INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact Factor: 6.078
(Volume 7, Issue 3 - V7I3-1172)
Available online at: https://www.ijariit.com

Co-Fuzzy Sub Ordered Finite Gamma Near Rings

D. Bharathi
bharathikavali@gmail.com
Sri. Venkateswara University, Tirupati, Andhra Pradesh, India

*Pakala Venkatrao**
pakalaupd@gmail.com
Sri. Venkateswara University, Tirupati, Andhra Pradesh, India

K. Balakoteswara Rao
balakoteswararaokedari@gmail.com
PRR and VS Government Degree College, Vidavalur, Andhra Pradesh

ABSTRACT

In this paper, we introduce and study the concept of Co-fuzzy sub ordered finite Γ- near rings and L-fuzzy sub ordered finite Γ- near rings with maximum and infimum conditions, where L stands for a complete lattice satisfying the infinite meet distributive law. We discuss the concept of intersection and union of co-fuzzy sub ordered finite Γ- near rings. We also prove theorems on homomorphism of finite Γ- near rings.

Keywords— *Fuzzy set, L – Fuzzy set, co-fuzzy sub ordered finite Γ- near ring, homomorphism of finite Γ- near rings.*

1. INTRODUCTION

We know the importance of fuzzy sets. It is being used in different areas like physics, Electronics, Robotics, Computer Languages, Military control, Artificial intelligence, Law, Psychology, Economics and Medical sciences.

The notion of fuzzy set was introduced by Zadeh L.A [12] in 1965 as function from a nonempty set X into $[0,1]$.Goguen[5] further generalized Zadeh definition of a fuzzy subset of X as a mapping of X into a Lattice L. Further more so many authors introduced the fuzzy set concept in algebraic structures, like groups, semi groups, rings and $\Gamma-$near rings etc. The concept of a Gamma – ring, a generalization of a ring was introduced by Nobusawa[4] and was generalized by Barnes. The concept of near ring was introduced by Pilz.G[6]. The common generalization of concepts of Near ring and $\Gamma-$near ring, namely $\Gamma-$near ring was introduced by Satyanarayana. Bh[9][10] and the ideal theory of $\Gamma-$near rings was studied by Satyanarayana.Bh and Booth.G.L[2]. Fuzzy ideals in rings were introduced by Liu.W and they have been studied by several authors.

In this paper, we define Co-Fuzzy sub ordered finite $\Gamma-$near ring. We prove the intersection and union of two Co-Fuzzy sub ordered finite $\Gamma-$near rings theorems. We also prove the homomorphic image and pre-image of Co-Fuzzy sub ordered finite $\Gamma-$near ring is again a Co-Fuzzy sub ordered finite $\Gamma-$near rings. We also prove some necessary and sufficient conditions for a Fuzzy set to be Co-Fuzzy sub ordered finite $\Gamma-$near ring. Throughout this chapter $\mathcal{N}$ stands for finite $\Gamma-$near ring.

2. PRELIMINARIES

In this section we recall some of the fundamental definitions, which are necessary for this paper.

Definition 2.1: A triplet $(\mathcal{N},+,\cdot)$ is said to be a near ring if

1. $(\mathcal{N},+)$ is a group (Not necessarily abelian group)
2. $(\mathcal{N},\cdot)$ is a semi-group
3. $(a+b)\cdot c = a\cdot c + b\cdot c \ \forall\, a,b,c \in \mathcal{N}$ (Right distribution law).

Definition 2.2: A near ring $(\mathcal{N},+,\cdot)$ is said to be finite near ring if $\mathcal{N}$ has finite Number of elements.

Definition 2.3: Let $\mathcal{N}$ be a non-empty finite set. If

1. $(\mathcal{N},+)$ is a group. (Not necessarily abelian group)
2. Γ is the set of binary operations such that $(\mathcal{N},+,\alpha)$ is a near ring.

Where $\alpha\in\Gamma$.

3.$a\alpha(b\beta c) = (a\alpha b)\beta c, \forall\, a,b,c \in \mathcal{N}$ and $\alpha,\beta \in \Gamma$.

Then $(\mathcal{N}, +, \Gamma)$ is a finite Γ - near ring.

Example 2.4: Let G be any finite non-abelian group and $X(\neq \phi)$ be any finite set.

Let N $= \{ f / f : X \to G \text{ is a function} \}$

Then $(\mathcal{N}, +)$ is a group with respect to point wise addition of functions, but $(\mathcal{N}, +)$ is not commutative. Let us choose $a, b \in G$ such that $a + b \neq b + a$.

