International Journal Of Advance Research, Ideas And Innovations In Technology

IJARIIT

ISSN: 2454-132X

Impact Factor: 6.078

(Volume 7, Issue 3 - V7I3-2139)

Available online at: https://www.ijariit.com

Dynamic response of a single pile subjected to a follower load

Vincent Ifeanyi Ike
ike.vincente@gmail.com
Nigeria Maritime University, Okerenkoko, Nigeria

ABSTRACT

The Dynamic response of a single pile following leipholz's rod has been solved. The analysis attempts to provide the solution of the pile if the model of the system is taken to have a non-self adjoint follower load under the influence of a heterogeneous winkler soil model. The method of laplace transform is used to deal with the non-conservative initial-boundary-value problems. In this paper, a close form solution has been developed to deal with the concave singularity function of the soil subgrade modulus if the admissible function for the pile is satisfied when upset by a time dependent lateral force. This research domain is still active as research engineers and mathematicians are engaged in developing a deeper understanding of dealing with the singularity. The result obtained is compared with those given by static (finite element) analysis.

Keywords: *Dynamic Analysis; Non-Conservative System; Follower Forces; Singularity Function; Heterogeneous Soil Profile; Green's Function.*

1. INTRODUCTION

The dynamic behaviour of a pile can be argued from the principle of generalized adjointness and exchanging of energy by different functional [1-3] due to the fact that piles are inherent to axial load from the superstructure. This approach has been seen to handle non-conservative systems and capable of dealing with the unique set(s) of admissible functions under tangential follower forces. The established variational principle in [1], this time by a definite linear functional provided for the initial and boundary conditions of the system is applied for a single pile stiffened by singularity brought into play by the nonlinear subgrade modulus as in figure 1.0. A simple analogy by Leipholz [4] developed the green function for the associated differential equation. This study aims at developing an approximate analytic solution capable of handling this inhomogeneous soil in a simpler manner.

2. PROBLEM FORMULATION

The problem may be simply expressed as the system below

$$Dw^{IV} + q(l-x)w'' + k_s w + \mu\ddot{w} = K(x,t) - c\dot{w} = P(x,t) \qquad \text{1a}$$

$$w(x,0) = \dot{w}(x,0) = 0; \qquad \text{1b}$$

$$w(0,t) = w'(0,t) = w''(l,t) = 0; \quad Dw'''(l,t) = Kw''(l,t); \qquad \text{1c}$$

where w is the deflection perpendicular to the middle-surface of the pile, where D is the flexural stiffness or linear self-adjointness, k_s is the nonhomogeneous soil stiffness on the pile, q is a kind of linear non-selfadjoint load, μ is the mass density per unit surface, K is the distributed perturbing force and c is the damping coefficient.

$$Dw^{IV} + q(l-x)w'' + \left(k_s - \mu\omega^2\right)w = P \qquad \text{2a}$$

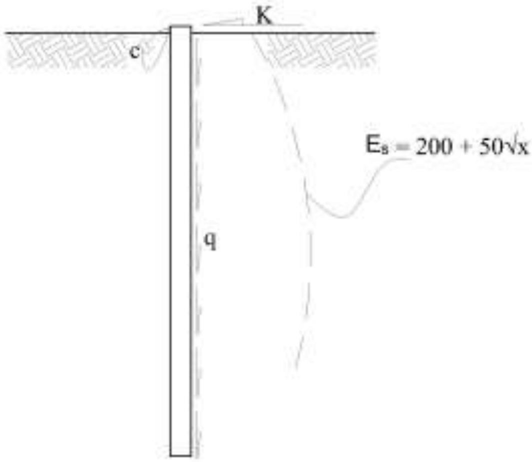

Fig. 1: Pile Foundation

Assuming that the variable is separable in the manner

$$w(x,t) = w(x)e^{iwt} \tag{2b}$$

by Laplace transformation

$$\bar{w} = \int_0^\infty e^{-st} w(x,t)\,dt, \qquad \bar{P} = \int_0^\infty e^{-st} P(x,t)\,dt \tag{3}$$

$$D\bar{w}^{iv} + q(l-x)\bar{w}'' + \left(k_s - \mu s^2\right)\bar{w} = \bar{P} \tag{4}$$

