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# Comparison between sliding mode and PID controller for fully interaction three-tank system

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## ABSTRACT

***Regulating the amount of liquids is a critical condition in many manufacturing processes. The tanks are also so linked together that the levels communicate and display a nonlinear conduct. The sliding mode control (SMC) is used to regulate the level of the coupling tank structure. We initially developed a mathematical model for a nonlinear multi input single output system. A simulation to track a non-linear three tank system model is performed using MATLB / SIMULINK. The performance of SMC is compared to PID controller.***

***Keywords***: *SMC, PID, Sliding surface, Chattering.*

## 1. INTRODUCTION

The sliding mode controller (SMC) is intended to be considered for hydraulic systems used in the pharmaceutical or chemical industries. Chemical reactions are expected to occur around pre-defined operating points in these systems, so liquid level controlling in these industries is a critical process. Sliding Mode Control (SMC) is a Variable Structure Control (VSC) derived technique. Originally, this control technique was studied by [1]. A twin tank system was studied [2] [3] [4] the liquid is delivered to the first tank and a method for the mathematical representation of the coupled tank system is suggested by the second author and then SMC is built for the operation. The SMC and then close loop performance of the system is contrasted with the PID controller performance in [5] writers. Three sliding mode control systems for the coupled tank system were suggested in [6] by the authors and the performance of the controlled system is then studied under system variations. Parameters and in the presence of intrusion from the outside the approach to minimizing chattering and testing robustness for three tank systems is studied in [7].

[8] For a non-linear three-tank configuration, the authors developed a sliding mode control technique. The key contribution in this paper is to develop the plant 's state model and transfer mechanism and a liquid level control sliding mode controller. The document is structured as the three-tank system is defined in section 2 and includes one subsection for the mathematical modeling of the three tank system. Section 3 explains the configuration of the SMC and PID controller for three tank systems, which is followed for observation by Section 5, Section 6, results and conclusion.

## 2. THREE TANK SYSTEM MODELLING

### 2.1 System description

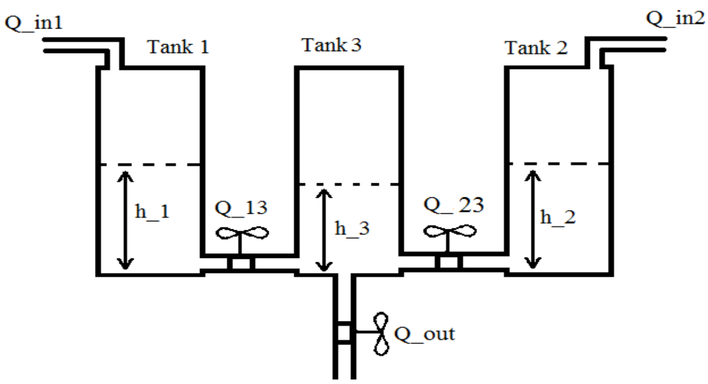


**Fig. 1: Three tank system**

Three coupled tanks as shown in Fig (1) are completely communicating with the system considered; tanks 1 and 2 are supplied by Qin1 and Qin2 and tanks 1 and 2 are supplied by the third by two cross-sectional pipes Sp. The liquid flows out through a third pipe of the Sp cross sectional area. Between tanks 1 and 2 (Q12) and tank 3 (Q23), and at the output of tank 3(Q out), manual valves are accessible. The liquid level is affected by the flows of input and output in any tank. As the device communicates fully with the liquid level in the other tanks, the liquid level in the other tanks is also affected. The main objective is to achieve the desired level in tank 3 by regulating the rate of input in tanks 1 and 2. The parameters in table 1 are given.

**Table 1: Parameters Values of the Three Tank System**

| Symbol | Parameters | Value |
|---|---|---|
| $Q_{in1}$ | Input to Tank 1 | 50 ml/sec |
| $Q_{in2}$ | Input to Tank 3 | 50 ml/sec |
| A | Area of tank | $0.0154 m^2$ |
| $S_p$ | Cross section of connecting pipes | $5 * 10^{-5} m$ |
| $C_1$ | Out flow co-efficient | 1 |
| $C_2$ | Out flow co-efficient | 1 |
| $C_3$ | Out flow co-efficient | 0.8 |
| g | Acceleration due to gravity | $9.81\ m/sec^2$ |
| $h_1$ | liquid level in tank 1 | 0.22 m |
| $h_2$ | liquid level in tank 2 | 0.22 m |
| $h_3$ | liquid level in tank 3 | 0.20 m |