Define $f_a : X \to G$ and

$f_b : X \to G$ such that

$f_a(x) = a$ & $f_b(x) = b\ \forall x \in X$.

$$Then\ (f_a + f_b)(x) = f_a(x) + f_b(x)$$

$$= a + b \text{.............................}(2.4.1)$$

But

$$(f_b + f_a)(x) = f_b(x) + f_a(x)$$

$$= a + b \text{.............................}(2.4.2)$$

From the equations (2.4.1) and (2.4.2),

$$(f_a + f_b)(x) \neq (f_b + f_a)(x)$$

$$f_a + f_b \neq f_b + f_a$$

Thus $(\mathcal{N}, +)$ is a not commutative group.

Now take $\Gamma = \{ g / g : G \to X \text{ is a function} \}$

Then by defining $\Gamma : \mathcal{N} \times \mathcal{N} \to \mathcal{N}$ by $f_1 g f_2$ as composition of mappings. Then $(\mathcal{N}, +, \Gamma)$ is a finite Γ - near ring.

Clearly,

$$((f_1 + f_2) g f_3)(x) = (f_1 + f_2)(g(f_3(x)))$$

$$= f_1(g(f_3(x))) + f_2(g(f_3(x)))$$

$$= (f_1 g f_3)(x) + (f_2 g f_3)(x)$$

$$= (f_1 g f_3 + f_2 g f_3)(x)$$

$\forall x \in X, f_1, f_2, f_3 \in \mathcal{N}$ and $g \in \Gamma$

i.e., $(f_1 + f_2) g f_3 = f_1 g f_3 + f_2 g f_3$

i.e., the right distribution law hold in $\mathcal{N}$.

But left distribution law is not satisfied in $\mathcal{N}$.

Let us take $z_0 (\neq 0) \in G$

Define $f_1 : X \to G$ such that $f_1(x) = z_0, \forall x \in X$ *and* $g : G \to X$ *such that* $g(x) = u, \forall x \in G$

Where $u \in X$

Now $(f_1 g(f_2 + f_3))(x) = z_0 \text{......}(2.4.3)$

But

$$(f_1 g f_2 + f_1 g f_3)(x) = (f_1 g f_2)(x) + (f_1 g f_3)(x)$$

$$= f_1(g(f_2(x))) + f_1(g(f_3(x)))$$

$$= f_1(u) + f_1(u) = z_0 + z_0 \text{------------}(2.4.4)$$

From equations (2.4.3) and (2.4.4),

$(f_1 g(f_2 + f_3))(x) \neq (f_1 g f_2 + f_1 g f_3)(x)$.

Clearly

$f_1 g_1 (f_2 g_2 f_3) = (f_1 g_1 f_2) g_2 f_3$ holds in $\mathcal{N}$ with respect to composition of functions

Hence $(\mathcal{N}, +, \Gamma)$ is a finite Γ – near ring.

Definition 2.5 : Let $(\mathcal{N}, +, \Gamma)$ is a finite Γ –near ring. Then a subset S of $\mathcal{N}$ is said to be sub Γ –

near ring of $(\mathcal{N}, +, \Gamma)$, if

1. $x + y \in S$
2. $x\alpha y \in S, \forall x, y \in S \text{ and } \alpha \in \Gamma$

Definition 2.6: A non-empty set P is said to be POSET (partial ordering set), if there exists a relation $\leq$ on P such that

1. $a \leq a$
2. $a \leq b \text{ and } b \leq a \Rightarrow a = b$
3. $a \leq b, b \leq c \Rightarrow a \leq c \ \forall a, b, c \in \text{P}$

Definition 2.7 : Finite Γ - near ring $\mathcal{N}$ is said to be finite ordered Γ - near ring , if $(\mathcal{N}, \leq)$ is a POSET.

Definition 2.8: A POSET L is said to be a lattice if every two elements $a, b \in L$ have suprimum and infimum. Generally, we denote

$$\sup\{a, b\} = a \vee b$$

$$Inf\{a, b\} = a \wedge b$$

Where $\vee, \wedge$ are respectively called join and meet.

Definition 2.9: If every non empty sub set of lattice L has suprimum and inifimum then L is called complete lattice.

Definition 2.10: If X be a non-empty set and $f : X \to [0,1]$ is a mapping then the pair (X, f) is called fuzzy set and f is called fuzzy sub set of X.If $[0,1] = L$ where L stands for lattice then f is called L - fuzzy sub set of X. It is denoted by f_L.

Definition 2.11: Let X be a non empty set and L is a lattice. If $f_L : X \to L$ is a mapping then f_L is Called L-fuzzy sub set of X.

Definition 2.12: Let f_L be a L - fuzzy sub set of X. The set $\{x \in X / f_L(x) < s\}$.Where $s \in L$ is called level sub set of f_L. It is denoted by f_s.