The series expansion becomes

$$\bar{w}(s,x) = \sum_i A_i(s) W_i(x), \qquad \bar{P}(s,x) = \sum_i B_i(s) W_i(x) \tag{5}$$

Using (5) in (2)

$$\sum_i A_i \left\{ D W_i^{iv} + q(l-x) W_i'' + \left(k_s - \mu s^2\right) W_i \right\} = \sum_i B_i W_i(x) \tag{6}$$

By considering (2) and (4)

$$\sum_i A_i \left\{ \mu(s^2 + \omega_i^2) \right\} W_i(x) = \sum_i B_i W_i(x) \tag{7}$$

$$A_i = B_i / \mu(s^2 + \omega_i^2) \tag{8}$$

From (4)

$$B_i(s) = \int_0^l \bar{P}(s,x) W_i(x)\,dx \tag{9}$$

$$A_i(s) = \frac{1}{\mu(s^2 + \omega_i^2)} \int_0^l \bar{P}(s,x) W_i(x)\,dx \tag{10}$$

The solution becomes

$$w(s,x) = \sum_i \frac{W_i(x)}{\mu(s^2 + \omega_i^2)} \int_0^l \bar{P}(s,\xi) W_i(\xi)\,d\xi \tag{11}$$

By inversion, the Laplace transform the solution becomes

$$w(x,t) = \sum_i \int_0^t \int_0^l \bar{P}(\xi,\tau) \sum_i \frac{W_i(\xi) W_i(x)}{\mu\omega_i} \sin\omega_i (t-\tau)\,d\xi\,d\tau \tag{12}$$

$$G(x,\xi,t,\tau) = \sum_i \frac{W_i(\xi) W_i(x)}{\mu\omega_i} \sin\omega_i (t-\tau) \tag{13}$$

Equation (13) is the unsymmetric green's function of the problem. The control response of the pile due to damping is

$$\bar{w}(x,t)=\int_0^t\int_0^l G(x,\xi,t,\tau)\left[K(\xi,\tau)-c\dot{w}(\xi,\tau)\right]d\xi\, d\tau \qquad 14$$

2.1 Eigenfrequency

From equation (1), $W_i(x)$ is chosen in terms of the eigenfunctions of the self-adjoint auxiliary problem or follower load problem given below

$$DW_i^{iv}+q(l-x)W_i''+\left(k_s-\mu\omega_i^2\right)W_i=0 \qquad 15a$$

$$W_i(0,t)=W_i'(0,t)=W_i''(l,t)=W_i'''(l,t)=0; \qquad 15b$$

To attempt the problem, an admissible function for the vibrational response of the pile under subjective follower load must be chosen for the pile to satisfy the above boundary conditions, following the variational principle [1]

$$F_{mn}=\sum_m a_m\int_0^l\left[D\psi_m''''+q(l-x)\psi_m''+\left(k_s-\mu\omega_i^2\right)\psi_m\right]\psi_n''(l-x)\,dx \qquad 16$$

If we consider a bilinear functional below

$$W_i=a_1\psi_1+a_2\psi_2 \qquad 17a$$

Where,

$$\psi_1(x)=1-\cos\frac{\pi x}{2l},\quad and \quad \psi_2(x)=1-\cos\frac{3\pi x}{2l} \qquad 17b$$

The function $\psi_m(x)$ and $\psi_n(l-x)$ thus satisfy the boundary conditions and the expression for generalized adjointness, so that

$$F_{mn}=\sum_m a_m\left[D\int_0^l\psi_m'''\psi_n'''dx+\int_0^l q(l-x)\psi_m''\psi_n''dx+\int_0^l K_s\psi_m'\psi_n'dx-\mu\omega_i^2\int_0^l\psi_m'\psi_n'dx\right]=0 \qquad 18$$