**2.2 Mathematical Modelling of Three Tank System**

Assuming that $h_1 = h_2$, $h_1$and $h_2 > h_3$.The three-tank system is represented using the mass balance as given in Equation (1), (2), (3)

$$A\left(\frac{dh_1}{dt}\right) = Q_{in1} - Q_{13} \qquad (1)$$

$$A\left(\frac{dh_3}{dt}\right) = Q_{13} + Q_{23} - Q_{out} \qquad (2)$$

$$A\left(\frac{dh_2}{dt}\right) = Q_{in2} - Q_{23} \qquad (3)$$

Where $Q_{13} = S_p C_1(\sqrt{2g(h_1 - h_3)}$

$$Q_{23} = S_p C_2(\sqrt{2g(h_2 - h_3)}$$

$$Q_{out} = S_p C_3(\sqrt{2g(h_3)}$$

$$\frac{dh_1}{dt} = \frac{Q_{in1} - S_p C_1(\sqrt{2g(h_1-h_3)}}{A} \qquad (4)$$

$$\frac{dh_3}{dt} = \frac{S_p C_1(\sqrt{2g(h_1-h_3)} - S_p C_2(\sqrt{2g(h_2-h_3)} - S_p C_3(\sqrt{2g(h_3)}}{A} \qquad (5)$$

$$\frac{dh_3}{dt} = \frac{Q_{in2} - S_p C_2(\sqrt{2g(h_2-h_3)}}{A} \qquad (6)$$

Square-root nonlinearities are involved in the three-tank device equations and the flow rates become proportional to the square root of the tank level. A typical system operation may be around an equilibrium point in control engineering, and the signals may be viewed as small signals around the equilibrium. However, if the system operates around a point of equilibrium and if the signals involved are small signals, a linear system may be used to approximate the non-linear system. Such a linear system is similar to that considered within a narrow operating range by a non-linear system (Ogata 2004).

**2.3 Linear Representation**

Using Jacobian linearization, a linear model can be built around an equilibrium point. The continuous LTI representation describes the linearized framework. Where y and u represent variations around a pair-defined operating point (U0, Y0). The goal is to monitor the device around the operating point to which it is attached (U0, Y0).$U_o = [50 \quad 50]^T$ ml/ sec

$$Y_o = [0.22 \quad 0.20 \quad 0.22]^T \text{ m}$$

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (7)$$

$$y(t) = Cx(t) + Du(t) \qquad (8)$$

In order to generate matrices A and B

$$A=\begin{bmatrix} \frac{\partial f1}{\partial h1} & \frac{\partial f1}{\partial h2} & \frac{\partial f1}{\partial h3} \\ \frac{\partial f2}{\partial h1} & \frac{\partial f2}{\partial h2} & \frac{\partial f3}{\partial h3} \\ \frac{\partial f3}{\partial h1} & \frac{\partial f3}{\partial h2} & \frac{\partial f3}{\partial h3} \end{bmatrix} \quad B=\begin{bmatrix} \frac{\partial f1}{\partial Q_{in1}} & \frac{\partial f1}{\partial Q_{in2}} \\ \frac{\partial f2}{\partial Q_{in1}} & \frac{\partial f2}{\partial Q_{in2}} \\ \frac{\partial f3}{\partial Q_{in1}} & \frac{\partial f3}{\partial Q_{in2}} \end{bmatrix} f1 = \frac{dh1}{dt},\ f2 = \frac{dh2}{dt},\ f3 = \frac{dh3}{dt}$$

Obtained continues time state space model is represented below.

$$A=\begin{bmatrix} -0.07981 & 0 & 0.07981 \\ 0 & -0.07981 & 0.07981 \\ 0.07981 & 0.07981 & -0.2394 \end{bmatrix}$$

$$B=\begin{bmatrix} 64.94 & 0 \\ 0 & 64.94 \\ 0 & 0 \end{bmatrix}$$

$$C=[0 \quad 0 \quad 1]$$

$$D=[0 \quad 0]$$

In terms of the state space model, the transfer function matrix of the plant can be derived using the formula

$$\bar{T}=\begin{bmatrix} T_1(s) \\ T_2(s) \end{bmatrix}=\begin{bmatrix} \frac{T_3(s)}{V_1(s)} \\ \frac{T_3(s)}{V_2(s)} \end{bmatrix}=C(sI_3-A)^{-1}+D$$

Where $T_3(s)$, $V_1(s)$ and $V_2(s)$ are the Laplace transforms of the height $h_3(t)$ and the voltages $v_1(t)$ and $v_2(t)$.

