Multiplier and dual multiplier on BCC/weak BCC algebras

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ABSTRACT

In this paper, we present some significant properties of multiplier and dual multiplier defined on BCC/Weak BCC algebras.

Keywords - BCC/Weak BCC Algebra, Multiplier, Dual Multiplier

1. INTRODUCTION

First we present some basic definition for easy reference.

Definition (1.1)
A BCC – algebra is a system (X, *, 0) consisting of a non-empty set X, a binary operation * and a fixed defined element 0 satisfying following conditions:

(BCC 1) ((a * b) * (c * b)) * (a * c) = 0,

(BCC 2) 0 * a = 0,

(BCC 3) a * 0 = a,

(BCC 4) a * b = 0 and b * a = 0 \implies a = b

for all a, b, c \in X.

Some properties of a BCC – algebra are given below:

Lemma (1.2)
If (X, *, 0) is a BCC - algebra then following are true

(BCC 5) a * a = 0,

(BCC 6) (a * b) * a = 0

for all a, b \in X.

Definition (1.3)
In a system (X, *, 0) satisfies only conditions (BCC 1) (BCC 3) (BCC 4) and (BCC 5) then it is called a weak - BCC - algebra.

Definition (1.4)
The Dudek’s map \( \phi \) is defined on a weak BCC – algebra \( X \) as \( \phi (a) = 0 \ast a \).

We have the following properties:

Theorem (1.5)
Let (X, *, 0) be a weak BCC – algebra. Then
\[ \phi^2(a) \leq a \]

\[ a \phi b \Rightarrow \phi(a) = \phi(b) \]

\[ \phi^1(a) \leq \phi(a) \]

\[ \phi^2(a, b) = \phi^2(a) \phi^2(b) \]

for all \( a, b \in X \)

**Multiplier and Dual Multiplier**

**Definition (2.1)**

Let \((X, *, 0)\) be a BCC – algebra and let \(f : X \rightarrow X\) be a function. Then \(f\) is called

(i) a multiplier, if \(f(x * y) = f(x) * y\)

(ii) a dual multiplier, if \(f(x * y) = x * f(y)\)

for all \(x, y \in X\).

**Notation (2.2)**

For any function \(f : X \rightarrow X\), let \(N_f\) and \(B_f\) be defined as

\[ N_f = \{x \in X : f(x) = 0\}, \]

\[ B_f = \{x \in X : f(x) = x\}. \]

**Example (2.3)**

Let \(X = \{0, a, b, c, d\}\) and let binary operation \(\ast\) be defined on \(X\) as follows:

<table>
<thead>
<tr>
<th>(\ast)</th>
<th>0</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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<td>0</td>
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</tbody>
</table>

Table (4.1.1)

Then \((X, *, 0)\) is a BCC – algebra.

We consider a function \(f : X \rightarrow X\) defined as

\[ F(0) = 0, f(a) = f(d) = a, f(b) = f(c) = c. \]

Now

\[ f(0 * x) = f(0) = 0 \text{ and } f(0) * x = 0 * x = 0 \]

and

\[ f(x * 0) = f(x) \text{ and } f(x) * 0 = f(x). \]

for all \(x \in X\).

Thus \(f\) is a multiplier on \(X\). This can be seen by simple computation.

**Theorem (2.4)**

Let \((X, *, 0)\) be a weak BCC - algebra and let \(f : X \rightarrow X\) be a multiplier. If \(f(0) = c\) then

\[(c \ast z) \ast \phi(z) = c\]

for all \(z \in X\) where \(\phi(z) = 0 \ast z\).
Proof
Since \((X, *, 0)\) is a BCC-algebra, for \(x, y, z \in X\) we have
\[
((x * y) * (z * y)) * (x * z)) = 0.
\]
Since \(f\) is a multiplier, we have
\[
f \left( ((x * y) * (z * y)) * (x * z) \right) = f(0)
\]
\[
\Rightarrow ((x * y) * (z * y)) * (x * z) = c
\]
Putting \(x = 0\), we get
\[
((f(0) * y) * (z * y)) * (x * z) = c.
\]
Again, putting \(y = 0\), we get
\[
((f(0) * y) * (z * y)) * (x * z) = c.
\]
for all \(z \in X\).

Corollary (2.5)
If \(f\) is a multiplier on a BCC-algebra
\[
F(0) = 0.
\]
Proof
We have \(0 * 0 = 0 \Rightarrow f(0) * 0 = 0\)
\[
\Rightarrow f(0) \leq 0
\]
Also \(0 \leq f(0)\). So \(f(0) = 0\).

Theorem (2.6)
\((X, *, 0)\) be a weak BCC-algebra and let \(f : X \rightarrow X\) be a multiplier such that \(f(0) = 0\). Then
\[
f(x) \leq x\text{ for all } x \in X,
\]
\[
x \leq y \Rightarrow f(x) \leq y,
\]
\(B_f\) is a subalgebra of \(X\).

Proof
The above results can be proved by simple computation.

Theorem (2.7)
Let \((X, *, 0)\) be a BCC-algebra and let
\(f : X \rightarrow X\) be a dual multiplier. Then \(f(0) = 0\).

Proof
Since \((X, *, 0)\) is a BCC-algebra, for \(x, y, z \in X\), we have
\[
((x * y) * (z * y)) * (x * z) = 0.
\]
gives
\[
f \left( ((x * y) * (z * y)) * (x * z) \right) = f(0).
\]
Since \(f\) is a dual multiplier, we have
\[
((x * y) * (z * y)) * f(x * z) = f(0).
\]
This
\[
((x * y) * (z * y)) * f(x * z) = f(0).
\]
Putting \(z = 0\), we get
\[
((x * y) * y) * (x * f(0)) = f(0).
\]
Again, putting \(x = 0\), we get
\[
(y * y) * f(0) = f(0)
\]
\[
\Rightarrow 0 * f(0) = f(0)
\]
We also have

\[ \Rightarrow 0 = f(0). \]

**Theorem (2.8)**

Let \((X, *, 0)\) be a BCC-algebra and let \(f : X \rightarrow X\) be a dual multiplier. Then

\[ x \leq f(x) \quad \text{for all } x \in X \]

\[ x \leq y \Rightarrow x \leq f(y) \]

\(B_f\) is a subalgebra of \(X\).

2. REFERENCES

