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Open Pyramid Graph and Euler Circuit

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ABSTRACT

The concept of open pyramid graph has been introduced and it has been established that it is closely related to BCK-algebra and Euler circuit.

Keywords— *Euler Circuit, Open Pyramid Graph, BCK Algebra, and Disjoint Elements*

1. INTRODUCTION

Definition (1.1) :- A BCK – algebra is a system (E, *, 0) having a non empty set E, a binary operation * and a fixed element 0 such that elements x, y, z $\in$ E satisfy the conditions:

(i) $0 * x = 0$ (ii) $x * 0 = x$

(ii) $((x * y) * (x * z) * (z * y) = 0$

(iii) $(x * ((x * y)) * y = 0$

(iv) $x * y = 0 = y * x \Rightarrow x = y$.

Definition (1.2) :- A pair {x, y} of distinct elements of E is said to be

(a). mutually disjoint if $x * y = x$ and $y * x = y$.

(b) Semi mutually disjoin if either $x*y = x$, $y*x=0$,

or $y*x = y$, $x*y=0$

Rashmi Rani and Puja (2021) has established the following useful result

Theorem (1.3) :- Let $E = \{ 0 \equiv u_0 , u_1,\ldots, u_{n-1} \}$ and let * be a binary operation defined on E such that

$$0 * u_i = 0,\ u_i * 0 = u_i ,\ u_i * u_i = 0 \qquad (1.1)$$

For i=1,2,3………..,n-1

We also define $u_i * u_j = u_i$ and $u_j * u_i = 0$ for $i < j$

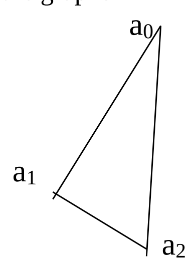
(or $u_i * u_j = 0$ and $u_j * u_i = u_j$) i , j = 1, 2, ….., n-1. Then

(E, *, 0) is a BCK algebra .

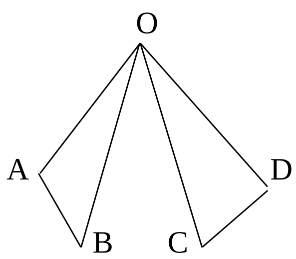
Corollary (1.4) Under the conditions of theorem (1.3) if we take some pairs as mutually disjoint then the result also hold.

Definition(1.5):- Let G = (V, E) be a graph. A path in G is called an Euler path if it includes every edge exactly once. Further, if it is also a circuit then it is called an Euler cicuit.

Example(1.6):- We consider the graphs



Graph (1.1)



Graph (1.2)

For the graph (1.1) Euler circuit is

$$a_1\ (a_1\ a_2)\ a_2\ (a_2\ a_0\ )\ a_0\ (a_0\ \ a_1)\ a_1 \qquad (1.2)$$

For the graph (1.2), we have Euler circuits as

$$A\ (AB)\ B\ (BO)\ O\ (OC)\ C\ (CD)\ D\ (DO)\ O\ (OA)\ A \qquad (1.3)$$

$$\text{and } A\ (AO)\ O\ (OD)\ D\ (DC)\ C\ (CO)\ O\ (OB)\ B\ (BA)\ A \qquad (1.4)$$

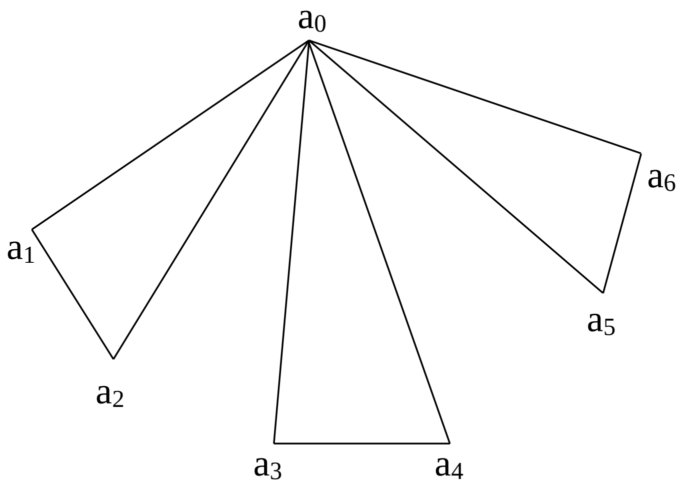
So Euler circuit of a graph is not unique.