$$\frac{T_3(s)}{V_1(s)}=\frac{5.1823s+0.4136}{s^3+0.3990s^2+0.0318s+0.0005}$$

$$\frac{T_3(s)}{V_2(s)}=\frac{5.1823s+0.4136}{s^3+0.3990s^2+0.0318s+0.0005}$$

Since equations (4) and (6) are the same as Qin1 = Qin2, C1 = C2 and h1 = h2, it can now be claimed that equations (4 and 6) are the same and can be rewritten as equations (9).

$$\frac{dh}{dt}=\frac{Q_{in}-S_pC(\sqrt{2g(h-h_3)}}{A} \qquad (9)$$

$$\frac{dh_3}{dt}=\frac{Q_{in2}-S_pC_2(\sqrt{2g(h_2-h_3)}}{A} \qquad (10)$$

After linearization around an operating point

$$A=\begin{bmatrix} -0.07981 & 0.07981 \\ 0.07981 & -0.1596 \end{bmatrix} B=\begin{bmatrix} 64.94 \\ 0 \end{bmatrix} \; C=[0 \quad 1] \quad D=[0 \quad 0]$$

$$\text{Now obtained transfer function is } T(s)=\frac{5.1823s+0.4136}{s^3+0.3990s^2+0.0318s+0.0005}$$

## 3. CONTROLLER DESIGN

### 3.1 Sliding mode controller design

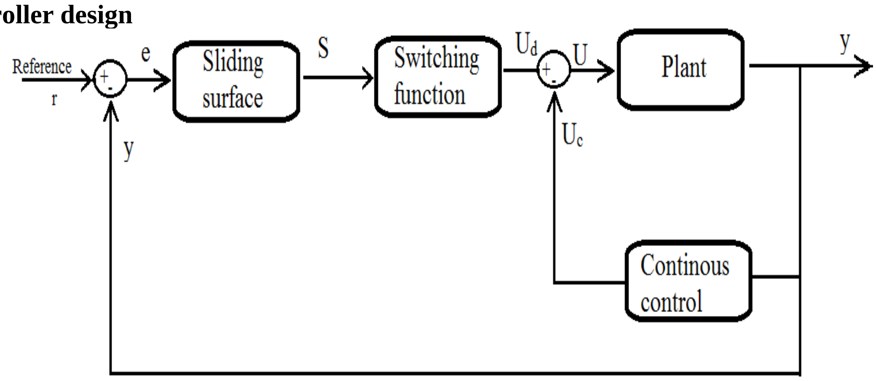


**Fig. 2: Closed loop block diagram for SMC three tanks system**

Figure (2) above represents the Closed Loop Block Diagram for the three tank method of SMC inaccuracy in modeling may have a strong impact on non-linear systems. Particular designs must specifically discuss them. The architecture of the sliding mode controller offers a comprehensive approach to the problem of preserving stability and output quality. SMC's idea is to follow a sliding surface over which the device will slide to its desired final value, as seen in Figure (3). A sliding surface has initially been selected in SMC, then an acceptable control law is built so that the control variable is pushed to its reference value. Two main components are based on SMC U (t).

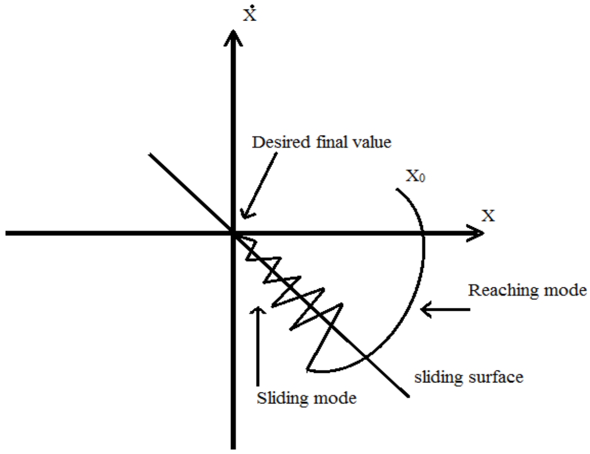


**Fig. 3: State trajectory and sliding surface in SMC**

1. A continuous component
2. Discontinuous section

$$U(t) = Uc(t) + Ud(t)$$

Uc(t) = Ueq(t) is the dominant equivalent controller that represents the continuous portion of the controller that holds the system output confined to the sliding surface.

SMC's Ud(t) consists of a non-linear portion comprising the switching portion of the controller's Ud control rule that is discontinuous across the sliding surface.

The goal is to make the error and derivative of error equal to zero in SMC. Error is defined mathematically as the difference between actual height and desired height e(t) = hd(t) − h3(t) where hd is the desired level of liquid and h3 is the level of liquid in tank 3 as the objective is to maintain the desired level of liquid in tank 3.

