives in G[N[9] U N[7] U N[4]] in the graph in figure 2. Furthermore, each of the remaining ɤ(G) - 3 domination balls are ignited by a corresponding grouped ingited vertex group. This observation proves that the number of vertices required to burn the entire graph is at most the number of vertices in the domination set.

We will now present examples of graphs that are disconnected or have a small domination number, demonstrating that the domination number in these cases is greater than the domination number previously discussed. We will now provide examples of graphs that are disconnected or have a small domination number, and demonstrate that the domination number is greater than or equal to the burning number. To begin, let us consider a graph represented by a disjoint union of edges, as shown in Figure 3. In this case, the domination number of the graph is equal to the number of edges, while the burning number is one more than the number of edges. Furthermore, let's visualize a path graph of length 2. Here, the domination number of the graph is 1, as one vertex can effectively dominate the entire graph. However, the burning number for this graph increases to 2, indicating that two vertices are needed to ensure the entire graph is reached. These examples help illustrate the relationship between domination number and burning number in different graph structures.

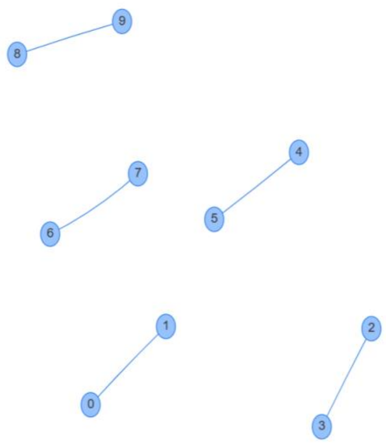

Figure 3

3.CONCLUSION

In conclusion, we have determined that in a connected graph, when the domination number is at least 6, the burning number of the graph is limited to being at most equal to the domination number. Moving forward, our goal is to explore additional relationships between these two graph characteristics. Additionally, we plan to compare the burning number of the graph with other relevant graph parameters to gain further insights.

4. REFERENCES

[1] Anthony Bonato., Jeannette Janssen., and Elham Roshanbin, How to burn a graph. arXiv:1507.06524, July 2015.

