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TDA layer: Impact of persistent homology on the performance of Convolutional Neural Networks

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ABSTRACT

In this work, we introduce the topological data analysis layer that estimates persistent homology on attributes extracted from convolutional layers for image classification. This method shows that topological information can be utilized to upgrade network performance. This work focuses on applying persistent images on the deep convolutional layer to learn topological features and also exploring the behavior of topological data analysis on various convolutional neural network architectures like sequential architecture and extended width architecture. Based on our empirical analysis, we exhibit the significance of topological data analysis on convolutional neural networks by attaining reliable scores on classification tasks on benchmark datasets.

Keywords— *Convolutional Neural Networks (CNN), Topological Data Analysis (TDA), Persistent Homology, Persistent Representations*

1. INTRODUCTION

Convolutional neural networks are one of the prominent deep learning algorithms utilized in the process of image recognition, object detection, edge detection, image segmentation, and neural style transfer and retrieval relevant tasks. The potential of convolutional neural networks lies in their capability to automatically learn information from the data utilizing its numerous feature extraction levels. The structure of CNN comprises a mixture of convolutional layers, subsampling layers, and fully connected layers [1]. It is a feed-forward system, where each level utilizes an assortment of filters and computes collective mutations. Convolution behavior aids in the derivation of different features from the input image. The yield of convolutional tasks is then passed to indiscriminate activation functions. The return of the non-linear processing unit is succeeded by subsampling, which alters the features to geometrical distortions invariant. The uniqueness of CNN's is because of automated feature extraction, sparse connectivity, multi-tasking, and shared weights. To improve the correctness of the CNN model at decreased training steps, the notion of topological data analysis has been applied.

TDA is a mathematical topology and computational geometry method [2], which gives algorithmic and statistical procedures to examine and elucidate tangled geometric and topological formation of features that are portrayed as point clouds in common metric or Euclidean spaces. TDA can be applied to any sort of data and it's vigorous to noise. TDA offers a strategy titled persistent homology which empowers us to pursue topological changes of data at multiple scales. At various scales, topological features like holes, loops, spheres and connected components appear and vanish. These topological aspects have a lifespan which is addressed by their birth and death time. Their life expectancy is portrayed by the barcode or persistence diagram. The persistence diagram involves an unordered cluster of intervals on the real line, where a long bar demonstrates the presence of an attribute that lives over an enormous range of values and its real, and short bars are the commotions. These are mentioned as topological signatures. However, the persistence diagrams are regular descriptors of topological data inspection, its metric space is not Hilbert i.e. the space of persistence illustrations intensely needs structure, diverse persistence graphs may have a distinctive group of points, and different rudimentary functions are not transparent, like summation and vector multiplication. It makes it bother on expert systems mechanism. To defeat these restrictions, this task presents a futuristic TDA layer for deep convolutional neural networks that takes a topological representation i.e. persistence image fused with convolutional feature maps to perform classification tasks. To exhibit the versatility of this strategy, we have constructed distinct CNN models alongside the TDA layer to accomplish the classification endeavor on the benchmark dataset.

The paper is structured in the accompanying order; Section 2 addresses several works on the enactment of topological data analysis to artificial intelligence models. The proposed TDA layer has been explored in Section 3 and the experimental probe of TDA combined various CNN architectures are displayed in Section 4. The Section 5 examines our future work and also concludes our work.

2. RELATED WORKS

To conquer the limitation of utilizing topological data analysis on the learning process, various methodologies have been proposed. Mathieu et al proposed a method depending on heat kernel signature and extended persistence hypothesis to learn vectors from persistent graphs in [3]. Rickard et al introduced a distinguishable topology layer in [4] that assesses persistent homology dependent on level set and edge-based filtrations. The topological data analysis has been applied to the parameters of CNN to illuminate topological structures that are coupled with the network's capacity to generalize to concealed data and in addition to rise networks performance by Gunnar et al in [5]. To illustrate the stability of topological data analysis, Alexander et al [6] examined topological features on numerous computer vision issues like image classification, shape recognition, and video summarization relevant to biomedical images. Alternatively, Christoph et al presented a methodology that permits us to input uncommonly organized topological signatures to deep neural networks and acquire an ideal portrayal during training in [7].

