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Dominator coloring of certain graphs

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ABSTRACT

A graph has a dominator coloring if it has a proper coloring in which each vertex of the graph dominates every vertex of some color class. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G. In this paper the dominator chromatic number for comb graph, jewel graph, flower graph is studied and also the dominator coloring of corona graph of path with triangular snake graph and double triangular snake graph is studied.

Keywords: *Comb Graph, Flower Graph, Jewel Graph, Corona Product Graph, Dominator Coloring, Dominator Chromatic Number*

1. INTRODUCTION

A dominating set *S* is a subset of the vertices in a graph such that every vertex in the graph either belongs to *S* or has a neighbor in *S*. The domination number, $\gamma(G)$, is the order of a minimum dominating set.

A proper coloring of a graph *G* = (*V*(*G*), *E*(*G*)) is an assignment of colors to the vertices of the graph, such that any two adjacent vertices have different colors. The chromatic number is the minimum number of colors needed in a proper coloring of graph.

One of the important areas in graph theory is graph coloring which is used in many applications like Map coloring, Sudoku, Register Allocations, Mobile radio frequency assignment and Making schedule or time table and it has played a major role in the development of graph theory and, more generally, discrete mathematics and combinatorial optimization.

A dominator coloring of a graph *G* is a proper coloring in which each vertex of the graph dominates every vertex of some color class. The dominator chromatic number $\chi_d(G)$ is the minimum number of color classes in a dominator coloring of a graph *G*. This concept was introduced by Ralucca Michelle Gera in 2006 [4].

The dominator chromatic number of cartesian products of P_2 and P_3 by arbitary paths and cycles was determined in [2, 3]. Dominator colorings of mycielskian graphs were investigated in [1]. The dominator coloring of bipartite graph, star and double star graphs, central and middle graphs, fan, double fan, helm graphs, prism graph etc., were also studied in various papers [5 - 9].

2. DOMINATOR COLORING OF COMB, JEWEL AND FLOWER GRAPHS

In this section we have obtained the dominator chromatic number of comb graph P_n^+, Jewel graph J_n, Flower graph F_n.

2.1 Dominator coloring of comb graph

Definition 2.1.1. Let P_n be a path graph with *n* vertices. The comb graph is defined as $P_n \odot K_1$. It has 2*n* vertices and 2*n*-1 edges.

Theorem 2.1.1. For a comb graph P_n^+ with 2*n* vertices, $\chi_d(P_n^+) = n + 1$.

Proof. Let P_n^+ be a comb graph with 2*n* vertices. Label the vertices of P_n^+ as follows:

Path vertices are labelled as $\{v_1, v_2, \dots, v_n\}$ and pendent vertices are labelled as $\{v_{n+1}, v_{n+2}, \dots, v_{2n}\}$ where

$V(G) = \{v_1, v_2, \ldots, v_n\} \cup \{v_{n+1}, v_{n+2}, \ldots, v_{2n}\}$. Now choose the vertices of path given by $\{v_1, v_2, \ldots, v_n\}$. It yields the minimum dominating set for P_n^+.

We now claim the $\chi_d(P_n^+) = n + 1$.

Suppose $\chi_d(P_n^+) \neq n + 1$. Let $\chi_d(P_n^+) \leq n$.

Assume $\chi_d(P_n^+) = n$. Define a function $C: V(G) \to \{1, 2, \ldots, n\}$ such that the path vertices will receive $\{1, 2, \ldots, n\}$ colors and the vertex v_{n+1} will not receive the color 1 which contradicts the coloring property of G and the vertex v_{n+1} will not colored with colors $\{2, 3, \ldots, n\}$ that will contradict the dominator coloring condition i.e. each vertex of the graph dominates every vertex of some color class.

Hence $\chi_d(P_n^+) \neq n$.

Thus, $\chi_d(P_n^+) \geq n + 1$. Let us assign the colors $c_1, c_2, \ldots, c_n$ to the vertices $\{v_1, v_2, \ldots, v_n\}$ and assign the color $c_{n+1}, \ldots$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the dominating set of P_n^+ dominates its own color class itself and other vertices also dominates all the other color classes (refer to Illustration 2.1.1).

Hence $\chi_d(P_n^+) = n + 1$. ▯

Illustration 2.1.1. Dominator coloring of comb graph P_4^+.

