INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X

Impact Factor: 6.078

(Volume 9, Issue 6 - V9I6-1172)

Available online at: https://www.ijariit.com

Fibonacci sequence in music

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ABSTRACT

Music, with its harmonious melodies and rhythmic patterns, has the power to soothe the mind and delight the ear. Within the realm of Hindustani classical music, the core elements include raags, sargams, alaps, and tans, which have played a significant role in not only shaping my life but also in creating some of my happiest memories. For approximately six years, I have dedicated my time to studying various musical genres, with a specific focus on Hindustani classical music. I derive immense enjoyment from this passion, occasionally picking up the guitar and keyboard, and actively participating in my school's choir. However, music is much more than what meets the eye; it encompasses a broad spectrum of sounds, from the tinkling of chimes to the soothing rush of crashing waves.

Keywords: *Mathematics, Music, Golden Ratio, Fibonacci Sequence*

I. INTRODUCTION

Mathematics has always piqued my interest due to its inherent rationality. To simplify mathematical concepts, I often seek to apply them to everyday experiences, such as understanding the geometry of a circular staircase, measuring the distance between electrical poles, or comprehending the global distribution of electrical wires. After years of attempting to unravel its complexity, I've come to appreciate the beauty of mathematics. Mathematics challenges the mind uniquely and nurtures the ability to objectively approach and judiciously solve real-life problems. Similar to music, numbers (the "raags" of math) interact in numerous ways and permeate even the smallest aspects of our lives.

II. HISTORICAL CONNECTIONS BETWEEN MUSIC AND MATH

Mathematics has constituted an integral component of human civilization. Over time, various scientific theories, such as the plum pudding model in chemistry, the luminiferous aether theory in physics, and the contributions of numerous eminent scholars, have undergone revisions and refutations as fresh concepts emerged, sparking revolutionary discoveries. To the ancient Greeks, it was evident that mathematics and music were inextricably intertwined. Early Greek mathematicians, including Pythagoras, Plato, and Aristotle, along with their respective schools of thought, not only recognized but also emphasized the profound link between mathematics and music. The Greek curriculum divided mathematics into four fundamental subsections: Number Theory, Music, Astronomy, and Geometry, placing music on par with Arithmetic, Astronomy, and Geometry.

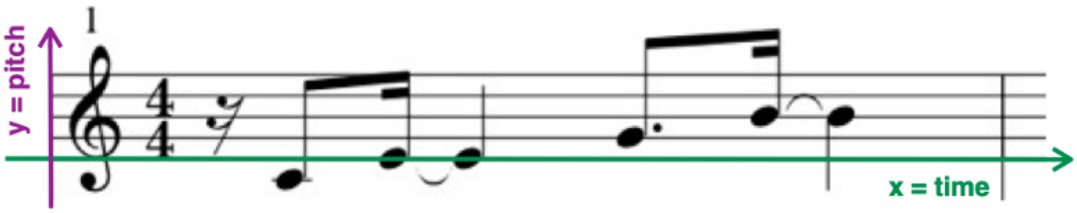
Pythagoras, one of the most prominent Greek mathematicians, made a groundbreaking discovery when he established the mathematical relationships between pleasing musical intervals, such as the octave, fifth, and fourth. His revelation came during a chance encounter near a blacksmith's shop, where the sound of pounding hammers intrigued him. Curious, Pythagoras inquired about the weight of the hammers, revealing weight ratios of 12:9:8:6. Armed with this knowledge, he returned to his community and conducted experiments using a monochord, an instrument with only one string.

The Pythagoreans further explored this concept by dividing a monochord string at two different places and plucking it, resulting in distinct sounds for each case. When applied to the monochord string, the weight ratios of the blacksmith's hammers produced the

same musical consonance that Pythagoras had heard from the pounding hammers. Simultaneously sounded, the vibrating strings demonstrated that specific ratios generated harmonious sounds, ultimately allowing Pythagoras to identify three musical ratios still used by musicians today: 2:1, 3:2, and 4:3. For instance, the ratio of 2:1 defines an octave, signifying the distance between two notes whose frequencies are double or half of each other. Similarly, the pitch ratio of 3:2 characterizes a perfect fifth, where the upper note completes three vibrations in the time it takes the lower one to make two. Likewise, a perfect fourth corresponds to the pitch ratio of 4:3. Plato, another renowned Greek philosopher, believed that geometry and mathematics existed in an ideal world and that specific shapes, now known as the Platonic solids, were associated with the classical elements that constituted the world: earth, fire, air, water, and the universe.