In the line of research, William et al [8] use algebraic topology as the measure for data intricacy, and dependent on it, the appropriate architecture that best fits on unrevealed data is resolved. Topological data analysis was carried out on the interior states of the neural nets to recognize it and also to construct generalized neural network structures by Gunnar et al in [9]. Gregory et al had tackled the mysteries behind the work of non-linear activation capabilities and also the role of deep layered neural networks in solving complex issues utilizing topological data investigation in [10]. A syndicate multichannel topological CNN is created by Zixuan et al in [11], where feature-specific persistent homology has been utilized to examine protein-ligand unbreakable bonds and protein steadiness commute upon transformation. Another application of topological data analysis in classifying biological repeated measurements data is invented by Henri et al in [12]. To analyze the commitment of topological information to the performance of convolutional neural networks, we performed our trials on a few CNN architectures utilizing the standard benchmark datasets.

3. TDA LAYER

In this part, we present our technique for embedding topological data analysis on convolutional neural networks with the proposed TDA layer. Our architectures comprise convolutional layers, non-linear activation functions, and topological data analysis (TDA) layers, subsampling layers, and fully connected layers. In TDA layers, the topological depiction of convolution feature maps is computed and passed on to consequent layers on architecture for further processing. The persistent image acquired by performing the persistent homology concept on convolutional feature maps is considered a topological feature map.

3.1 Concepts behind TDA layers

Homology [2,3&5] is a procedure from algebraic topology offering an amazing asset to standardize and deal with the idea of geometrical attributes of a simplicial complex or a topological extent in a mathematical manner. For some extent K, the K-dimensional *holes* are addressed by a vector span H_k, whose element is naturally the quantity of such self-sufficient features. For instance, the 0-dimensional homology set H_0 addresses the coupled components of the aggregation, the 1-dimensional homology category H_1 depicts the 1-dimensional spirals, and the 2-dimensional homology group H_2 addresses the 2-dimensional holes, etc. Homology in dimension K is characterized as underneath.

$$H_k(x) = \ker \partial_k / im\ \partial_{k+1}$$

Persistent homology is an incredible asset to analyze and inscribe proficiently multiscale topological characteristics of encapsulated groups of topological and simplicial spaces. It inscribes the advancement of the homology gatherings of the clustered complexes across the dimensions. It figures out how homology changes over an expanding succession of complexes, also described as filtration on X. Each section in the filtration also makes or demolishes homology when it emerges. The full information about how homology is born and expires over the filtration can be addressed as a multi-set of sets (b. d) where b is the birth constant of a homology class, and d is the death constant of that class (d = ∞ it is as yet remains in X). This multiset of sets for homology in aspect K is noted as the k-dimensional persistence outline of the process, $PD_k(x_0) = \{(b_i, d_i)\}_{i \in T_k}$ or the K-dimensional barcode of X_0. As persistence graphs are a crowd of points in R^2, there are numerous concepts of distances among cost functions on diagrams that rely upon the points and representations. The loss functions comprise of three parameters,

$$\epsilon(\mathrm{p}, \mathrm{q}, \mathrm{i}_0; \mathrm{PD}_\mathrm{k}) = \sum_{\mathrm{i}=\mathrm{i}_0}^{|\mathrm{T}_\mathrm{k}|} |\mathrm{d}_\mathrm{i} - \mathrm{b}_\mathrm{i}|^\mathrm{p} (\frac{\mathrm{d}_\mathrm{i} + \mathrm{b}_\mathrm{i}}{2})^\mathrm{q}$$