Definition(1.7):- Let G =(V, E) be a simple graph where V = { a_0, a_1, a_2,…, a_n}, n = 2m. If there exists an element a_0 such that $d(a_0) = n$ and $d(a_i) = 2$ for I = 1, 2, …., n then G is called an open pyramid graph of order (n + 1) with vertex a_0 and base points a_0, a_1, a_2,…, a_n. In this case edges of an open pyramid graph are

$a_0\ a_1$, $a_o\ a_2$,......, $a_0\ a_n$, $a_1\ a_2$, $a_3\ a_4$,...., $a_{n-1}\ a_n$

Here n must satisfy $n \geq 4$.

Example(1.8):- For n = 6, the open pyramid graph is as follows:



Graph(1.3)

Here V = { a_0, a_1, a_2, a_3, a_4, a_5, a_6} where n = 6 and m = 3

This open pyramid graph has Euler circuit as

$$a_1\ (a_1\ a_2)\ a_2\ (a_2\ a_0\ )\ a_0\ (a_0\ \ a_3)\ a_3\ (a_3\ a_4)\ a_4\ (a_4\ a_0\ )\ a_0\ (a_0\ \ a_5)\ \ a_5\ (a_5\ a_6)\ a_6\ (a_6\ a_0\ )\ a_0\ (a_0\ \ a_1)\ a_1 \qquad (1.5)$$

Here $d(a_0) = 6$, $d(a_i) = 2$ for i = 1, 2, …, 6.

2. MAIN RESULTS

Here we prove some results relating to open pyramid graph and BCK algebras.

Theorem (2.1):- For a given open pyramid graph of order n + 1 where n = 2m there exists a BCK – algebra on a set of n + 1 elements such that the simple graph associated with mutually disjoint elements coincide with the given open pyramid graph.

Proof:- Let a_0 be the vertex and $a_1, a_2, \ldots, a_n$. be base points of an open pyramid graph of order n + 1. Let E = { $a_0, a_1, a_2, \ldots, a_n$}. Then as given in definition (1.7) edges of open pyramid graph are $a_0a_1, a_0a_2, \ldots, a_0a_n, a_1a_2, a_3a_4, \ldots, a_{n-1}a_n$.

We consider a binary operation '*' in E such that a_0 is taken as zero element and an edge connects two distinct points of the open pyramid graph iff the points are mutually disjoint. This means that pairs

$$\{a_0, a_1\}, \{a_0, a_2\}, \ldots, \{a_0, a_n\},\{ a_1, a_2\},\{ a_3, a_4 \}, \ldots, \{a_{n-1}, a_n\}.$$

contain mutually disjoint elements. So binary operation '*' must satisfy

$$a_0 * a_1 = a_0 * a_2 = \ldots. = a_0 * a_n = a_0 \qquad (2.1)$$

$$a_1 * a_0 = a_1 ,\ a_2 * a_0 = a_2,\ \ldots. ,\ a_n * a_0 = a_n \qquad (2.2)$$

$$a_1 * a_2 = a_1 ,\ a_2 * a_1 = a_2,\ \ldots. ,\ a_{n-1} * a_n = a_{n-1} \qquad (2.3)$$

$$a_n * a_{n-1} = a_n$$

We also assume that $a_i * a_i = a_0$ for i = 0, 1, 2, …, n.

For pairs { a_2, a_3 }, { a_4, a_5}, … , { a_{n-2}, a_{n-1}} which are not connected by an edge, we define

$$(a_i * a_j = a_i \text{ and } a_j * a_i = a_0,\ i < j\ ) \qquad (2.4)$$

$$\text{or } (a_i * a_j = a_0 \text{ and } a_j * a_i = a_j,\ i < j\ ) \qquad (2.5)$$

In other words, the elements of these pairs are taken as semi mutually disjoint. Now using theorem (1.3) and corollary (1.4) we see that $(E, *, a_0)$ is a BCK – algebra.

If two points of E are connected by an edge iff they are mutually disjoint then the simple graph so obtained coincides with given open pyramid graph.

Hence the result.

Theorem(2.2) For a given open pyramid graph of order n + 1 where n = 2m there exists a BCK - algebra on a set of n + 1 elements such that the simple graph associated with semi mutually disjoint elements coincide with the given open pyramid graph.