**3.2 Sliding function**

The construction of the sliding function S(t) is the most important step in the SMC design. The sliding feature for the nth order system is written as follows.

S(t) is the time varying surface, then S(x;t)=0 is the scalar function where the time varies.

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{(n-1)} e \qquad (11)$$

In equation(11), n represents the system 's order as the second order of the plant transfer function is n=2 and the second order system's sliding function can be expressed as given in equation(12).

$$S(t) = \left(\frac{d}{dt} + \lambda\right)^{1} e \qquad (12)$$

$$= \dot{e} + \lambda e$$

Where $\lambda$=0 is the slope of sliding surface.

**3.3 Stability condition**

Consider lyapnov function V= 1/2 s^2 where V ?? is negative definite, the device trajectory will be guided and the sliding surface will be attracted towards and stays sliding on it until asymptotically the origin is reached. The appropriate condition for the system's stability is now Now $\dot{V}$=S$\dot{S}$ .

$$V = \frac{1}{2}\frac{d}{dt}s^2 <= -|S| \qquad (13)$$

After substituting equation (12) in (13)

$$\frac{1}{2}\frac{d}{dt}(\dot{e} + \lambda e)^2 <= -|\dot{e} + \lambda e|$$

The basic discontinues control law of SMC is given by

$$U_d = Ksgn(S)$$

Where K is constant manual tuning parameters and responsible for reaching mode the disadvantage of sliding mode controller is chattering effect, toavoid chattering effect $U_d$ is designed as.

$$U_d = K\frac{S}{|S| + \delta} \qquad (14)$$

Where chattering is solved by $U_d$ and $\delta$ is chattering suppuration factor and is adjusted to eliminate chattering. When system remains on sliding surface that means e(t) is zero all times.

**3.4 Continues control law** $U_c(t)$

Consider equation (9 and 10)

$$\frac{dh}{dt} = \frac{Q_{in}-Q_{13}}{A} \quad (15)$$

$$\frac{dh_3}{dt} = \frac{Q_{23}+Q_{13}-Q_{out}}{A} \quad (16)$$

Where

$$Q_{13} = SpC\left(\sqrt{2g(h-h3)}\right) \text{ for } h > h_3$$

$$Q_{out} = SpC_2\left(\sqrt{2g(h3)}\right) \text{ for } h_3 > 0$$

Where h and $h_3$ are the liquid level of Tank 1, 2 where($h_1 = h_2$) and Tank 3, respec- tively, $Q_in$ is the inlet flow rate, A is the cross-section area of Tank. $Q_in > 0$ means that pump can only force water into the tank. Let

$$Z_1 = h_3 > 0, Z_2 = h - h_3 > 0$$

$$C_3 = SpC_2\left(\sqrt{2g}\right), \text{C= SpC}\left(\sqrt{2g}\right)$$

The dynamic model in equation (15 and 16) can be written as

$$\dot{Z}_1 = -C\sqrt{Z_1} + C_3\sqrt{Z_2} \quad (17)$$

$$\dot{Z}_1 = -C\sqrt{Z_1} + C_3\sqrt{Z_2} + \frac{Q_{in}}{A} \quad (18)$$

$$Y_1 = Z_1 \quad (19)$$

Then the goal is to regulate the system output ($h_3(t)$) to the desired value ($h_d$) Now sliding function S can be defined as follows.

From equation(12) that is$S(t) = \dot{e} + \lambda e$ where e is the error that is difference between desired value($h_d$) and present value($h_3$).continues control law ($U_c$) as follows.

$$S(t) = \dot{Z}_1 + \lambda(Z_1 - h_d) \quad (20)$$

By taking the time derivative of both sides of (20),

$$S(\dot{t}) = \dot{Z}_1 + \lambda(\dot{Z}_1) \quad (21)$$

Now by using equation (17) in (21).

$$S(\dot{t}) = -C\sqrt{\dot{Z}_1} + C_3\sqrt{\dot{Z}_2} + \lambda(\dot{Z}_1) \quad (22)$$

After substituting equation (17, 18) in equation (22)

$$\dot{S} = \frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \frac{C_3}{\sqrt{Z_2}}\frac{Q_{in}}{A} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}] \quad (23)$$

$$\frac{-C_3}{\sqrt{Z_2}}\frac{Q_{in}}{A} = \frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}] - \dot{S} \quad (24)$$