A valuable portrayal of this homological information is a persistence diagram (PD). Persistence diagrams are changed over into limited dimensional vector depictions called persistence images. The persistence image is acquired by spinning by -pi/4, compacting Gaussian functions on all representation points normally weighted by a parameter function like the squared distance to the slanted, and adding every one of these Gaussians. This gives a 2D function that is pixelated into an image. Let B be a PD in birth-death coordinates. Let T: $R^2 \to R^2$ be the linear operator T(x, y) = (x, y-x), and let T (B) be the altered multiset in birth-persistence coordinates, where each point (x, y) € B corresponds to a point (x, y-x) € T(B). Let $\emptyset_u$: $R^2 \to R$ alone a differentiable likelihood distribution with mean $u = (u_x, u_y) \in R^2$. the standardized symmetric Gaussian $\emptyset_u = g_u$ with mean u and variance σ^2 characterized as

$$g_u(x, y) = \frac{1}{2\pi\sigma^2} e^{-\left|(x-u_x)^2 + (y-u_y)^2\right| / 2\sigma^2}$$

These concepts are applied on convolutional layer feature maps to induce geometric and topological signatures, which are then fused with their corresponding activation maps and fed into the next convolutional layer on hierarchy for learning.

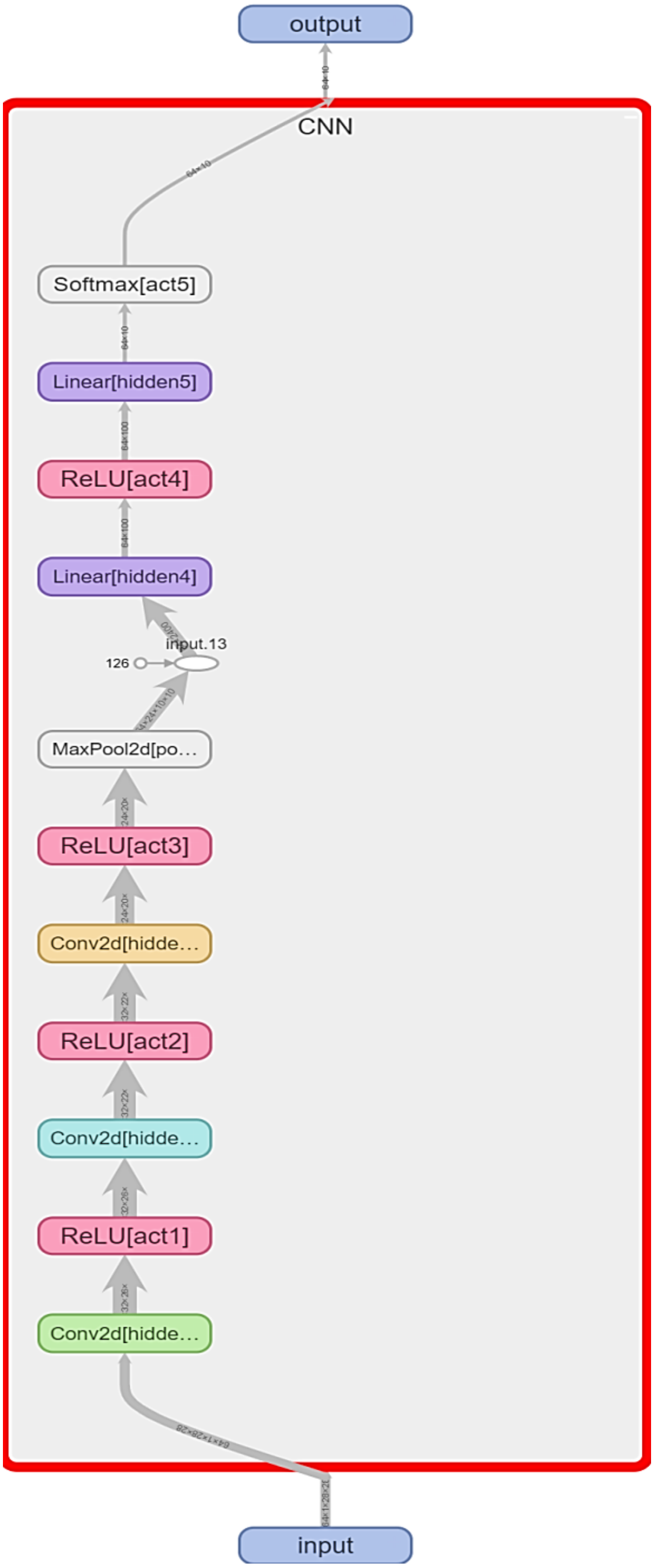

Figure 1: Sequential CNN Model

4. EXPERIMENTATION AND ANALYSIS

To exemplify the robustness of the preferred approach, we present an assessment with two distinct categories of CNN architectures: 1) Sequential (Fig.1) and 2) Expanded Width architectures (Fig.2 &3). In both cases, the learning task is classification. The model architectures can be found in figures 1 & 2. We performed the study utilizing the standard MNIST repository, which comprises 70,000 samples of scripted digits (zero to nine) that have been size-standardized and focused in a square grid of pixels. Each picture has a resolution of 28 × 28 in which each pixel represents grey scale intensities spanning from 0 (black) to 1 (white). The objective data comprises of one-hot binary vectors of size 10, comparing to the digit classification categories zero through nine. The GUDHI library has been utilized to compute higher-dimensional geometry of shapes that exists in data.