This f_s is called s - cut of f_L.

$$\therefore f_s = \{x \in X / f(x) < s\}$$

Definition 2.13: Let $f_L \text{ and } g_L$ be two L-fuzzy sub sets of X. If $f_L(x) \leq g_L(x)$, $\forall x \in X$ then f_L is said to be contained in g_L. It is denoted by $f_L \subseteq g_L$.

3. CO-FUZZY SUB ORDERED FINITE Γ - NEAR RING

In this section we define the intersection and union of co-fuzzy sub sets of non-empty set X. We prove the intersection of two co-fuzzy sub ordered finite Γ - near rings is also co-fuzzy sub ordered finite Γ - near ring. But the union of two co-fuzzy sub ordered finite Γ - near rings need not be co-fuzzy sub ordered finite Γ - near ring and also, we give suitable illustrations.

Definition 3.1: Let $f \text{ and } g$ be two fuzzy sub sets of X. Then their intersection and union are denoted by $f \cap g \text{ and } f \cup g$ respectively and defined as follows

$$(f \cap g)(x) = \min\{f(x), g(x)\}$$

and

$$(f \cup g)(x) = \max\{f(x), g(x)\}$$

Definition 3.2: For any sub set A of a set X, the co-fuzzy characteristic set δ_A is defined as follows

$$\delta_A = \begin{cases} 0 & if \ x \in A \\ 1 & if \ x \notin A \end{cases}$$

Definition 3.3: Let $M_1 \text{ and } M_2$ be two non-empty sets and $\mu : M_1 \to M_2$ is a mapping. If f is a fuzzy sub set of M_1 then g be a fuzzy sub set of M_2 defined by

$$g(y) = \begin{cases} \underset{z \in \mu^{-1}(y)}{Inf} f(z) & if \ \mu^{-1}(y) \neq \phi \\ 0 & if \ \mu^{-1}(y) = \phi \end{cases}$$

Where $\mu^{-1}(y)=\{x\in M_1 / \mu(x)=y\}$

If g is fuzzy sub set of M_2 then f be a fuzzy sub set of M_1 defined by

$$f(x)=g(\mu(x)),\ \forall x\in M_1$$

Definition 3.4: Let f be a fuzzy sub set of finite ordered Γ - near ring $(\mathcal{N},+,\Gamma)$. Then f is said to be co-fuzzy sub ordered finite Γ - near ring of $(\mathcal{N},+,\Gamma)$, if

1. $f(x+y)\le \max\{f(x),f(y)\}$
2. $f(x\alpha y)\le \max\{f(x),f(y)\}$
3. $x\le y\Rightarrow f(x)\le f(y), \forall x,y\in \mathcal{N}$ and $\alpha\in\Gamma$

Definition 3.5: Let f_L be a L-fuzzy sub set of a finite ordered Γ - near ring $(\mathcal{N},+,\Gamma)$. Then f_L is Said be a co-fuzzy sub ordered finite Γ - near ring of $(\mathcal{N},+,\Gamma)$, If

1. $f_L(x+y)\le f_L(x)\vee f_L(y)$
2. $f_L(x\alpha y)\le f_L(x)\vee f_L(y)$
3. $x\le y\Rightarrow f_L(x)\le f_L(y)$, $\forall x,y\in \mathcal{N}$ and $\alpha\in\Gamma$.

Example 3.6: Let $S=\{1,2,3,4\}$ and $\mathcal{N}=P(S)$ is the power set of S .Then $\mathcal{N}$ is a group with respect to symmetric difference of two sets.
Where the symmetric difference of two sets A and B is defined as

$$A\Delta B=(A\cup B)-(A\cap B)$$

Let us take $L=\Gamma=\{\{1\},\{2\},\{1,2\},\{1,2,3\}\}$

Define the mapping $\mathcal{N}\times\Gamma\times\mathcal{N}\to\mathcal{N}$, by $A\alpha B$ is the intersection of the sets A,α,B. Where $A,B\in\mathcal{N}$ and $\alpha\in\Gamma$.
It is clear that 1. $(\mathcal{N},\Delta,\Gamma)$ is finite Γ –near ring.
2. $(\mathcal{N},\subseteq)$ is a poset and $(L,\subseteq)$ is lattice.

Then $(\mathcal{N},\Delta,\Gamma)$ is finite ordered $\Gamma-$ near ring.

Now define $f_L:\mathbf{N}\to L$ such that

$$f_L(A)=\begin{cases}\{1,2,3\} & if \quad A\neq\phi\\ \phi & if \quad A=\phi\end{cases}$$ Where $A\in\mathcal{N}$.