Take,

$$f_{mn}=D\int_0^l\psi_m'''\psi_n'''dx+\int_0^l q(l-x)\psi_m''\psi_n''dx+\int_0^l K_s\psi_m'\psi_n'dx-\mu\omega_i^2\int_0^l\psi_m'\psi_n'dx \qquad 19$$

and solving the characteristic equation below to obtain the vibration features

$$\det\left|f_{mn}\right|=0 \qquad 20$$

3. NUMERICAL EXAMPLE

Consider a system as in [5, 6] having,

$$l=19m,\ D=200\times10^9\times508\times10^{-6}\,Nm^2,\ G_s=200+50\sqrt{x}\ kN/m^3$$

$$A=22.2\times10^{-3}m^2,\ \mu=174\ kg/m,\ \text{and}\ K_s=AG_s,\ and\ q=26\ kN/m\le q_{\max}$$

$q_{\max}$ is a control parameter of stability for the auxiliary problem, for which the eigenvalues must remain real [7].

Equation (18) reduces for m,n $\in\{1,2\}$ to

$$\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix}\begin{Bmatrix}\alpha_1\\ \alpha_2\end{Bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

By applying conditions, the eigenvalues $(rad/\sec)^2$ are obtained as

$$\omega_1^2=81.212,\ \omega_2^2=887.57$$

one obtains the coordinate functions as

$$W_1(x)=-0.99485\times\left[1-\cos\frac{\pi x}{2l}\right]-0.10138\times\left[1-\cos\frac{3\pi x}{2l}\right]$$

$$W_2(x)=-0.89989\times\left[1-\cos\frac{\pi x}{2l}\right]+0.43612\times\left[1-\cos\frac{3\pi x}{2l}\right]$$

consider a lateral disturbance at the top of the pile, such that

$$K(x,t)=\begin{cases} K_0\delta(x-l)\sin\Omega t, & 0<t<T \\ 0, & T<t<0 \end{cases}$$

$$\text{given } K_0 = 5078/19 = 2.6726 kN/m,\ T = 0.3\text{sec},\ \Omega = 2\pi/T\ rad/\text{sec}$$

Substituting parameters in equation (12) becomes

$$w(x,t)=\int_0^l \sum_i \frac{K_0\delta(\xi-l)W_i(\xi)W_i(x)}{\mu\omega_i}d\xi\int_0^t \sin\Omega t\sin\omega_i(t-\tau)d\tau$$

$$w(x,t)=\sum_i\left\{\begin{array}{l}\dfrac{K_0}{\mu\omega_i^2}(a_{i1}+a_{i2})\times\left[a_{i1}\left(1-\cos\dfrac{\pi x}{2l}\right)+a_{i2}\left(1-\cos\dfrac{3\pi x}{2l}\right)\right] \\ \qquad\qquad\qquad\qquad\times\dfrac{1}{1-\left(\Omega/\omega_i\right)^2}\left[\sin\Omega t-\dfrac{\Omega}{\omega_i}\sin\omega_i t\right]\end{array}\right\}$$

By introducing a control system at the pile head as in [7, 8], displacement is reduced as

$$\bar{w}(x,t)=w(x,t)-\int_0^t\int_0^l G(x,\xi,t,\tau)c\frac{\partial\bar{w}}{\partial\tau}\delta(\xi-l)d\xi\,d\tau$$

$$\bar{w}(x,t)+\int_0^t\int_0^l G(x,\xi,t,\tau)i2\mu\xi\omega_i^2\bar{w}\delta(\xi-l)d\xi\,d\tau=w(x,t)$$

$$\left|\bar{w}(x,t)\right|=\left[1+2\xi\sum_i\left|W_i(l)\right|\left|W_i(x)\right|\times\omega_i\left|\cos\omega_i t-1\right|\right]^{-1}\times\left|w(x,t)\right|$$

3. RESULT AND DISCUSSION

A computer-aided (MATLAB) procedure is applied. The maximum displacement of the pile is 0.1568 m as seen in Table 1.0, which is more than twice the static result of 0.06206 m and 0.0622 m in [5, 6] respectively. Pile foundations can experience high natural frequencies and displacement, hence amplitude. The result emphasizes the importance of dynamic analysis to foundation design problems.

