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The effect of overtaking disturbances on the motion of strong plane shock wave in a highly viscous medium

Dr. Kamlesh Kumar
krkamlesh06@gmail.com
Agra College, Agra, Uttar Pradesh

Dr. Arvind Kumar
arvindphy80@gmail.com
Daryao Singh Rathore Memorial Degree College, Gularia, Uttar Pradesh

ABSTRACT

The effect of overtaking disturbances, the motion of strong plane shock wave in highly viscous medium has been investigated by Chester-Chisnell- Whitham method. It is applied to obtain the analytical relations for shock velocity and shock strength for freely propagation of shock. These relations are modified in presence of overtaking disturbances. The obtained expansions are computed and discussed through table.

***Keywords**- Shock Wave, CCW Method and Viscosity*

1. INTRODUCTION

Shock waves arise in a wide range of physical phenomena such as gas dynamics, nuclear explosions, shallow water flows, supernovae, stellar winds, traffic flows, quantum fluids, and many others. The theory of shock waves has a rich history beginning with the fundamental contributions by Riemann in the mid of the 19th century. In fact, all natural fluids admit some compressibility and therefore support shock waves. Shock waves can only develop in a medium which behaves like a fluid. Shock waves may be produced in fluids such as sea water by a variety of natural and artificial mechanisms.

In determining the structure and thickness of shock front, the dissipative processes such as viscosity and thermal conduction play key role within the shock front as the gradients are very or almost infinitely steep there. It is also worth mentioning that the viscosity plays a major role in the mechanism of shock compression rather than the heat conduction, as the viscosity of the gas causes the scattering of directed momentum of incident gas and the conversion of kinetic energy of directed molecular motion into the kinetic energy of random motion, i.e., the conversion of mechanical energy into heat energy. It is notable that the heat conduction only indirectly affects the conversion of mechanical energy due to the redistribution of the pressure. The thickness of viscous shock front is proportional to the coefficient of viscosity which in turn is proportional to the molecular mean free path l.

Shock processes can naturally occur in various astrophysical situations, for example, photo ionized gas, stellar winds, supernova explosions, collisions between high velocity clumps of interstellar gas, etc. Shock phenomena, such as a global shock resulting from a stellar pulsation or supernova explosion passing outward through a stellar envelope or perhaps a shock emanating from a point source such as a man-made explosion in the Earth's atmosphere or an impulsive flare in the Sun's atmosphere, have tremendous importance in astrophysics and space sciences. Shock waves are common in the interstellar medium because of a great variety of supersonic motions and energetic events, such as cloud-cloud collision, bipolar outflow from young protostellar objects, powerful mass losses by massive stars in a late stage of their evolution (stellar winds), supernova explosions, central part of star burst galaxies, etc. Shock waves are also associated with spiral density waves, radio galaxies and quasars. Similar phenomena also occur in laboratory situations, for example, when a piston is driven rapidly into a tube of gas (a shock tube), when a projectile or aircraft moves supersonically through the atmosphere, in the blast wave produced by a strong explosion, or when rapidly flowing gas encounters a constriction in a flow channel or runs into a wall. The explanation and analysis for the internal motion in stars is one of the basic problems in astrophysics.

A few approximate methods, which give fellow conditions just behind the shock as it propagates, have also been proposed but the fellow in the entire region behind the shock is not determined. Two method in this category, which have been widely used, are **Chester (1954), Chisnell (1955)** and **Whitham (1958)** method and **Brinkley-Krikwood (1947)**. **Vishwakarma (2000), Vishwakarma and Nath (2008)** and **Vishwakarma et al. (2008)** have discussed the propagation of shock waves in a medium where density varies exponentially and obtained similarity and non-similarity solutions. **K. Zumbrun (2010)** the existence and effects of the viscous forces for the similarity solutions to shock wave problems were studied. **G. Nath (2010)** Magnetogas dynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of perfect gas with variable density. **Xiaojuan et al. (2010)** studied the effects of viscosity on shock induced damping of initial sinusoidal disturbances. **Anand et al. (2013)** the effects of Viscosity on the Structure of Shock Waves in a Non-Ideal Gas. **Ramu et al. (2016)** Similarity solution of spherical shock waves - effect of viscosity. **Kumar Arvind et al. (2020)** the motion of weak spherical shock wave in highly viscous medium.