Proof: Let a_0 be the vertex and $a_1, a_2, \ldots, a_n$. be base points of an open pyramid graph of order n + 1. Let E = { $a_0, a_1, a_2, \ldots, a_n$}. Then as given in definition (1.7) edges of open pyramid graph are $a_0\ a_1, \ldots a_0 a_n, a_1 a_2, a_3 a_4, \ldots, a_{n-1}a_n$.

We consider a binary operation '*' in E such that a_0 is taken as zero element and an edge connects two distinct points of the open pyramid graph iff the points are semi - mutually disjoint. This means that pairs

$$\{a_0, a_1\}, \{a_0, a_2\}, \ldots, \{a_0, a_n\}, \{a_1, a_2\}, \ldots, \{a_{n-1}, a_n\}$$

contain semi mutually disjoint elements. So binary operation '*' must satisfy

$$a_0 * a_1 = a_0, a_1 * a_0 = a_1; a_0 * a_2 = a_0, a_2 * a_0 = a_2, \ldots a_0 * a_{n-1} = a_0, a_{n-1} * a_0 = a_{n-1}, a_0 * a_n = a_0, a_n * a_0 = a_n. \quad (2.6)$$

$$a_1 * a_2 = a_1, a_2 * a_1 = a_0; a_3 * a_4 = a_3, a_4 * a_3 = a_0, \ldots a_{n-1} * a_n = a_{n-1}, a_n * a_{n-1} = a_0 \quad (2.7)$$

We also assume

$$a_i * a_i = 0 \text{ for } i = 0, 1, 2, \ldots, n.$$

For pairs { a_2, a_3}{a_4, a_5}, ….,{ a_{n-2}, a_{n-1}} which are not connected by an edge, we define

$$a_2 * a_3 = a_2, a_3 * a_2 = a_3, \ldots, a_{n-2} * a_{n-1} = a_{n-2},$$
$$a_{n-1} * a_{n-2} = a_{n-1} \quad (2.8)$$

In other words, the elements of these pairs are taken as mutually disjoint. Using corollary (1.4), we see that (E, *, a_0) is a BCK – algebra. Further, if two points of E are connected by an edge iff they are semi – mutually disjoint then the simple graph so obtained coincides with the given open pyramid graph.
Hence the result.
We also have the following useful result.

Theorem (2.3):- Every open pyramid graph with vertex a_0 and base points $a_1, a_2, \ldots, a_n$; n = 2m has an Euler circuit.
Proof :- Suppose that P(m) stands for the statement given in the theorem (2.3). From examples (1.6) and (1.8) we see that P(1), P(2), and P(3) are satisfied. We assume that P(m – 1) is true. Then as explained definition (1.7) we have Euler circuit as

$$a_1 (a_1 a_2) a_2 (a_2 a_0) a_0 (a_0 a_3) a_3 \ldots a_{2m-2} (a_{2m-2} a_0) a_0 (a_0 a_1) a_1 \quad (2.9)$$

Now we introduce two points a_{2m-2} and a_{2m} .

These points are connected with a_0 as ($a_{2m-1} - a_0$) and ($a_{2m}\ a_0$). Also a_{2m-1} and a_{2m} are connected by ($a_{2m-1} a_{2m}$).
Now we change the circuit of (2,9) by ommiting $a_0(a_0 a_1) a_1$ and introducing $a_0(a_0 a_{2m-1}) a_{2m-1} (a_{2m-1} a_{2m}) a_{2m} (a_{2m} a_0) a_0 (a_0 a_1) a_1$.
This gives a circuit in the open pyramid graph with vertex a_0 and base points $a_1, \ldots, a_n$ where n = 2m. In other words P(m) is satisfied.
So using Principles of Mathematical induction we see that P(m) is satisfied for all positive integers m.

REFERENCES

[1] Y. Imai , and K. Iseki, On axiom systems of propositional Calculi, XIV Proc. Japan. Acad.42 (1966), 19 – 22.
[2] Rashmi. Rani, Disjoint elements in a BCK – algebras, Acta Ciencia Indica , vol.XXXIX M No. 4 (2013) , 459 – 462.
[3] Rashmi. Rani, R.L.Prasad. Disjoint elements in some specific Shadab Ilyas BCK – algebras, Acta Ciencia Indica . Vol.XL M No. 4 (2014) , 527 – 532.
[4] Rashmi Rani , Puja, BCK /BCH – algebras with mutually disjoint elements, ICCI ASH – 2020.
[5] Rashmi Rani , Puja, BCK /BCH – algebras with semi mutually disjoint elements IEASMA 2021