Let us take $A,B\in\mathcal{N}$ and $\alpha\in\Gamma$

Case (i): *Let* $A\neq\phi, B\neq\phi, A\Delta B\neq\phi$, *and* $A\alpha B\neq\phi$

$f_L(A)=\{1,2,3\}$ *and* $f_L(B)=\{1,2,3\}$

and $f_L(A\Delta B)=\{1,2,3\}$

$f_L(A\alpha B)=\{1,2,3\}$ *and* $f_L(A)\vee f_L(B)=\{1,2,3\}$.

Case (ii): *Let* $A\neq\phi, B=\phi$ *then* $A\Delta B=A$, *and* $A\alpha B=\phi$

$f_L(A)=\{1,2,3\}$ *and* $f_L(B)=\phi$

$f_L(A\Delta B)=\{1,2,3\}$

$f_L(A\alpha B)=\phi$ *and* $f_L(A)\vee f_L(B)=\{1,2,3\}$

Case (iii): *Let* $A\neq\phi, B\neq\phi$ *and* $A\Delta B=\phi$, *and* $A\alpha B\neq\phi$

$f_L(A)=\{1,2,3\}$ *and* $f_L(B)=\{1,2,3\}$

$\therefore f_L(A\Delta B)=\phi, f_L(A\alpha B)=\{1,2,3\}$ *and* $f_L(A)\vee f_L(B)=\{1,2,3\}$

$Case\ (iv): Let\ A=\phi, B=\phi, Then\ A\Delta B=\phi,\ and\ A\alpha B=\phi$

$$f_L(A)=\phi\ \ and\ \ f_L(B)=\phi$$

$$\therefore f_L(A\Delta B)=\phi, f_L(A\alpha B)=\phi\ \ and\ \ f_L(A)\vee f_L(B)=\phi.$$

$Case\ (v): Let\ A\neq\phi, B\neq\phi\ and\ A\Delta B=\phi, A\alpha B=\phi$

$$\therefore f_L(A)=\{1,2,3\}\ \ and\ \ f_L(B)=\{1,2,3\}$$

$$\therefore f_L(A\Delta B)=\phi, f_L(A\alpha B)=\phi\ \ and\ \ f_L(A)\vee f_L(B)=\{1,2,3\}$$

$Case\ (vi): Let\ A\neq\phi, B\neq\phi\ then\ A\Delta B\neq\phi,\ and\ A\alpha B=\phi$

$$f_L(A)=\{1,2,3\}\ \ and\ \ f_L(B)=\{1,2,3\}$$

$$\therefore f_L(A\Delta B)=\{1,2,3\} f_L(A\alpha B)=\phi\ \ and\ \ f_L(A)\vee f_L(B)=\{1,2,3\}$$

In above all the cases,

1. $f_L(A\Delta B)\leq f_L(A)\vee f_L(B)$
2. $f_L(A\alpha B)\leq f_L(A)\vee f_L(B)$

and it is clear that $A\subseteq B\Rightarrow f_L(A)\subseteq f_L(B)$

Hence f_L is a L - fuzzy sub ordered finite Γ - near ring of $(\mathcal{N}, \Delta, \Gamma)$.

Now, we prove a necessary and sufficient condition for f to be a co-fuzzy sub ordered finite $\Gamma-$ near ring.

Theorem3.7: Let f be a fuzzy sub set of finite ordered Γ - near ring $(\mathcal{N}, +, \Gamma)$. Then f is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$ if and only if for each $\delta\in(0,1]$, the δ -cut

$f_\delta=\{x\in\mathcal{N}\ /f(x)<\delta\}$ is a sub ordered Γ - near ring of $\mathcal{N}$.

Proof:

Let f be a co-fuzzy sub ordered Γ - near ring of $\mathcal{N}$. We need to prove that the δ - cut $f_\delta=\{x\in\mathcal{N}\ /f(x)<\delta\}$ is a sub ordered Γ - near ring of $\mathcal{N}$.

$$Let\ x,y\in f_\delta$$

$$\therefore f(x)<\delta\ \ and\ \ f(y)<\delta$$

$$Now\ \ f(x+y)\leq\max\{f(x),f(y)\}<\delta$$

$$\therefore x+y\in f_\delta$$

$$Let\ \ x,y\in f_\delta\ \ and\ \ \alpha\in\Gamma$$

$$\therefore f(x\alpha y)\leq\max\{f(x),f(y)\}<\delta$$

$$\therefore x\alpha y\in f_\delta$$

Thus f_δ is a sub ordered Γ - near ring of $\mathcal{N}$.

Conversely assume that f_δ is a sub ordered Γ - near ring of $\mathcal{N}$. for all $\delta\in(0,1]$

We have to prove that f is co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$.

