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# Ant Colony Optimization Algorithms for the Knapsack and traveling salesman problems

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**ABSTRACT**

***This paper is a simple tutorial for researchers interested in ant colony optimization (ACO) in general and max-min ant system (MMAS) in particular. The paper compares the differences in implementing these algorithms to solve sequencing and selection problems. For selection problems, we use the famous knapsack problem (KP) to demonstrate how MMAS can be used, wherease, for the sequencing problem, we use the famous travelling salesman problem (TSP). Results from the literature shows how a MMAS algorithm can outperform other meta-heuristics in solving these two types of problems.***

***Keywords:*** *Max-min ant system, Ant Colony Optimization, Knapsack roblem, Travelling Salesman Problem, Algorithms*

## 1. INTRODUCTION

Ant colony optimization (ACO) [1] algorithm have been extensively used by researcher to solve several problems like the travelling salesman problem (TSP) [2], knapsack problem (KP) [3], and several other problems [3]—[9]. A major difference between TSP and KP is that TSP is a sequencing problem whereas KP is a selection problem.

In this paper, we compare the implementation a special type of the ACO algorithms, the max-min ant system (MMAS) [10], to solve both problems. In our comparison, we show that the number of indices used to denote the pheromone trails are two for the sequencing problem while it is one for the selection problem. This difference would lead to more memory needs for the sequencing problem. Thus, research might need to use selection lists in the sequencing problem [10].

## 2. KNAPSACK PROBLEM

The KP problem is about selecting a set of items such that there consumption of a single or multiple resources is limited by the availability of this resource. The objective function is to maximize the function represented in Equation 1. In this equation, $w_i$ and $x_i$ represet the number of benefit of including item $i \in N$, where $N$ is the set of items. Each item $i \in N$ consumes $t_i$ units of a resource that has a capacity of T. Eequation 2 represents this constraint. The decision variable of the KP model is $x_i$ which has a value of 1 if item $i \in N$ is selected.

$$\max Z = \sum_{i=1}^{N} w_i x_i \qquad (1)$$

$$\sum_{i=1}^{N} t_i x_i \leq T \qquad (2)$$

$$x_i \epsilon Z$$

### 2.1 A max-min ant system for KP

Ants in nature have collective intelligence. To find the shortest route from point A to point B, ants lay a chemical called pheromone to know their path their nest. Pheromones have an evaporation rate; thus, if the path is long, time for pheromone evaporation increase and not much chemicals are laid along this path. When an ant from a later bach wants to select its path, it selects paths with more pheromones with higher probabilities. So, pheromone trails along short paths tend to increase and a long short paths tend to decrease. After a while, ants abandon the long path and only follow the short paths. This foreaging ant behavior is imitated in our MMAS algorithm.

 

A typical implementation of a MMAS starts with developing a construction graph as shown in Figure 1. Ant are divided into G generations. An ant in the first generation, go through the construction graph and at each step, the ant selects an item. This item reduces the available constraint capacity. Thus, an ant keeps adding items as long as capacity allows. Once all ants from the first generation find a time feasible solution, the best solution, the one that maximizes the objective function shown in Equation 1, increases the pheromone trails along the path it took while pherome trails along other paths are reduced as shown in Equation 2. The $\tau_{is}$ is the pheromone trail algong the path to node representing topic $i \in N$, whereas $0 < \rho < 1$ represent the evaporation rate. The indicator function $1.\{condition\}$ has a value of 1 if the condition is satisfied. Thus, the first term of Equation 1 shows that $\Delta\tau$ pheromones are added to arcs belonging to the best solution.

$$\tau_{is} = \Delta\tau 1.\{if\ topic\ i\ is\ selected\ in\ step\ s\ by\ the\ best\ ant\} + \rho\tau_{is} \quad 5$$