The aim of the present part is to study the propagation of strong plane shock waves propagating in a uniform medium. When shock moves freely and the effect of overtaking disturbances. The shock strength, pressure and particle velocity both decreases as plane shock. The effect of overtaking disturbances is to inhance the values.

2. BASIC EQUATIONS

The general equations of exploding shock waves in presence of uniform viscous medium

$$\frac{\delta u}{\delta t} + u\frac{\delta u}{\delta r} + \frac{1}{\rho}\frac{\delta P}{\delta r} - \frac{4}{3}\mu\frac{\delta u}{\delta r} = 0$$

$$\frac{\delta \rho}{\delta t} + u\frac{\delta \rho}{\delta r} + \rho\frac{\delta \rho}{\delta t} + \frac{\alpha \rho u}{r} = 0$$

$$\frac{\delta P}{\delta t} + u\frac{\delta P}{\delta r} - a^2\left[\frac{\delta r}{\delta t} + u\frac{\delta \rho}{\delta r}\right] = 0$$

$$\frac{\delta P}{\delta t} + u\frac{\delta P}{\delta r} + a^2\rho\left[\frac{\delta r}{\delta t} + \frac{\alpha u}{r}\right] = 0$$

Where, u(r, t), P(r, t) and ρ (r, t) denote particle velocity, pressure, density at a distance r from the origin at time t, γ is the adiabatic index of gas, μ is the coefficient of viscosity and $\alpha = 0$ for plane shock waves.

3. BOUNDARY CONDITIONS

Let P_0 and ρ_0 denotes the unperturbed values of pressure and density in front-

$$P = {a_0}^2\rho_0\left[\frac{2M^2}{(\gamma+1)} - \frac{(\gamma-1)}{(\gamma+1)}\right]$$

$$\rho = \rho_0\left[\frac{(\gamma+1)M^2}{(\gamma-1)M^2+2}\right].$$

$$U = \frac{2a_0}{(\gamma+1)}\left[M - \frac{1}{M}\right]$$

$$a = a_0\sqrt{\frac{[2\gamma M^2-(\gamma-1)]\,[(\gamma-1)M^2+2]}{(\gamma+1)}}$$

where, $M = \frac{U}{a_0}$ is Mach number, U is the shock velocity, a and a_0 are the sound velocity in disturbed and undisturbed medium respectively.

4. STRONG SHOCK WAVES

For strong shock waves i.e. ($U \gg a_{\square}$) the boundary conditions are reduce-

$$P = \frac{2\rho_0 U^2}{(\gamma+1)}$$

$$\rho = \rho_0\left[\frac{(\gamma+1)}{(\gamma-1)}\right]$$

$$u = \frac{2U}{(\gamma+1)}$$

$$S = \sqrt{\frac{2\gamma}{(\gamma-1)}}$$

Characteristic Equation for Freely Propagation of Shock Waves:

The characteristic equation for exploding shock is given as-

$$dP + \rho a\,du + \frac{\alpha\rho a^2 u}{r}\frac{dr}{(u+a)} - \frac{4}{3}\frac{\mu\rho a\,du}{(u+a)} = 0$$

Now using boundary conditions and solving the characteristic equation-

$$\frac{dU}{U} + \frac{S^2(\gamma-1)\alpha}{[2+S(\gamma-1)]\left[2+S-\frac{4\mu S(\gamma-1)}{3[2+S(\gamma-1]}\right]}\frac{dr}{r} = 0$$

The expression for shock velocity may be written as-

$$U = k\, r^{-2\gamma\alpha/[2+S(\gamma-1)][(2+S)-4\mu S(\gamma-1)/3[2+S(\gamma-1)]]} \quad (1)$$

The expression for shock strength may be written as-

$$M = \frac{U}{a_0} = \frac{\rho_0}{\gamma P_0}\, k\, r^{-2\gamma\alpha/[2+S(\gamma-1)][(2+S)-4\mu S(\gamma-1)/3[2+S(\gamma-1)]]} \quad (2)$$