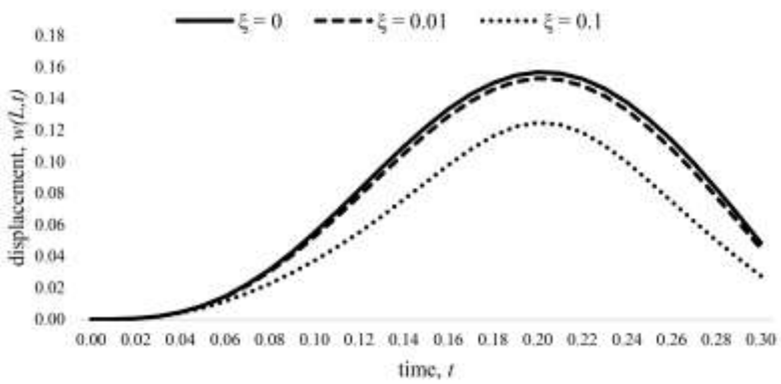

Fig. 2: Uncontrolled and Controlled (damped) displacement

Table-1: Uncontrolled and controlled (damped) displacement

Time (secs)	$\xi = 0$	$\xi = 0.01$	$\xi = 0.1$
0.02	5.9995e-04	5.9770e-04	5.7816e-04
0.04	4.6178e-03	4.5546e-03	4.0549e-03
0.06	1.4611e-02	1.4225e-02	1.1491e-02
0.18	1.5001e-01	1.4579e-01	1.1636e-01

0.2	1.5681e-01	1.5289e-01	1.2482e-01
0.22	1.5264e-01	1.4837e-01	1.1857e-01
0.24	1.3776e-01	1.3269e-01	9.9671e-02
0.28	8.3009e-02	7.7894e-02	5.0105e-02
0.3	4.8488e-02	4.5072e-02	2.7584e-02

4. CONCLUSION

Soil-pile system under heterogeneous soil mass as well as dynamic load is investigated. The Green's function of the system makes the solution applicable to any kind of dynamic load as well as the shape function. Static finite element results compared showed the uniqueness of the approach. The results obtained can help improve developed dimensionless parameters in appropriate ranges for other complex approaches.

5. REFERENCES

[1] Leipholz, H., *On a variational principle for the clamped-free rod subjected to tangential follower forces.* Mechanics Research Communications, 1978. 5(6): p. 335-339.
[2] Leipholz, H., *On an energylike functional for the calculation of buckling loads of completely supported rods subjected to follower forces.* Mechanics research communications, 1986. 13(1): p. 33-37.
[3] Leipholz, H., *On principles of stationarity for non-selfadjoint rod problems.* Computer methods in applied mechanics and engineering, 1986. 59(2): p. 215-226.
[4] Leipholz, H., *On the buckling of thin plates subjected to nonconservative follower forces.* Transactions of the Canadian Society for Mechanical Engineering, 1975. 3(1): p. 25-34.
[5] Bowles, J.E., *Foundation Analysis and Design, The McGrawHill Companies.* Inc., Singapore, 1996: p. 950.
[6] Dinev, D., *Analytical solution of beam on elastic foundation by singularity functions.* Engineering Mechanics, 2012. 19(6): p. 381-392.
[7] Leipholz, U. and M. Abdel-Rohman, *Control of structures*. Mechanics of Elastic Stability. Vol. 11. 1986: Springer Netherlands.
[8] Leipholz, H., *Distributed control of elastic plates.* Mechanics Research Communications, 1982. 9(2): p. 133-136.