If possible, let $\exists\ \ x_0, y_0\in\mathcal{N}\ \ \ni$

$$f(x_0+y_0)>\max\{f(x_0),f(y_0)\}$$

$$Choose\ \delta=\frac{f(x_0+y_0)+\max\{f(x_0),f(y_0)\}}{2}$$

$$\therefore f(x_0+y_0)>\delta\ and\ \max\{f(x_0)f(y_0)\}<\delta$$

$$\therefore x_0,y_0\in f_\delta\ \ and\ \ x_0+y_0\notin f_\delta$$

This is a contradiction to the fact that f_δ is a sub ordered Γ - near ring.

$\therefore f(x+y)\leq\max\{f(x),f(y)\}\ \ \forall x,y\in\mathcal{N}$.

Again, if possible let $\exists\ x_0, y_0 \in \mathcal{N}$. $\ni$

$$f(x_0\alpha y_0) > \max\{f(x_0), f(y_0)\}\ for\ some\ \alpha \in \Gamma$$

$$Choose\ \delta = \frac{f(x_0\alpha y_0) + \max\{f(x_0), f(y_0)\}}{2}$$

$$\therefore f(x_0\alpha y_0) > \alpha\ and\ \max\{f(x_0), f(y_0)\} < \delta$$

$$\therefore x_0\alpha y_0 \notin f_\delta\ and\ x_0, y_0 \in f_\delta$$

Which is a contradiction

$\therefore f(x\alpha y) \le \max\{f(x), f(y)\}\ \forall x, y \in \mathcal{N}$ and $\alpha \in \Gamma$

$Let\ x, y \in \mathcal{N}$ and $x \le y$

If possible, let $f(x) > f(y)$

$\therefore$ choose a $\delta \in (0,1)$ such that $f(x) > \delta > f(y)$

$\therefore x \notin f_\delta\ and\ y \in f_\delta$

Which is a contradiction

Hence $x \le y \Rightarrow f(x) \le f(y)$

$\therefore$ f is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$.

Theorem3.8: The intersection of two co-fuzzy sub ordered finite Γ - near rings of a finite ordered Γ - near ring $(\mathcal{N}, +, \Gamma)$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$.

Proof:

Let f and g are two co-fuzzy sub ordered finite Γ - near rings of finite ordered Γ - near ring $(\mathcal{N}, +, \Gamma)$.

We have to show that $(f \cap g)$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$.

Let $x, y \in \mathcal{N}$ and $\alpha \in \Gamma$.

$$i. (f \cap g)(x+y) = \min\{f(x+y), g(x+y)\}$$
$$\le \min\{\max\{f(x), f(y)\}, \max\{g(x), g(y)\}\}$$
$$= \max\{(f \cap g)(x), (f \cap g)(y)\}$$

$$\therefore (f \cap g)(x+y) \le \max\{(f \cap g)(x), (f \cap g)(y)\}$$

$$ii. (f \cap g)(x\alpha y) = \min\{f(x\alpha y), g(x\alpha y)\}$$
$$\le \min\{\max\{f(x), f(y)\}, \max\{g(x), g(y)\}\}$$
$$= \max\{(f \cap g)(x), (f \cap g)(y)\}$$

$$\therefore (f \cap g)(x+y) \le \max\{(f \cap g)(x), (f \cap g)(y)\}$$

$iii. Let\, x, y \in$ N and $x \le y$

$$\therefore f(x) \le f(y)\ and\ g(x) \le g(y)$$

$$Now\ (f \cap g)(x) = \min\{f(x), g(x)\}$$
$$\le \min\{f(y), g(y)\}$$
$$= (f \cap g)(y)$$

Hence $f \cap g$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$.

Theorem 3.9: The union of two co-fuzzy sub ordered finite Γ - near rings of a finite ordered Γ - near rings $\mathcal{N}$ is a co-fuzzy sub ordered finite Γ - near ring if one is contained in other.

Proof: Let f and g be two co-fuzzy sub ordered finite Γ - near rings of $\mathcal{N}$ such that $f \subseteq g$ or $g \subseteq f$.

With out less of generality, assume that $f \subseteq g$

i. Let $x, y \in \mathcal{N}$

$\therefore f(x) \leq g(x)$ *and* $f(y) \leq g(y)$

$Now\ (f \cup g)(x+y) = \max\{f(x+y), g(x+y)\}$

$\leq \max\{\max\{f(x), f(y)\}, \max\{g(x), g(y)\}\}$

$\leq \max\{\max\{f(x), g(x)\}, \max\{f(y), g(y)\}\}$

$= \max\{(f \cup g)(x), (f \cup g)(y)\}$

ii. Let $x, y \in \mathcal{N}$ and $\alpha \in \Gamma$

$\therefore (f \cup g)(x\alpha y) = \max\{f(x\alpha y), g(x\alpha y)\}$

$\leq \max\{\max\{f(x), f(y)\}, \max\{g(x), g(y)\}\}$

$\leq \max\{\max\{f(x), g(x)\}, \max\{f(y), g(y)\}\}$

$= \max\{(f \cup g)(x), (f \cup g)(y)\}$

iii. let $x, y \in \mathcal{N}$ and $x \leq y$

$\therefore (f \cup g)(x) = \max\{f(x), g(x)\}$

$\leq \max\{f(y), g(y)\}$

$= (f \cup g)(y)$

Hence $f \cup g$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}$.