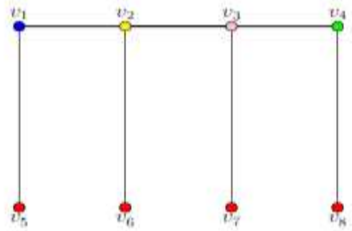

Fig. 2.1: Dominator coloring of P_4^+

The color classes are $C_1 = \{v_1\}$, $C_2 = \{v_2\}$, $C_3 = \{v_3\}$, $C_4 = \{v_4\}$, $C_5 = \{v_5, v_6, v_7, v_8\}$.

The vertices $\{v_1, v_2, v_3, v_4\}$ will dominates its own color class and the vertex v_5 will dominate the color class 1, the vertex v_6 will dominate the color class 2, the vertex v_7 will dominate the color class 3, and the vertex v_8 will dominate the color class 4 and hence by the definition of dominator coloring, each vertex of the graph dominates every vertex of some color class.

Therefore, the dominator chromatic number for comb graph P_4^+ is 5.

2.2 Dominator coloring of jewel graph

Definition 2.2.1. The jewel graph J_n is a graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and an edge set $E(J_n) = \{ux, vy, xy, yv, uu_i, vv_i : 1 \leq i \leq n\}$.

Theorem 2.2.1. For a jewel graph J_n, $\chi_d(J_n) = 3$.

Proof. Let J_n be a Jewel graph with n vertices. Let $V = \{u, v, x, y, u_1, u_2, \ldots, u_n\}$ be the vertex set. We only need two vertices to form a minimum dominating set. From Figure 2.2 we know that $D = \{u, v\}$ form a minimum dominating set.

We now claim the $\chi_d(J_n) = 3$.

Suppose $\chi_d(J_n) \neq 3$ and let $\chi_d(J_n) \leq 2$.

Assume $\chi_d(J_n) = 2$. Define a function $C: V(J_n) \to \{1, 2\}$ such that the dominating set $D = \{u, v\}$ vertices will receive color 1 and the vertices $\{x, u_1, u_2, \ldots, u_n\}$ will receive the color 2. Since the vertex y cannot be colored with colors 1 or 2 which contradicts the coloring property of G.

Thus $\chi_d(J_n) \neq 2$ (refer to the Illustration 2.2.1). Hence $\chi_d(J_n) = 3$. ▯

Illustration 2.2.1. Dominator coloring of jewel graph J_4.

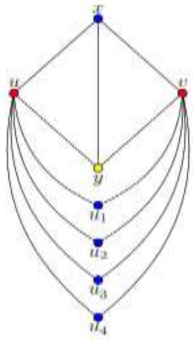

Fig. 2.2: Dominator coloring of J_4

The color classes are $C_1 = \{u, v\}$, $C_2 = \{x, u_1, u_2, u_3, u_4\}$, $C_3 = \{y\}$.

The vertices $\{u_1, u_2, u_3, u_4, x\}$ will dominates the color class 1 and the vertices $\{u, v\}$ will dominate the color class 2 and 3, the vertex y will dominate the color class 1 and hence by the definition of dominator coloring, each vertex of the graph dominates every vertex of some color class.
Therefore, the dominator chromatic number for jewel graph J_4 is 3.

2.3 Dominator coloring of flower graph

Definition 2.3.1. A Flower graph F_n is a graph obtained from a Helm by joining each pendent vertex to the central vertex of helm.

Theorem 2.3.1. For any flower graph F_n, $\chi_d(F_n) = 3, n \geq 4$

Proof. Let $V(F_n) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and $E(F_n) = \{vv_i, v_jv_{j+1}, v_nv_1, vu_i : 1 \leq i \leq n, 1 \leq j \leq n-1\}$. First select the central vertex v of F_n, this will dominate all other vertices of F_n.

We now claim the $\chi_d(F_n) = 3$.

Suppose $\chi_d(F_n) \neq 3$, let $\chi_d(F_n) \leq 2$.

Assume $\chi_d(F_n) = 2$. Define a function C: $V(F_n) \rightarrow \{1, 2\}$ such that the central vertex v will receive color 1 and so that all the other vertices cannot receive the color 1 since it is adjacent with all the other vertices. The vertex v_1 will receive the color 2 so that the vertex u_1 cannot be colored using 1 or 2 which contradicts the coloring property of G.