Therefore, increment in particle velocity may be written as-

$$du_+ = \frac{2}{(\gamma+1)} dU_+ = \frac{2}{(\gamma+1)} dk\, r^{-2\gamma\alpha/[2+S(\gamma-1)][(2+S)-4\mu S(\gamma-1)/3[2+S(\gamma-1)]]}$$

5. EFFECT OF OVERTAKING DISTURBANCES

To estimate the effect of overtaking disturbances, we have taken differential equation valid across C_- characteristics, as

$$dP - \rho a\, du + \frac{\alpha\rho\, a^2 u}{r}\frac{dr}{(u-a)} - \frac{4}{3}\frac{\mu\rho a\, du}{(u-a)} = 0$$

Now using boundary conditions for strong shock and substituting respective values in this equation, then

$$\frac{dU}{U} + \frac{S^2(\gamma-1)\alpha}{[2-S(\gamma-1)]\left[2-S-\frac{4\mu S(\gamma-1)}{3[2-S(\gamma-1]}\right]}\frac{dr}{r} = 0$$

Solving this equation-

$$U = k\, r^{-2\gamma\alpha/[2-S(\gamma-1)][(2-S)-4\mu S(\gamma-1)/3[2-S(\gamma-1)]]}$$

Therefore, increment in particle velocity may be written as-

$$du_- = \frac{2}{(\gamma+1)} dU_- = \frac{2}{(\gamma+1)} dk\, r^{-2\gamma\alpha/[2-S(\gamma-1)][(2-S)-4\mu S(\gamma-1)/3[2-S(\gamma-1)]]}$$

In presence of overtaking disturbances, the result and increment in particle velocity will be-

$$du_+ + du_- = dU^*$$

Substituting the value of $du_+ + du_-$ and then solving-

$$dU^* = \frac{A\,k}{r^{A+1}} - \frac{B\,k}{r^{B+1}}$$

Solving this equation, we get-

$$U^* = k\left[\frac{1}{r^A} + \frac{1}{r^B}\right] + k \quad (3)$$

Shock strength may be written as-

$$M^* = \frac{U^*}{a_0} = \frac{k\left[\frac{1}{r^A}+\frac{1}{r^B}+1\right]}{\sqrt{\frac{\gamma P_0}{\rho_0}}} \quad (4)$$

Where –

$$A = \frac{2\gamma\alpha}{[2+S(\gamma-1)]\left[2+S-\frac{4\mu S(\gamma-1)}{3[2+S(\gamma-1]}\right]} \quad \text{and} \quad B = \frac{2\gamma\alpha}{[2-S(\gamma-1)]\left[2-S-\frac{4\mu S(\gamma-1)}{3[2-S(\gamma-1]}\right]}$$

6. RESULTS

Strong Plane Shock Waves:

Expression (1) and (2) represents the shock velocity (U) and shock strength (M) for the freely propagation of strong shock, in uniform medium and equation (3) and (4) represent the effect of overtaking disturbances. Shock strength is a function of propagation distance r, adiabatic index γ and viscosity coefficientμ. It is concluded that shock strength, pressure and particle velocity decreases for both the freely propagating shock and with the represent the effect of overtaking disturbances as plane shock advances.

Table1: Variation of variable with adiabatic index for strong plane shock waves

$(r = 2, \mu = 0.000172 , \alpha = 0 \; and \; \rho = 1.29)$

Adiabatic index (γ)	Shock velocity (U)	Shock velocity (U*)	Shock strength (M)	Shock strength (M*)	Pressure (P)	Pressure (P*)	Particle velocity (u)	Particle velocity (u*)
1.33	13.6905	27.381	13.483	26.966	207.351	829.967	11.6868	23.471
1.40	13.6905	27.381	13.142	26.283	201.252	805.709	11.3426	22.785
1.66	13.6905	27.381	12.105	24.210	182.073	729.505	10.2619	20.629
1.69	13.6905	27.381	11.996	23.993	180.009	721.313	10.1458	20.398
1.75	13.6905	27.381	11.754	23.508	175.365	702.891	9.88463	19.878
1.80	13.6905	27.381	11.589	23.179	172.188	690.293	9.70609	19.522

Note- * represent the effect of overtaking disturbances.