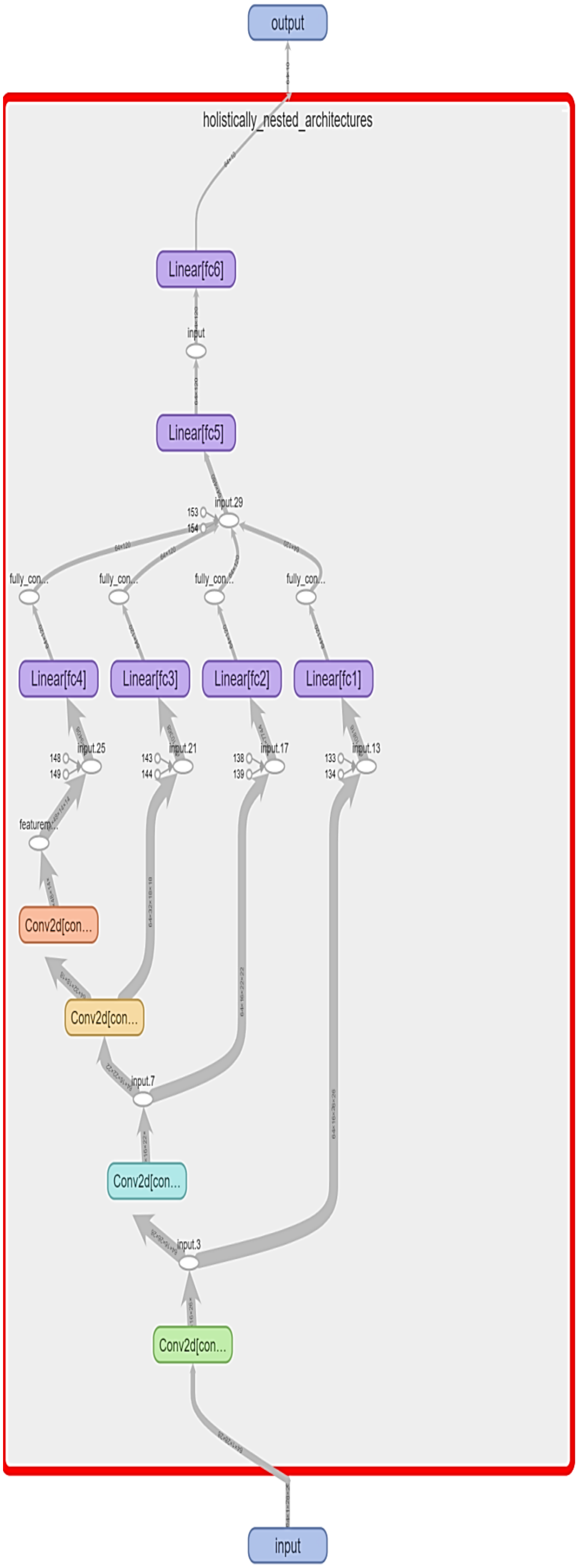

Figure-2 Holistically CNN Model

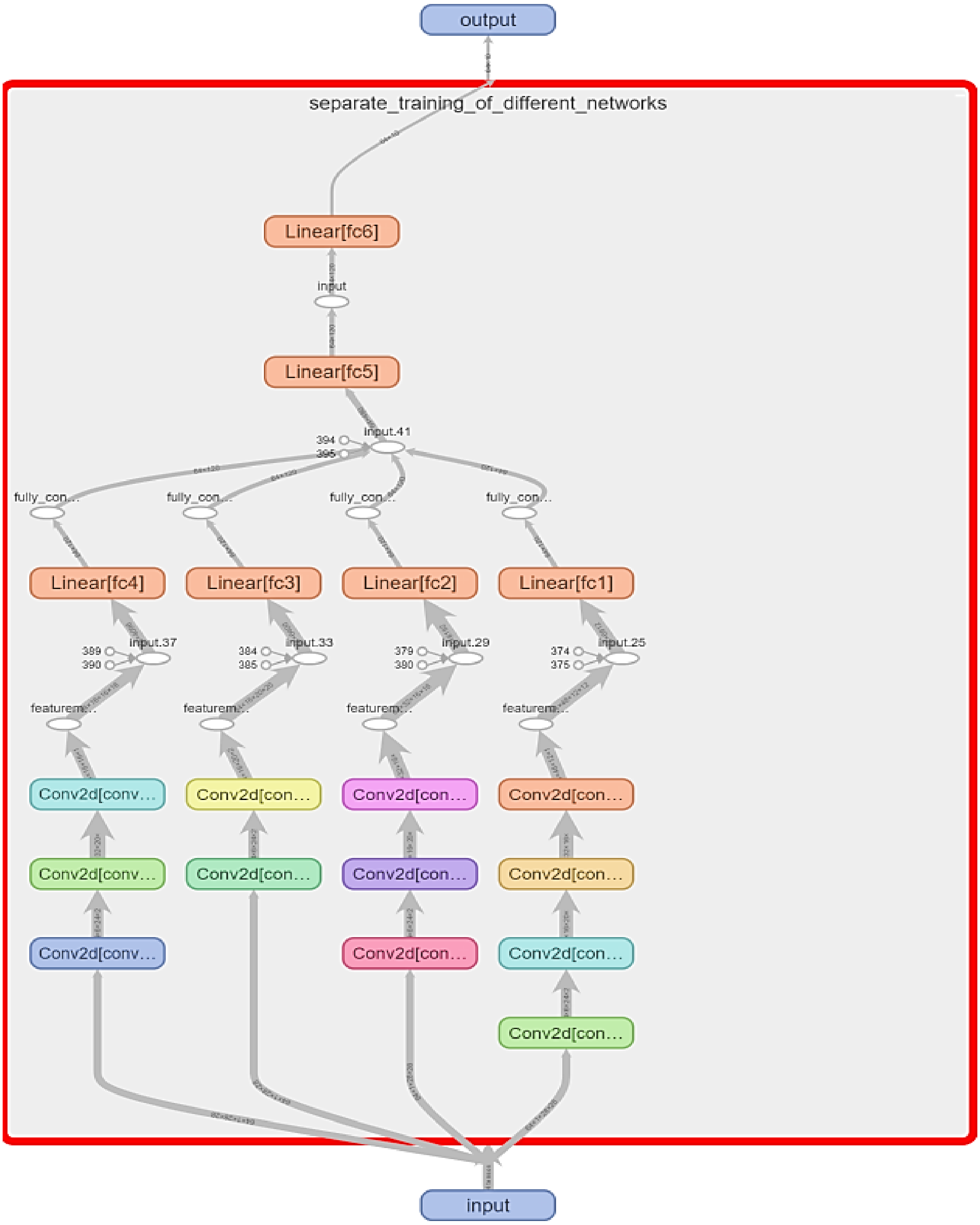

Figure 1: Single Inputs on Different Networks Model

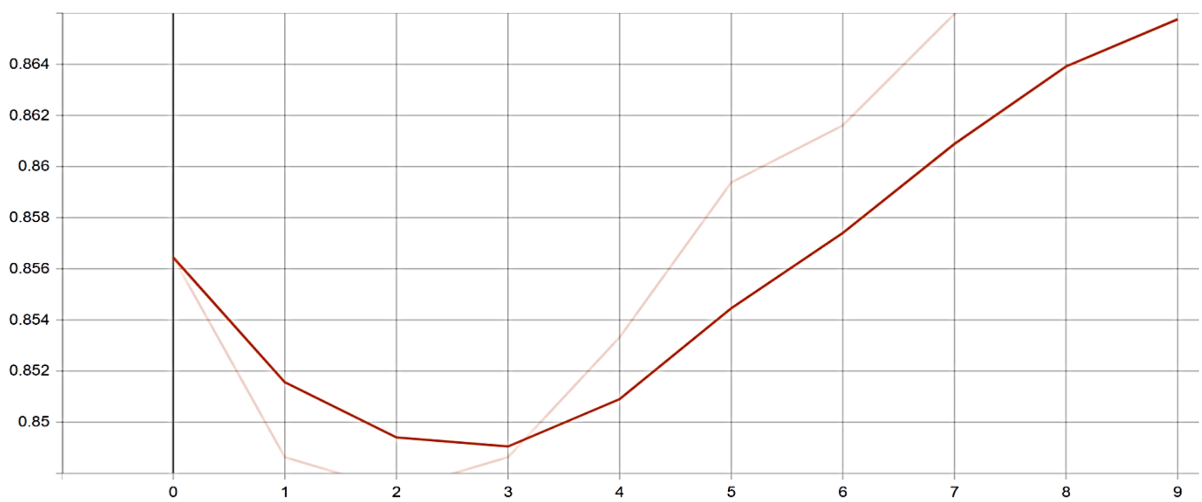

Figure 2: Accuracy of Sequential CNN model

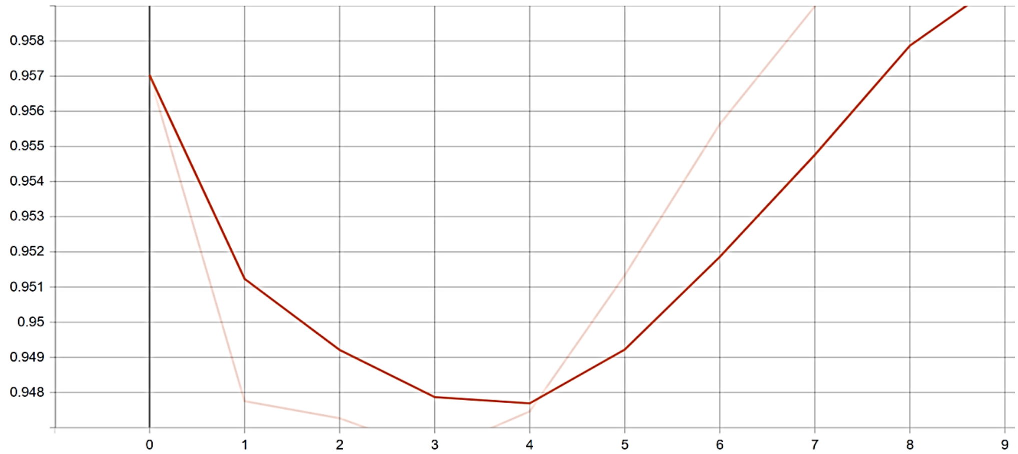

Figure 5: Accuracy of TDA-Sequential CNN Model

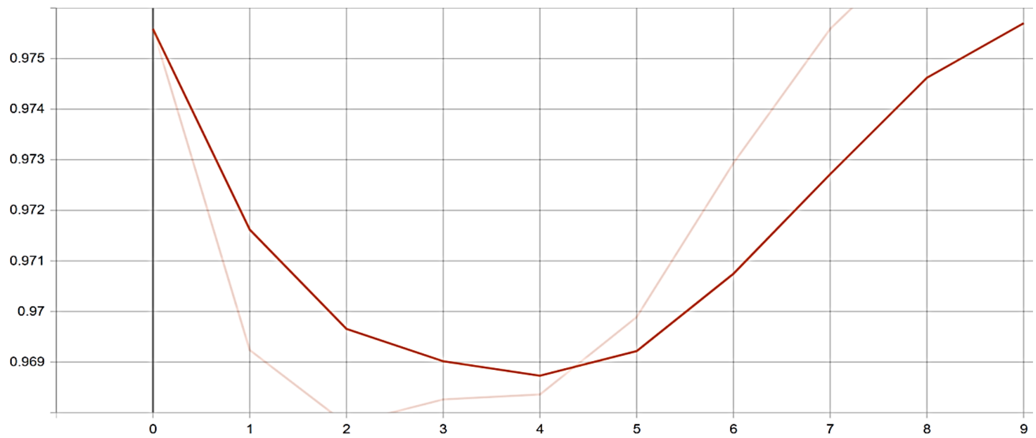