Note 3.10:

The converse of the above theorem need not be true, i.e., even if the union of two co-fuzzy sub ordered finite Γ - near rings of finite ordered Γ - near ring $\mathcal{N}$ is a co-fuzzy sub ordered finite Γ - near rings of $\mathcal{N}$, one may not contained in the other.

Example 3.11: Let $S = \{1,2,3,4\}$ then $(P(S), \Delta, \Gamma)$ is a finite ordered Γ - near ring. Where $\Gamma = \{\{1\}, \{1,2\}\}$

Define $f : P(S) \to [0,1]$ and $g : P(S) \to [0\,1]$ such that

$$f(A) = \begin{cases} 0.5 & if \quad A \neq \phi \\ 0.4 & if \quad A = \phi \end{cases}$$

and

$$g(A) = \begin{cases} 0.6 & if \quad A \neq \phi \\ 0.3 & if \quad A = \phi \end{cases}$$ are two co-fuzzy sub ordered finite Γ - near rings of $(P(S), \Delta, \Gamma)$.

Then clearly $(f \cup g)$ is a co-fuzzy sub ordered finite Γ - near ring of $(P(S), \Delta, \Gamma)$. But one is not contained in the other.

i.e., $f \not\subseteq g$ and $g \not\subseteq f$.

Theorem 3.12:

The necessary and sufficient condition for two level subsets f_{δ_1} and f_{δ_2} of a finite ordered Γ - near ring $(\mathcal{N}, +, \Gamma)$ are equal is there exists, no $x \in \mathcal{N}$ such that $\delta_1 \leq f(x) < \delta_2$.

Where $0 < \delta_1 < \delta_2 \leq 1$

Proof:

Let the two level subsets f_{δ_1} and f_{δ_2} are equal.

Suppose, let $\exists x \in \mathcal{N}$, $\ni$ $\delta_1 \leq f(x) < \delta_2$

$\therefore x \notin f_{\delta_1}$ *and* $x \in f_{\delta_2}$

This is a contradiction to $f_{\delta_1} = f_{\delta_2}$

$\therefore \exists$ no $x \in \mathcal{N}$ $\ni$ $\delta_1 \leq f(x) < \delta_2$

Conversely assume that $\therefore \exists$ no $x \in \mathcal{N}$ $\ni$ $\delta_1 \leq f(x) < \delta_2$

Suppose let $f_{\delta_1} \neq f_{\delta_2}$

$\therefore f_{\delta_1} \subset f_{\delta_2}$

$\Rightarrow$ There exist $x \in f_{\delta_2}$ such that $x \notin f_{\delta_1}$

$\Rightarrow \delta_1 \leq f(x) < \delta_2$

This is a contradiction to our assumption

Hence $f_{\delta_1} = f_{\delta_2}$. □

4. HOMOMORPHISM OF FINITE ORDERED $\Gamma-$ NEAR RINGS

In this section we define homomorphism between two finite ordered $\Gamma-$ near rings and prove that homomorphic image and pre image of co-fuzzy sub ordered finite $\Gamma-$ near rings are also co-fuzzy sub ordered finite $\Gamma-$ near rings.

Definition 4.1: Let ($\mathcal{N}_1$,+,Γ) and($\mathcal{N}_2$,+,Γ) be two finite ordered Γ - near rings and $f : \mathcal{N}_1 \to \mathcal{N}_2$ be a mapping. Then f is said to be homomorphism if

1. $f(a+b) = f(a)+f(b)$
2. $f(a\gamma b) = f(a)\gamma f(b)$
3. $a \leq b \Rightarrow f(a) \leq f(b)$,$\forall a,b \in \mathcal{N}_1$ and $\gamma \in \Gamma$

Definition 4.2: Let ($\mathcal{N}_1$,+,Γ_1) and($\mathcal{N}_2$,+,Γ_2) be two finite ordered Γ - near rings. $f : \mathcal{N}_1 \to \mathcal{N}_2$ and $g : \Gamma_1 \to \Gamma_2$ be two mappings. Then (f,g) is said to be homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$ if

1. $f(a+b) = f(a)+f(b)$
2. $f(a\gamma b) = f(a)g(\gamma)f(b)$
3. $a \leq b \Rightarrow f(a) \leq f(b)$,$\forall a,b \in \mathcal{N}_1$ and $\gamma \in \Gamma_1$

Definition 4.3: Let ($\mathcal{N}_1$, +, Γ) and ($\mathcal{N}_2$, +, Γ) be two finite ordered Γ - near rings and the function $f : \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$. If μ is a fuzzy subset of $\mathcal{N}_1$.