Thus $\chi_d(F_n) \neq 2$ (refer to the Illustration 2.3.1). Hence $\chi_d(F_n) = 3$.

Illustration 2.3.1. Dominator coloring of flower graph F_4.

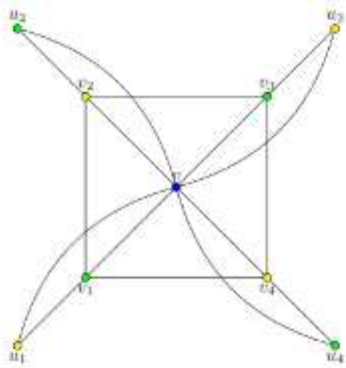

Fig. 2.3: Dominator coloring of F_4

The color classes are $C_1 = \{v\}$, $C_2 = \{u_1, u_3, v_2, v_4\}$, $C_3 = \{u_2, u_4, v_1, v_3\}$.

The vertices $\{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$ will dominates the color class 1 and the vertex v will dominate its own color class and hence by the definition of dominator coloring, each vertex of the graph dominates every vertex of some color class. Therefore, the dominator chromatic number for flower graph F_4 is 3.

3. DOMINATOR COLORING ON CORONA GRAPH OF PATH WITH SOME GRAPHS

In this section we obtain the dominator chromatic number on corona graph of path with triangular snake graph T_n, corona graph of path with double triangular snake graph $D(T_n)$.

Definition 3.1. Let G_1 and G_2 be two graphs on disjoint sets of n and m vertices, respectively. The corona graph $G_1 \circ G_2$ of G_1 and G_2 is defined as the graph obtained by taking one copy of G_1 and n copies of G_2, and then joining the ith vertex of G_1 to every vertex in the ith copy of G_2.

Example 3.1. Let $G_1 = C_4$, the cycle of order 4 and $G_2 = K_2$. The corona $G_1 \circ G_2$ of graphs of G_1 and G_2 are shown in Figures 3.1 and 3.2.

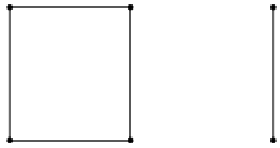

Fig. 3.1: Graphs G_1 and G_2

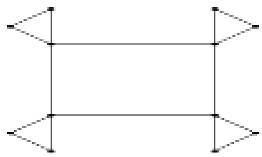

Fig. 3.2: Corona graph $G_1 \circ G_2$

Definition 3.2. A walk in which no vertex appears more than once is called a simple path or a path, that is each vertex in the path is distinct. Path on n vertices is P_n.

3.1 Dominator chromatic number on corona graph of path with triangular snake graph

Definition 3.1.1. A triangular snake graph T_n is obtained from the path P_n by replacing each edge by a triangle C_3.

Theorem 3.1.1. Let P_n and T_n denote the path and triangular snake graph with n vertices then $\chi_d(P_n \circ T_n) = n + 3, n \geq 2$.

Proof. Let $V(P_n) = \{v_i : 1 \leq i \leq n\}$ and $V(T_n) = \{u_i : 1 \leq i \leq n\}$.

Let $V(P_n \circ T_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq i, j \leq n\}$.

By the definition of corona graph, each vertex of P_n is adjacent to every vertex of a copy of T_n (i.e) every vertex $v_i \in V(P_n)$ is vertex from the set $\{u_{ij} : 1 \leq j \leq n\}$. Now choose the vertices of the path $\{v_1, v_2, \ldots, v_n\}$. It yields a minimum dominating set D. We now claim that $\chi_d(P_n \circ T_n) = n + 3$, suppose $\chi_d(P_n \circ T_n) \neq n + 3$, let $\chi_d(P_n \circ T_n) \leq n + 2$.

Assume that $\chi_d(P_n \circ T_n) = n + 2$. Define a function $C : V(P_n \circ T_n) \to \{1, 2, \ldots, n+2\}$ such that the dominating set D will receive the color $\{1, 2, \ldots, n\}$. Now the vertex u_{i1} will receive the color $n + 1$ and u_{i2} will receive the color $n + 3$ but u_{i3} cannot be colored using the color $n + 1$ or $n + 2$ or 1 which contradicts the coloring property and also u_{i3} cannot be colored using $\{1, 2, \ldots, n\}$ which will not satisfy the dominator coloring condition, that is each vertex of the graph dominates every vertex of some color class.