With the advent of the Renaissance movement in the 14th century, the traditional Greek approaches to mathematics began to wane, making way for new, non-Greek perspectives on the subject. Notable developments included René Descartes' introduction of algebraic geometry and Gerard Desargues' exploration of projective geometry. During this era, a more pragmatic mathematics curriculum was formulated, which excluded the rigors of hardcore algebra and arithmetic. Simultaneously, music transitioned from being a mere subpart of mathematics to becoming a subject in its own right. As we progressed into the 17th century, a significant shift occurred, leading to the integration of music and mathematics as intertwined professions. In contrast to earlier times when individuals like Pythagoras pursued geometry, number theory, musicology, and musical performance as separate vocations, the 17th century witnessed the convergence of these domains. This convergence marked the genesis of concepts intersecting and coalescing. René Descartes, a pivotal figure in this evolution, pioneered a completely new field known as analytical geometry, emphasizing an artist's perspective in mathematical exploration. As we entered the 21st century, the interdisciplinary connection between mathematics and music remained a flourishing area of exploration, resonating with the adage that "there is geometry in the humming of the strings... there is music in the spacing of the spheres."

III. PLOTTING MUSIC



When graphically representing a musical piece, the initial step involves indicating the clef, which serves as the key denoting which lines and spaces on the staff represent specific pitches. Subsequently, the time signature is established on the musical staff, typically taking the form of fractions like 2/4, 4/4, 3/4, and 6/8. In this notation, the denominator of the time signature designates the type of note used for beat counting, while the numerator specifies the number of beats within each measure.

A crucial tool for comprehending how music unfolds over time is the meter, typically indicated by the upper number in the time signature. Common meters, such as 2, 3, 4, 6, 9, and 12, convey the number of beats contained in a measure. For instance, in a time signature of 3/4, each measure corresponds to three quartet notes, where the numerator (3) denotes the count within each measure, specifically 1, 2, 3 beats. Here, the "1" represents the stressed pulse, while the "2" and "3" represent relaxed beats. A critical question underlying this exploration is whether mathematical theories can predict musical success which served as the hypothesis for this paper.

IV. THE MATH BEHIND HIT MUSIC

Fibonacci Sequence

The Fibonacci sequence is a series of numbers in which each number is the sum of the preceding two, such as 1, 1, 2, 3, 5, 8, and so on. The golden ratio, denoted by the Greek symbol phi (φ), is the geometric representation of these Fibonacci numbers. It is calculated by dividing a Fibonacci number by its predecessor, resulting in an approximate value of 1.618033988... The Fibonacci sequence has been employed consistently in both ancient and contemporary music to craft melodies.

Classical Tunes

In classical music, mathematical principles like the golden ratio have played a substantial role. For instance, the initial five measures of Beethoven's Fifth Symphony are reiterated 372 measures and 228 measures later, effectively dividing the composition into the golden ratio. There are 377 measures preceding the middle repetition and 233 measures following it, producing a ratio of approximately 1.618%. Moreover, approximately 82% of Mozart's renowned sonatas are divisible by the golden section, indicating its essential usage in Mozart's compositions (Posamentier & Lehmann, 2007, pp. 277-278).

In Handel's Messiah, the "Hallelujah" chorus consists of 94 bars, and the climactic moment when "king of kings" is chanted occurs between bars 57 and 58, representing approximately 8/13 or 0.61538 of the way through the chorus. Additionally, the introduction of the theme "The kingdom of glory" is aligned with the 8/13 mark of the first 57 bars of the composition (Perition, 2013). In the compositions of Chopin, there is a notable alignment between the climax of the music and the point where the golden ratio would naturally divide the piece's duration. This is particularly evident in his 34-measure Prelude No. 1 in C major, where the composition's climax transpires in measure 21, closely approximating the golden ratio of 0.618. This suggests that Chopin may have deliberately utilized the golden ratio to establish a sense of equilibrium and harmony in his music.

Furthermore, Stradivarius violins, among the most esteemed and costly violins globally, have been crafted using the golden ratio. The components of these violins are divided in proportions of 2, 4, 5, 8, and 13, forming a Fibonacci sequence. This application of mathematical principles contributes to their exceptional quality and sound. When examining a keyboard, the golden ratio can also be discerned in the arrangement of keys. The ratio between the width of the white keys and the black keys follows a pattern that closely resembles the golden ratio. This harmonious proportion is believed to enhance both the visual appeal and functionality of the keyboard. With thirteen keys per octave, eight notes are employed to construct a fundamental major scale in that octave, where the first, third, and fifth notes together generate the fundamental major chord.