Then $f(\mu)$ is a fuzzy subset of N_2 defined by

$$(f(\mu))(y) = \begin{cases} \inf\limits_{z \in f^{-1}(y)} \mu(z) & if \;\; f^{-1}(y) \neq \phi \\ 0 & if \;\; f^{-1}(y) = \phi \end{cases} \quad \forall y \in \mathcal{N}_2$$

Definition 4.4 : Let ($\mathcal{N}_1$,+,Γ) and($\mathcal{N}_2$,+,Γ) be two finite ordered Γ - near rings and the function $f : \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism from $\mathcal{N}_1$ to $\mathcal{N}_2$. If σ is a fuzzy subset of $\mathcal{N}_2$, then its inverse image $f^{-1}(\sigma)$ is a fuzzy subset of $\mathcal{N}_1$ defined by

$$(f^{-1}(\sigma))(x) = \sigma(f(x)) \;\; \forall x \in \mathcal{N}_1$$

Now, we prove some theorems related to homomorphism.

Theorem 4.5: Let ($\mathcal{N}_1$, +, Γ) and ($\mathcal{N}_2$,+,Γ) be two finite ordered Γ - near rings. The function $f : \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism. If σ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_2$, then its inverse image $f^{-1}(\sigma)$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_1$.

Proof:

Given $f : \mathcal{N}_1 \to \mathcal{N}_2$ is a homomorphism and $\sigma : \mathcal{N}_2 \to [0,1]$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_2$.

We have to prove that $f^{-1}(\sigma)$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_1$.

Let $x, y \in \mathcal{N}_1$ and $\alpha \in \Gamma$.

$$\begin{aligned} i.\ (f^{-1}(\sigma))(x+y) &= \sigma(f(x+y)) \\ &= \sigma(f(x)+f(y)) \\ &\leq \max\{\sigma(f(x)), \sigma(f(y))\} \\ &= \max\{(f^{-1}(\sigma))(x), (f^{-1}(\sigma))(y)\} \end{aligned}$$

$$(f^{-1}(\sigma))(x+y) \leq \max\{(f^{-1}(\sigma))(x), (f^{-1}(\sigma))(y)\}$$

$$ii. \left(f^{-1}(\sigma)\right)(x\alpha y) = \sigma\left(f(x\alpha y)\right)$$
$$= \sigma\left(f(x)\alpha f(y)\right)$$
$$\leq \max\left\{\sigma\left(f(x)\right), \sigma\left(f(y)\right)\right\}$$
$$= \max\left\{\left(f^{-1}(\sigma)\right)(x), \left(f^{-1}(\sigma)\right)(y)\right\}$$
$$\left(f^{-1}(\sigma)\right)(x\alpha y) \leq \max\left\{\left(f^{-1}(\sigma)\right)(x), \left(f^{-1}(\sigma)\right)(y)\right\}$$

$$iii. \quad x \leq y \Rightarrow f(x) \leq f(y)$$
$$\Rightarrow \sigma\left(f(x)\right) \leq \sigma\left(f(y)\right)$$
$$\Rightarrow \left(f^{-1}(\sigma)\right)(x) \leq \left(f^{-1}(\sigma)\right)(y)$$
$$x \leq y \Rightarrow \left(f^{-1}(\sigma)\right)(x) \leq \left(f^{-1}(\sigma)\right)(y)$$

Hence $f^{-1}(\sigma) : \mathcal{N}_1 \rightarrow [0,1]$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_1$.

Theorem 4.6: Let ($\mathcal{N}_1$,+, Γ) and ($\mathcal{N}_2$,+, Γ) be two finite ordered Γ - near rings. The function $f : \mathcal{N}_1 \rightarrow \mathcal{N}_2$ is an onto homomorphism. If μ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_1$ then it's image $f(\mu)$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_2$.

Proof:

Given $f : \mathcal{N}_1 \rightarrow \mathcal{N}_2$ is an onto homomorphism and $\mu : \mathcal{N}_1 \rightarrow [0\ 1]$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_1$.