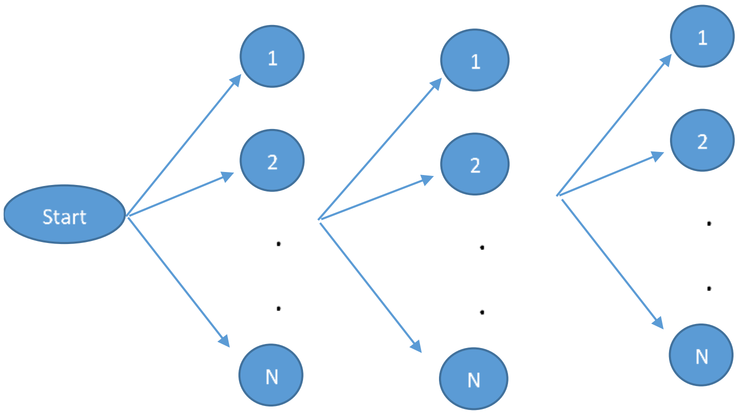


**Figure 1. A construction graph for the KP problem**

An ant belonging to a later generation probabilitically selects an arc as shown in Equation 6. The selecting item $i \in N$ is step s, $P_{is}$, depends on both the pheromone trails along this path $\tau_{is}$ and heuristic information about this path, $\pi_{is}$. In Equation 6, $\alpha$ and $\beta$ show the importance of the pheromone trail and heuristic information, respectively. To avoid convergence to a single solution and force ants to keep exploring new solutions, a MMAS enfoces a lower and upper limits for $\tau_{is}$, which we denote by $\rho_{min}$ and $\rho_{max}$, respectively.

$$P_{is} = \frac{\tau_{is}^{\alpha}\pi_{is}^{\beta}}{\sum_{i=1}^{N}\tau_{is}^{\alpha}\pi_{is}^{\beta}} \quad 6$$

## 3. TSP PROBLEM

The TSP problem is about a sales person that needs to find the shortest tour to stop by a set of potentional buyer. The distance between locations $i$ and $j$ is represent by $d_{ij}$ The objective function is to maximize the function represented in Equation 1. The decision variable of the TSP model is $x_{ij}$ which has a value of 1 if route between city $i$ and $j$ is selected where $i\ and\ j \in N$. We do not give the full formulation to the TSP problem and interested readers are referred to [3].

$$\max Z = \sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij} \quad x_{ij} \quad 7$$

$$x_i \epsilon Z \quad 8$$

### 3.1 A max-min ant system for TSP

The MMAS implementation to solve a TSP instance is similar to the KP implementation; however, the construction graph should show all the possible path between the cities. Moreover, Equation 6 above needs to be modified to take into account all possible paths between two consecutive cities.

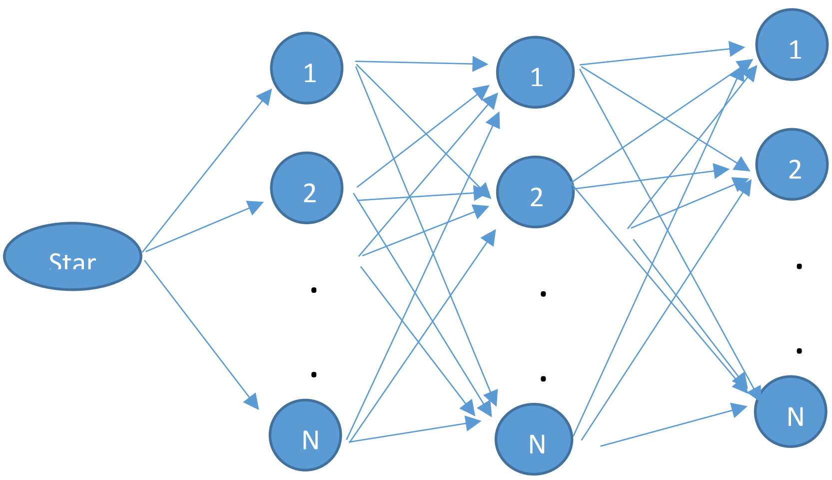


**Figure 2. A construction graph for the TSP problem**

 

$$P_{ijs} = \frac{\tau_{ijs}^{\alpha}\pi_{ijs}^{\beta}}{\sum_{i=1}^{N}\tau_{ijs}^{\alpha}\pi_{ijs}^{\beta}}$$

### 3.2 Comparison

Assume that N is equal to 100. Then for the KP problem we have 100 hundred variables; however, for the TSP problem we have 100*99=9,900 variables. This explosion in the number of variable is translated in an increase in the memory needs and algorithm complexity. Thus, in most implementations to solve sequencing problems, ants are equipped to with a selection list from which they can select the next path. For example, if each ant is equipped with a selection list of 20 cities, then the number of variables would drop to 2,000, which is much better than 9,900.

## 4. CONCLUSION AND FUTURE RESEARCH

ACO algorithms are very powerful meta-heuristics that can be used to solve combinatorial optimization problems. Two types of problems can be solved using ACO algorithms, selection problems like KP and sequencing problems like TSP. In terms of implementation, both alogirhtms are the same; however, the construction graph of the sequencing problem is more complicated. Thus, for a problem having N variables, we have N*N-1 possible decision variables. This increase in decision variables leads to the use of selection baskets to mininimze the memory needs and expedite solution time.

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