7. DISCUSSION

The shock strength varies from 13.483 to 11.589 the freely propagating shock and from 26.966 to 23.179 with the effect of overtaking disturbances. Then shock strength decreases for both cases with adiabatic index. Pressure varies from 207.351 to 172.188 the freely propagating shock and from 829.967 to 690.293 with the effect of overtaking disturbances as strong plane shock waves moves from adiabatic index 1.33 to 1.80. Particle velocity (u) varies from 11.6868 to 9.70609 the freely propagating shock and from 23.471 to 19.522 with the effect of overtaking disturbances with the variation of adiabatic index 1.33 to 1.80. These parameter decreases as plane shock advances in uniform medium. But shock velocity are constant for both cases (i) the freely propagating shock (ii) the effect of overtaking disturbances for adiabatic index (γ). Strong plane shock wave passes through the propagation distance and viscosity coefficients are constant. These tables are constant variable for shock velocity. But similar results are found for strong shock propagating in non- ideal gas **Vishwakarma et al. (2007)**.

8. CONCLUSIONS

It is concluded that shock strength, pressure and particle velocity decrease with adiabatic index and shock velocity is constant for adiabatic index. The present study provides results useful for future studies including dissipative shocks. These types of shock waves arise frequently in the cases where the velocity of fluid is greater than the local sound speed; they find an application in gas dynamics, fluid mechanics, aerodynamics, astrophysics, solar physics, and space physics, for both magnetised and unmagnetised fluid motions. It is planned to use the present algorithm and its results to be applied to solar coronal shock waves.

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10. REFERENCES

[1] Chester,W, Phil.Meg.,Soc.7 Vol.45,1293-1301,1954.
[2] Chisnell,R.F.Proc.Roy.Soc.A.232pp.350-370,1955.
[3] Whitham,G.B.,Jornal of fluid mech.,4,337,1958.
[4] Brinkley, S.R. and Krikwood, J.G.,Physics,Review,71,606,1947.
[5] J. P. Vishwakarma, “Propagation of shock waves in a dusty gas with exponentially varying density,” European Physical Journal B, vol. 16, no. 2, pp. 369–372, 2000.
[6] J. P. Vishwakarma, A. K. Maurya, and K. K. Singh, “Self-similar adiabatic flow headed by a magnetogasdynamic cylindrical shock wave in a rotating non-ideal gas,” Geophysical and Astrophysical Fluid Dynamics, vol. 101, no. 2, pp. 155–168, 2007.
[7] J. P. Vishwakarma and G. Nath, “Propagation of shock waves in an exponential medium with heat conduction and radiation heat flux,” Modelling, Measurement and Control B, vol. 77, no. 3-4, pp. 67–84, 2008.
[8] J. P. Vishwakarma, G. Nath, and K. K. Singh, “Propagation of shock waves in a dusty gas with heat conduction, radiation heat flux and exponentially varying density,” Physica Scripta, vol. 78, no. 3, article 035402, 11 pages, 2008.
[9] K. Zumbrun, “The refined inviscid stability condition and cellular instability of viscous shock waves”, Physica D: Nonlinear Phenomena., 239 (13), pp. 1180-1187, 2010.
[10] Xiaojuan ,M.A.,Fusheng, L.I.U. and Fuqian, J.I.N m.G, “the effects of viscosity on shock induced damping of an initial sinusoidal disturbances”Science China,Phys.,Mech.,Astron.Vol.53(5),2010.
[11] G. Nath, “Magnetogasdynamic shock wave generated by a moving piston in a rotational axisymmetric isothermal flow of perfect gas with variable density,” Advances in Space Research, vol. 47, no. 9, pp. 1463–1471, 2011.
[12] Anand R.K. and Yadav H.C., **“**The Effects of Viscosity on the Structure of Shock Waves in a Non-Ideal Gas” Acta Physica Polonica,Vol.296,pp.28-34,2013.
[13] A. Ramu, N. Dunna, D. K. Satpathi, Similarity solution of spherical shock waves - effect of viscosity, Journal of Mathematics Vol. 35, No 1, pp. 11-31, March 2016.
[14] Arvind Kumar, Kamlesh Kumar, H Singh and S Prakash **, “**the motion of weak spherical shock wave in highly viscous medium”, IJARIIT Vol. - 6 No- 4,pp.745-747,2020.