Thus $\chi_d(P_n \circ T_n) \neq n + 2$ (refer to the Illustration 3.1.1). Hence $\chi_d(P_n \circ T_n) = n + 3$. $\square$

Illustration 3.1.1. Dominator chromatic number on corona graph of path P_4 with triangular snake graph T_4.

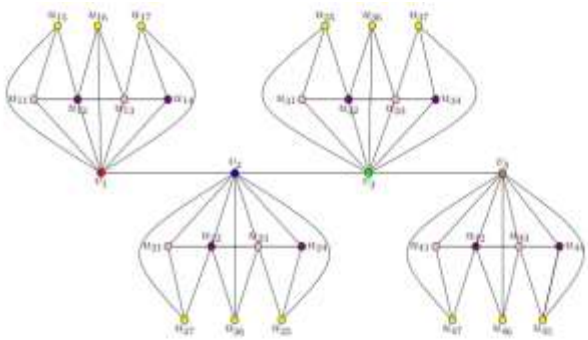

Fig. 3.3: Dominator coloring of $P_4 \circ T_4$

The color classes are $C_1 = \{v_1\}$, $C_2 = \{v_2\}$, $C_3 = \{v_3\}$, $C_4 = \{v_4\}$, $C_5 = \{u_{15}, u_{16}, u_{17}, u_{25}, u_{26}, u_{27}, u_{35}, u_{36}, u_{37}, u_{45}, u_{46}, u_{47}\}$, $C_6 = \{u_{11}, u_{13}, u_{31}, u_{33}, u_{21}, u_{23}, u_{41}, u_{43}\}$, $C_7 = \{u_{12}, u_{14}, u_{22}, u_{24}, u_{32}, u_{34}, u_{42}, u_{44}\}$.

The vertices $\{v_1, v_2, v_3, v_4\}$ will dominates its own color class 1, 2, 3 and 4 respectively. The vertices $\{u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}\}$ will dominates the color class 1. The vertices $\{u_{21}, u_{22}, u_{23}, u_{24}, u_{25}, u_{26}, u_{27}\}$ will dominates the color class 2. The vertices $\{u_{31}, u_{32}, u_{33}, u_{34}, u_{35}, u_{36}, u_{37}\}$ will dominates the color class 3 and the vertices $\{u_{41}, u_{42}, u_{43}, u_{44}, u_{45}, u_{46}, u_{47}\}$ will dominates the color class 4. Hence by the definition of dominator coloring, each vertex of the graph dominates every vertex of some color class.

Therefore, the dominator chromatic number for corona graph of path P_4 with triangular snake graph T_4 is 7 (i.e.) $\chi_d(P_n \circ T_n) = 7$.

3.2. Dominator chromatic number on corona graph of path with double triangular snake graph

Definition 3.2.1. A double triangular snake graph $D(T_n)$ is obtained from the path $u_1u_2 \ldots u_n$ by joining u_i, u_{i+1} with two new vertices v_i and w_i for $1 \le i \le n-1$.

Theorem 3.2.1. Let P_n and $D(T_n)$ denote the path and double triangular snake graph with n vertices then $\chi_d(P_n \circ D(T_n)) = n+3$.

Proof. Let $V(P_n) = \{v_i : 1 \le i \le n\}$ and $V(D(T_n)) = \{u_i : 1 \le i \le n\}$.

Let $V(P_n \circ D(T_n)) = \{v_i : 1 \le i \le n\} \cup \{u_{ij} : 1 \le i, j \le n\}$.

By the definition of corona graph, each vertex of P_n is adjacent to every vertex of a copy of $D(T_n)$, that is every vertex $v_i \in V(P_n)$ is adjacent to every vertex from the set $\{u_{ij} : 1 \le j \le n\}$. Now choose the vertices of the path $\{v_1, v_2, \ldots, v_n\}$. It yields a minimum dominating set D.

We now claim the $\chi_d(P_n \circ D(T_n)) = n+3$.

Suppose $\chi_d(P_n \circ D(T_n)) \ne n+3$, let $\chi_d(P_n \circ D(T_n)) \le n+2$.

