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Multiplier and dual multiplier on BCC/weak BCC algebras

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ABSTRACT

In this paper, we present some significant properties of multiplier and dual multiplier defined on BCC/Weak BCC algebras.

Keywords*- BCC/Weak BCC Algebra, Multiplier, Dual Multiplier*

1. INTRODUCTION

First we present some basic definition for easy reference.

Definition (1.1)
A BCC – algebra is a system (X, *, 0) consisting of a non-empty set X , a binary operation * and a fixed defined element 0 satisfying following conditions:

$$\text{(BCC 1)}\ \ ((a * b) * (c * b)) * (a * c) = 0,$$

$$\text{(BCC 2)}\ 0 * a = 0,$$

$$\text{(BCC 3)}\ a * 0 = a,$$

$$\text{(BCC 4)}\ a * b = 0 \text{ and } b * a = 0 \Rightarrow a = b$$

For all a, b, c $\in$ X.

Some properties of a BCC – algebra are given below:

Lemma (1.2)
If (X, *, 0) is a BCC - algebra then following are true

$$\text{(BCC 5)}\ \ a * a = 0,$$

$$\text{(BCC 6)}\ \ (a * b) * a = 0$$

for all a, b $\in$ X.

Definition (1.3)
In a system (X, * , 0) satisfies only conditions (BCC 1) (BCC 3) (BCC 4) and (BCC 5) then it is called a weak - BCC - algebra.

Definition (1.4)
The Dudek's map ϕ is defined on a weak BCC – algebra X as ϕ (a) = 0 * a.

We have the following properties :
Theorem (1.5)
Let (X, * , 0) be a weak BCC – algebra . Then

$$\phi^2 (a) \leq a$$

$$a \;\phi\; b \Rightarrow \phi (a) = \phi (b)$$

$$\phi^3 (a) \leq \phi (a)$$

$$\phi^2 (a , b) = \phi^2 (a) \; \phi^2 (b)$$

for all $a, b \in X$

Multiplier and Dual Multiplier

Definition (2.1)
Let $(X, *, 0)$ be a BCC – algebra and let $f : X \to X$ be a a function . Then f is called

(i) a multiplier, if $f (x * y) = f (x) * y$

(ii) a dual multiplier, if $f (x * y) = x * f (y)$

for all $x, y \in X$.

Notation (2.2)
For any function $f : X \to X$, let N_f and B_f be defined as

$$N_f = \{x \in X : f (x) = 0\},$$

$$B_f = \{x \in X : f (x) = x\}.$$

Example (2.3)
Let $X = \{ 0, a, b, c\ d\}$ and let binary

Operation ' *' be defined on X as follows:

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	a	0
b	b	a	0	0	b
c	c	a	0	0	c
d	d	0	d	d	0

Table (4.1.1)

Then $(X, *, 0)$ is a BCC – algebra.

We consider a function $f : X \to X$ defined as

$$F (0) = 0, f (a) = f (d) = a, f (b) = f (c) = c.$$

Now

$$f (0 * x) = f (0) = 0 \text{ and } f (0) * x = 0 * x = 0$$

and

$$f (x * 0) = f (x) \text{ and } f (x) * 0 = f (x) .$$

for all $x \in X$.

Thus f is a multiplier on X. This can be seen by simple computation.

Theorem (2.4)
Let $(X, * , 0)$ be a weak BCC - algebra and let $f : X \to X$ be a multiplier . If $f (0) = c$ then

$$(c * z) * \phi (z) = c$$

for all $z \in X$ where $\phi (z)=0*z$

Proof

Since (X, * , 0) is a BCC - algebra , for x, y , z $\in$ X we have

$$((x * y) * (z * y)) * (x * z)) = 0.$$

Since f is a multiplier, we have

$$f (((x * y) * (z * y)) * (x * z)) = f (0)$$

$$\Rightarrow f ((x * y) * (z * y)) * (x * z)) = c$$

$$\Rightarrow ((f (x * y) * (z * y)) * (x * z)) = c$$

$$\Rightarrow ((f (x) * y) * (z * y)) * (x * z)) = c.$$

Putting x = 0 , we get

$$((f (0) * y) * (z * y)) * \phi (z) = c.$$

Again Putting y = 0 , we get

$$(c * z) * \phi (z) = c$$

for all z $\in$ X.

Corollary (2.5)

If f is a multiplier on a BCC - algebra

$$F (0) = 0.$$

Proof

$$\text{We have } 0 * 0 = 0 \Rightarrow f (0) * 0 = 0$$

$$\Rightarrow f (0) \leq 0$$

$$\text{Also } 0 \leq f (0). \text{ So } f (0) = 0.$$

Theorem (2.6)

(X, * , 0) be a weak BCC - algebra and let f : X $\rightarrow$ X be a multiplier such that f (0) = 0. Then

$$f (x) \leq x \text{ for all } x \in X,$$

$$x \leq y \Rightarrow f (x) \leq y,$$

B_f is a subalgebra of X.

Proof

The above results can be proved by simple computation.

Theorem (2.7)

Let (X, * , 0) be a BCC - algebra and let

f : X $\rightarrow$ X be a dual multiplier . Then f (0) = 0.

Proof

Since (X, * , 0) is a BCC - algebra , for x, y , z $\in$ X, we have

$$((x * y) * (z * y)) * (x * z) = 0.$$

gives

$$f (((x * y) * (z * y)) * (x * z)) = f (0).$$

Since f is a dual multiplier, we have

This

$$((x * y) * (z * y)) * f (x * z) = f (0).$$

$$\Rightarrow ((x * y) * (z * y)) * (x * f (z)) = f (0).$$

Putting z = 0 , we get

$$((x * y) * y) * (x * f (0)) = f (0).$$

Again Putting x = 0 , we get

$$(y * y) * f (0) = f (0)$$
$$\Rightarrow 0 * f (0) = f (0)$$

$$\Rightarrow 0 = f(0).$$

We also have

Theorem (2.8)
Let (X, * , 0) be a BCC - algebra and let

$f : X \to X$ be a dual multiplier. Then

$$x \leq f(x) \text{ for all } x \in X$$

$$x \leq y \Rightarrow x \leq f(y)$$

B_f is a sub algebra of X.

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