V. MODERN MUSIC

In the realm of modern music, the influence of mathematical concepts persists. In 2001, the band Tool released the album "Lateralus," which prominently incorporates the Fibonacci sequence into its lyrics. The song "Lateralus" explicitly follows the sequence with the syllable counts in its lyrics, beginning with

1 - "Black"
1 - "Then"
2 - "White are"
3 - "All I see"
5 - "In My Infancy"
8 - "Red and yellow then came to be"
5 - "Reaching out to me"
3 - "Let's me see"

The song continues to follow the sequence, including a backward progression when it reaches the count of 8. Many recent hits also feature traces of the Fibonacci sequence and the golden ratio within their compositions, as illustrated by the excerpts from "The Golden Ratio and Fibonacci Sequence in Music" by Ryan Blankership.

VI. LIST OF TOP 25 SONGS IN THE PAST DECADE

Song	Tempo in BPM	Song length in bars	"Golden bar" or φ/1 point	Significance of φ/1 point in song	Climax / breakdown / build-up	Why is this the climax?	Distance from climax	Distance from breakdown	Distance from build-up
#1: Uptown Funk by Mark Ronson ft. Bruno Mars	115	129	80	During repetition of last line of second chorus	112 / 81-93 / 29-32 and 65-68	Screaming vocals, intense drum hit, extra horns brough in to emphasize, most emotion in voice	32 bars	1 bar	-51 and -15 bars
#2: Party Rock Anthem by LMFAO ft. Lauren Bennet, Goon Rock	130	143	88	During 2nd verse	106 / 91-98 / 99- 106	Snare hits at high speed, synth builds up and hits highest note, tension is released in bar 107	17 bars	3 bars	11 bars
#3: Shape of You by Ed Sheeran	96	94	58	During 2nd chorus	76 / 69-76 / no build-up	Emphasis on isolated vocals which are harmonized the most at this point	18 bars	11 bars	N/A

#4: Closer by The Chainsmoker s ft. Halsey	95	97	60	Bar before 2nd instrumental breakdown	84 / 69-76 / 77-84	Repeating vocal scream, snare build-up until bar 84 then a release in tension after	24 bars	9 bars	17 bars
#5: Girls Like You by Maroon 5 ft. Cardi B	125	133	82	During breakdown	107 / 77-85 / no build-up	Most elements to the chorus that follows this bar, larger emphasis on bass drum	25 bars	5 bars but includes golden bar	N/A

#6: We Found Love by Rihanna ft. Calvin Harris	128	114	70	During 2nd Pre-Chorus	96 / 69-80 / 29-36 and 89-96	Build-up is most intense here	26 bars	-1 bar but includes golden bar	-41 bars and 19 bars
#7: Old Town Road by Lil Nas X ft. Billy Ray Cyrus	136	89	55	During 2nd chorus	72 / 57-65 / no build-up	Vocal harmonization between both singers with almost a screaming type of voice	17 bars	-2 bars	N/A
#8: Somebody That I Used to Know by Gotye ft. Kimbra	129	132	82	Middle of instrumental breakdown	98 / 75-83 / 91-98	Most emotional scream in the song, during a build-up, isolated vocal for emphasis to release tension of build-up	16 bars	-7 bars but includes golden bar	9 bars
#9: Despacito by Luis Fonsi & Daddy Yankee ft. Justin Bieber	89	86	53	During breakdown	56 / 52-56 / no build-up	Culmination of singer's voices singing different parts at the same time ends breakdown tension, sweeping synth to a vocal isolation	3 bars	-1 bar but includes golden bar	N/A

#10: Rolling in the Deep by Adele	105	100	62	During 2nd Chorus	82 / 67-80 / 81-82	Long breakdown then build-up into the final chorus of the song that releases the tension	20 bars	5 bars	19 bars
#11: Sunflower by Post Malone & Swae Lee	90	59	36	During 2nd verse	46 / 43-46 / no build-up	Most emotional singing leading into an emotional chorus, short sweeping synth with vocal isolation, ends tension of breakdown	10 bars	7 bars	N/A
#12: Without Me by Halsey	136	114	70	During 2nd pre-chorus	96 / 89-96 / no build-up	Vocal isolation that ends the tension of the breakdown	26 bars	19 bars	N/A
#13: Call Me Maybe by Carly Rae Jepsen	120	97	60	During 2nd chorus	76 / 69-76 / no build-up	Sweeping synth that leads into vocal isolation, tension released then in final chorus after this part	16 bars	9 bars	N/A
#14: Blurred Lines by Robin Thicke ft. T.I., Pharrell	120	131	81	During 3rd verse	97 / 90-97 / no build-up	Many voice fluctuations that release tension of breakdown	16 bars	9 bars	N/A
#15: Perfect by Ed Sheeran	95	138	85	During 2nd verse	109 / 103-109 / no build-up	Vocal isolation that ends the tension of the breakdown	24 bars	18 bars	N/A
#16: Sicko Mode by Travis Scott	69.50 / 155	237	146	During verse in 3rd beat change	No climax / no breakdown / no build-up	N/A	N/A	N/A	N/A