Figure 6: Accuracy of Holistically CNN Model

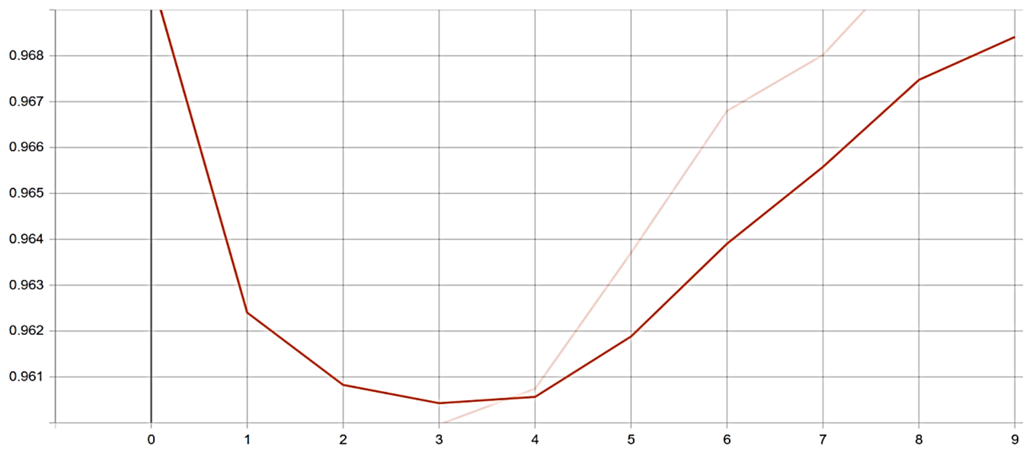

Figure 7: Accuracy of TDA-Holistically CNN Model

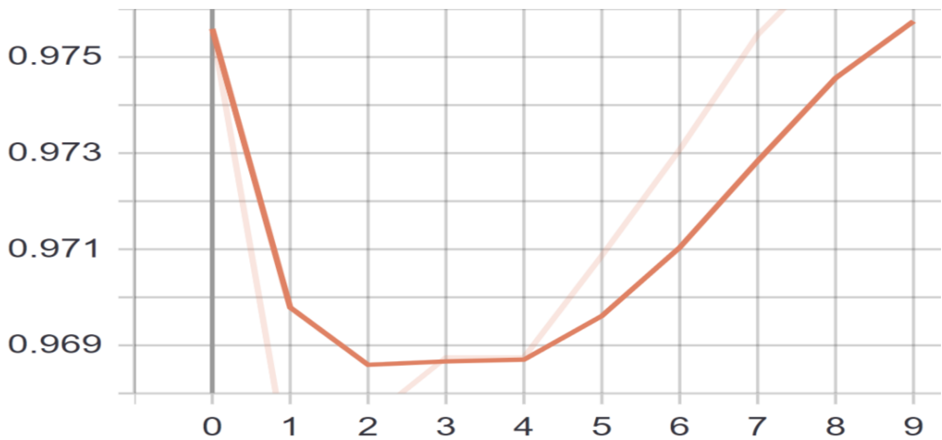

Figure 8: Accuracy of Single Input on Different Networks CNN Model

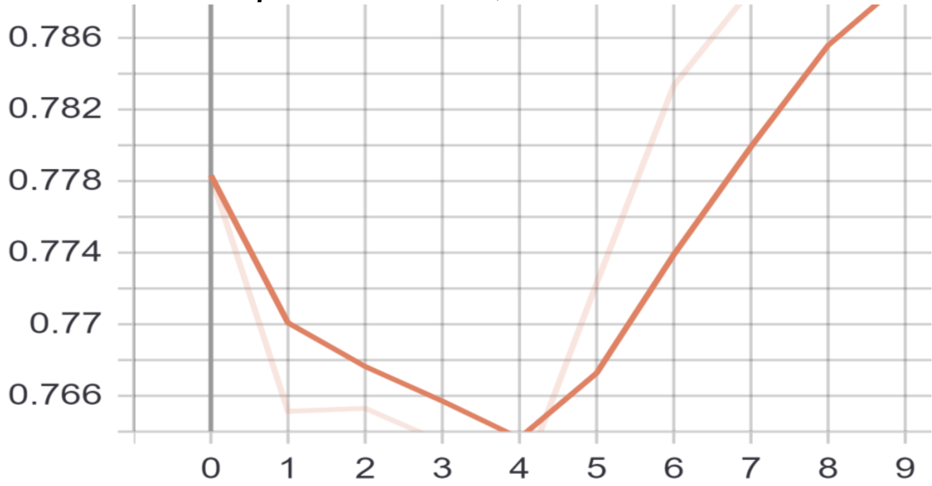

Figure 9: Accuracy of TDA-Single Input on Different Networks CNN Model

Table 1: Results on MNIST Digits Dataset

Model Name	Accuracy
Sequential CNN	0.86
TDA-Sequential CNN	0.95
Holistically CNN	0.97
TDA-Holistically CNN	0.97
SIDN-CNN	0.97
TDA-SIDN CNN	0.79

5. CONCLUSION

In this work, we have developed a topological data analysis integrated convolutional neural networks method to solve a classification problem. The trained model TDA-Sequential CNN model yielded high accuracy on the MNIST dataset when compared with its Sequential CNN counterpart. To understand the capability and behavior of topological features integrated with various CNN architectures, two commonly used CNN categories were examined. From our experimentation, it is found that the combination of topological features and convolutional features performs well on conventional sequential models, but it is not suitable for wide-width CNN architectures. And also the topological attributes could be utilized to determine which CNN architecture is suitable for specific data. In the future, we consider developing large-scale topological data analysis integrated learning methods for training the object contour detector on the BIPED dataset and applying the generated proposals for object instance segmentation.

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