Assume that $\chi_d(P_n \circ D(T_n)) = n+2$. Define a function $C : V(P_n \circ D(T_n)) \to \{1, 2, \ldots, n+2\}$ such that the dominating set D will receive the color $\{1, 2, \ldots, n\}$. Now the vertex u_{i1} will recieve the color $n+1$ and u_{i2} will receive the color $n+3$ but u_{i3} cannot be colored using the color $n+1$ or $n+2$ or 1 which contradicts the coloring property and also u_{i3} cannot be colored using $\{1, 2, \ldots, n\}$ which will not satisfy the dominator coloring condition (i.e.) each vertex of the graph dominates every vertex of some color class.

Thus $\chi_d(P_n \circ D(T_n)) \ne n+2$ (refer to the Illustration 3.2.1.). Hence $\chi_d(P_n \circ D(T_n)) = n+3$. $\square$

Illustration 3.2.1. Dominator chromatic number on corona graph of path P_4 with double triangular snake graph $D(T_4)$.

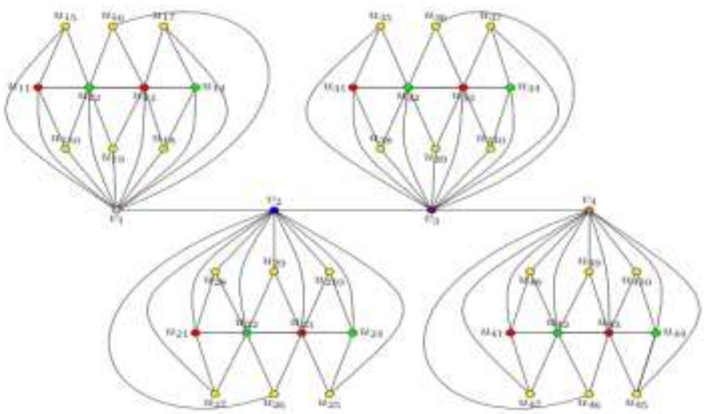

Fig. 3.4: Dominator coloring of $P_4 \circ D(T_4)$

The color classes are $C_1 = \{v_1\}$, $C_2 = \{v_2\}$, $C_3 = \{v_3\}$, $C_4 = \{v_4\}$, $C_5 = \{u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{110}, u_{25}, u_{26}, u_{27}, u_{28}, u_{29}, u_{210}, u_{35}, u_{36}, u_{37}, u_{38}, u_{39}, u_{310}, u_{45}, u_{46}, u_{47}, u_{48}, u_{49}, u_{410}\}$, $C_6 = \{u_{11}, u_{13}, u_{31}, u_{33}, u_{21}, u_{23}, u_{41}, u_{43}\}$, $C_7 = \{u_{12}, u_{14}, u_{22}, u_{24}, u_{32}, u_{34}, u_{42}, u_{44}\}$.

The vertices $\{v_1, v_2, v_3, v_4\}$ will dominates its own color class 1, 2, 3 and 4 respectively. The vertices $\{u_{11}, u_{12}, u_{13}, u_{14}, u_{15}, u_{16}, u_{17}, u_{18}, u_{19}, u_{110}\}$ will dominate the color class 1. The vertices $\{u_{21}, u_{22}, u_{23}, u_{24}, u_{25}, u_{26}, u_{27}, u_{28}, u_{29}, u_{210}\}$ will dominate the color class 2. The vertices $\{u_{31}, u_{32}, u_{33}, u_{34}, u_{35}, u_{36}, u_{37}, u_{38}, u_{39}, u_{310}\}$ will dominate the color class 3 and the vertices $\{u_{41}, u_{42}, u_{43}, u_{44}, u_{45}, u_{46}, u_{47}, u_{48}, u_{49}, u_{410}\}$ will dominate the color class 4. Hence by the definition of dominator coloring, each vertex of the graph dominates every vertex of some color class. Therefore, the dominator chromatic number for corona graph of path P_4 with double triangular snake graph $D(T_4)$ is 7 (i.e.) $\chi_d(P_4 \circ D(T_4)) = 7$.

4. CONCLUSION

In this paper we obtain the dominator chromatic number of comb graph, jewel graph and flower graph and also the dominator chromatic number of corona graph of path with triangular snake graph and corona graph of path with double triangular snake graph. This paper can be further extended by identifying some families of graph and corona graph of path with any graphs.

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