#17: All About That Bass by Meghan Trainor	134	106	66	During 2nd pre-chorus	82 / 73-82 / no build-up	New drum fill, vocal fluctuation over this fill, releases tension of breakdown	16 bars	7 bars	N/A
#18: Royals by Lorde	85	68	42	During 2nd pre-chorus	59 / 52-59 / no build-up	Vocal isolation, ends tension of breakdown, quick bass kick hits that build into chorus	17 bars	10 bars	N/A
#19: God's Plan by Drake	77.2	65	40	During 2nd chorus	29-30 / 21- 24 / 25-28	Vocal isolation that ends tension of build-up	-11 bars	-19 bars	-15 bars
#20: Moves Like Jagger by Maroon 5 ft. Christina Aguilera	128	108	67	During 2nd chorus	88 / 73-80 / no build-up	Emotional scream that is isolated	21 bars	6 bars	N/A
#21: Happy by Pharrell Williams	160	154	95	During 3rd chorus	No climax / 66-81 / 74- 81 and 114- 121	N/A	N/A	-29 bars	-21 bars and 19 bars
#22: Just the Way You Are by Bruno Mars	109	98	61	During 2nd chorus	73 / no breakdown / no build-up	High sustained vocal notes that have the most emotion	12 bars	N/A	N/A
#23: Rockstar by Post Malone ft. 21 Savage	160	145	90	During 2nd verse	No climax / no breakdown / no build-up	N/A	N/A	N/A	N/A
#24: Tik Tok by Ke$ha	120	101	62	During 2nd chorus	81-82 / 65- 72 / 73-80	Vocal isolation that leads into chorus with most layers of instruments to it, ends tension of build-up	19 bars	3 bars	11 bars
#25: See You Again by Wiz Khalifa ft. Charlie Puth	80	76	47	During 2nd pre-chorus	67 / 51-58 / no build-up	Highest vocal note in the song that leads into the close, ends tension of just piano and vocal bars that precede it	20 bars	4 bars	N/A

The presence of the golden bar in the majority of songs was not consistently aligned with the climax, as climactic moments typically tend to occur towards the conclusion of a song. Furthermore, there was no definitive evidence indicating that the golden bar could predict the initiation of a climax or breakdown within a composition. However, the ratio appeared to be more closely associated

with breakdowns than with climactic points, and in some instances, it coincided with breakdowns. For instance, in one song ("Despacito"), the climax transpired within 10 bars of the "golden bar," while in three songs, breakdowns were just 1 bar away from it. In nine songs, breakdowns were separated by 5 bars from the "golden bar." The analysis revealed that climaxes and breakdowns were present in 22 out of 25 tracks. Builds-ups were less frequent, occurring in only 9 out of 25 songs, with a single build-up being located at a distance of 10 bars from the "golden bar." Notably, "Call Me Maybe" and "Blurred Lines" exhibited breakdowns and climaxes at the same distance from the "golden bar," despite variations in song lengths and the same tempo. Additionally, "We Found Love" by Rihanna featuring Calvin Harris and "Uptown Funk" both featured two identical build-ups before the chorus. It's worth mentioning that, with the exception of "Perfect" by Ed Sheeran, all the songs analyzed adhered to a 4/4 time signature.

Isolated vocals were found to play a significant role in several of these songs' climactic moments, and many songs featured climaxes at the culmination of their build-ups, which may have contributed to their popularity. While the study concluded that the Fibonacci sequence does not guarantee a song's popularity, it underscored its significance as an integral component of musical compositions. The Fibonacci sequence and the golden ratio are inherently intertwined with music, existing within an octave, the fundamental unit of music and melody. Stradivarius, the renowned violin maker, incorporated these mathematical principles into the creation of the world's finest string instruments. Therefore, while they may not serve as predictors of a song's success, they undeniably hold a pivotal place within the realm of music.

VII. CONCLUSION

In conclusion, this research paper has explored the profound connection between mathematics and music, spanning historical links, musical notation, and the role of the Fibonacci sequence and the golden ratio in both classical and modern music. While these mathematical principles may not predict a song's popularity, they undeniably enrich the fabric of musical composition, emphasizing the inseparable bond between math and music.

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