Now we have to prove that $f(\mu)$ is a co-fuzzy sub ordered finite Γ - near ring of $\mathcal{N}_2$ defined by

$$\left(f(\mu)\right)(y) = \begin{cases} \inf\limits_{z \in f^{-1}(y)} \mu(z) & if \quad f^{-1}(y) \neq \phi \\ 0 & if \quad f^{-1}(y) = \phi \end{cases} \quad \forall y \in \mathcal{N}_2.$$

Let $x, y \in \mathcal{N}_2$ and $\alpha \in \Gamma$

Since f is onto from $\mathcal{N}_1$ to $\mathcal{N}_2$, $\exists$ $x^1, y^1 \in \mathcal{N}_1$, such that $f(x^1) = x$ and $f(y^1) = y$

$$\text{i. } \left(f(\mu)\right)(x+y) = \inf_{z \in f^{-1}(x+y)} \mu(z)$$
$$Let\ \mu(x_0) = \inf_{z \in f^{-1}(x)} \mu(z)\ and\ \mu(y_0) = \inf_{z \in f^{-1}(y)} \mu(z)$$
$$\therefore f(x_0) = x\ and\ f(y_0) = y$$
$$Now\ f(x_0 + y_0) = f(x_0) + f(y_0) = x + y$$
$$x_0 + y_0 \in f^{-1}(x+y)$$
$$\therefore \left(f(\mu)\right)(x+y) = \inf_{z \in f^{-1}(x+y)} \mu(z)$$
$$\leq \mu(x_0 + y_0)$$
$$\leq \max\left\{\mu(x_0), \mu(y_0)\right\}$$
$$= \max\left\{\inf_{z \in f^{-1}(x)} \mu(z), \inf_{z \in f^{-1}(y)} \mu(z)\right\}$$
$$= \max\left\{\left(f(\mu)(x)\right), \left(f(\mu)(y)\right)\right\}$$

$$\text{ii. } \left(f(\mu)\right)(x\alpha y) = \inf_{z \in f^{-1}(x\alpha y)} \mu(z)$$
$$\leq \mu(x_0 \alpha y_0)$$
$$\leq \max\left\{\mu(x_0), \mu(y_0)\right\}$$
$$= \max\left\{\inf_{z \in f^{-1}(x)} \mu(z), \inf_{z \in f^{-1}(y)} \mu(z)\right\}$$
$$= \max\left\{\left(f(\mu)\right)(x), \left(f(\mu)\right)(y)\right\}$$

iii. $x \leq y \Rightarrow f(x) \leq f(y)$

$$\Rightarrow \inf_{z \in f^{-1}(x)} \mu(z) \leq \inf_{z \in f^{-1}(y)} \mu(z)$$

$$\Rightarrow (f(\mu))(x) \leq (f(\mu))(y)$$

Hence $f(\mu): \mathcal{N}_2 \rightarrow [0,1]$ is a co-fuzzy sub ordered finite $\Gamma-$ near ring of ($\mathcal{N}_2$, +, Γ). □

5. CONCLUSION AND FUTURE SCOPE

In the paper, we proved that the homomorphic image and its pre-image of a co-fuzzy sub ordered finite $\Gamma-$ Near ring is also a co-fuzzy sub ordered finite $\Gamma-$ Near ring. Also the intersection and union of two co-fuzzy sub ordered finite $\Gamma-$ Near rings are co-fuzzy sub ordered finite $\Gamma-$ Near rings under certain conditions. Using this idea we have a scope to develop co-fuzzy ideals in ordered $\Gamma-$ Near rings.

6. REFFERENCES

[1]. Atanassuv.K.T,. (1986) institute Fuzzy sets. Fuzzy sets and systems Vol.20.87-96.
[2]. Booth.G.L., A note on G – near rings stud. Sci. Math.Hung 23(1988). 471-475.
[3]. Gouguen. J.A, L – Fuzzy sets. Journal of mathematical Analysis and applications Vol.18(1967), 145-174.
[4]. Nobusawa.N.,(1978) on generalization of the ring theory, Osaka Journal of mathematics, Vol.1, 185-190.
[5]. Ozturk M.A., Uokm.M.Jun : Y.B.,Fuzzy ideals in Gamma – Rings Turk J Math 27. (2003).369-374.
[6]. Pilz.a, Near rings, North Holland(1983).
[7]. Rosenfield.A., Fuzzy groups.J.Math.Anal.Appl.35-512-517(1973).
[8]. Raghamayi.S.,(2004). A study of L – Fuzzy and L – Vague structures on $\Gamma-$ near rings. Doctoral Thesis. JNT University, Hyderabad.
[9]. Satyanarayana.Bh.(1984) contributions to Near – ring Theory. Doctoral Thesis, Nagarjuna University, Guntur, India.
[10]. Satyanarayana.Bh., A note on G – near rings, Indian J – Math 41(3)(1999) 427 – 433.
[11]. Swamy K.L.N and Swamy U.M.,(1988) Fuzzy prime ideals of rings, Journal of mathematical Analysis and Applications Vol.134, No.1. 94-103.
[12]. Zadeh. L.A (1965) Fuzzy sets, Information and control Vol.8. 338-353.