Where

$$\dot{S} = -Ksig(s)$$

$$Q_{in} = \frac{A}{C_3}\sqrt{Z_2}\left[\frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}]\right] + Ksig(s) \quad (25)$$

Above equation (25) gives the continuous control law $U_c(t)$ Control variable

$$U(t) = U_c(t) + U_d(t)$$

$$Q_{in} = \frac{A}{C_3}\sqrt{Z_2}\left[\frac{C^2}{2} - C_3^2 - \frac{CC_3\sqrt{Z_2}}{\sqrt{Z_1}} + \lambda[2C\sqrt{Z_2} - C\sqrt{Z_1}]\right] + \frac{S}{|S|+\delta} \quad (26)$$

### 3.5 PID controller design

Proportional (P), Integral (I) and Derivative (D) gains consist of the PID controller. The feedback control scheme of the PID is shown in Figure.5. Ki is an integral gain and Kd is a derivative gain, where Kp is a proportional gain. It can be shown that the error signal e (t) is used in a PID controller to produce the proportional, integral, and derivative behavior, weighing and summing the resulting signals to form the u(t) control signal applied to the plant model. A mathematical description of the PID controller is.

$$U(t) = Kp\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)\,d\tau + T_d\,\frac{de(t)}{dt}\right]$$

Where u (t) is the input signal to the plant model, the error signal e(t) is defined as e(t) = r(t) − y(t), and r(t) is the reference input signal.

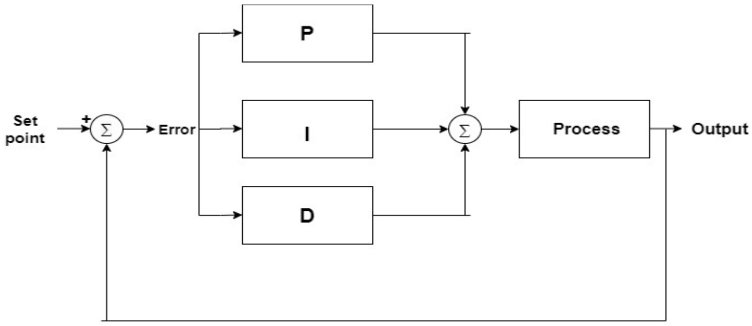


**Fig. 5: close loop Simulink model**

## 4. RESULTS

A demanding interaction approach is three tank level processes. The impact of tank-1 affects the shift in tank-2 and tank-3 levels. Likewise, the influence of tank-3 affects the variability of tank-1 and tank-2. In this phase, tank-2 is a level that interacts.

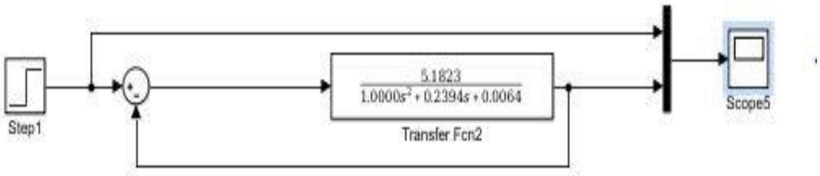


**Fig. 6: close loop Simulink model**

The close-loop simulated model of the three-tank system is described in figure (6) above. The response of the traditional controller close loop reference tracking re-response is shown in Figure (7) and the response of oscillations is observed in the initial re-response step. It suggests that, to minimize the oscillations, there is a need for an advanced controller. Therefore, SMC has been added to the device to eliminate these oscillations.

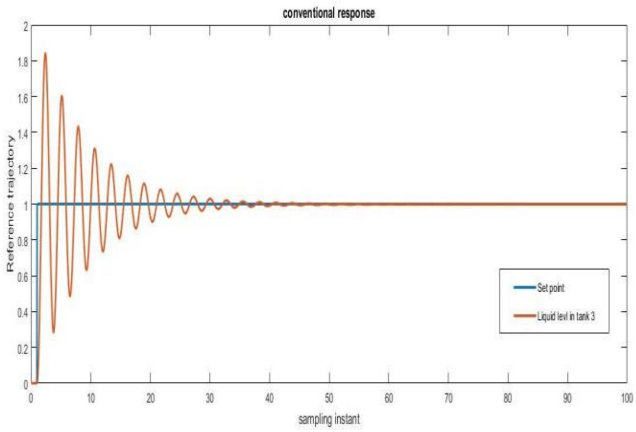


**Fig. 7: close loop response**

One of the former control methods is PID (proportional integral derivative) control. It has a basic system of control that was understood by the plant operators. For the three tank system shown in Figure 8 and the response of the PID controller as shown in Figure 9, Ziegler Nicholas open loop tuning is used.

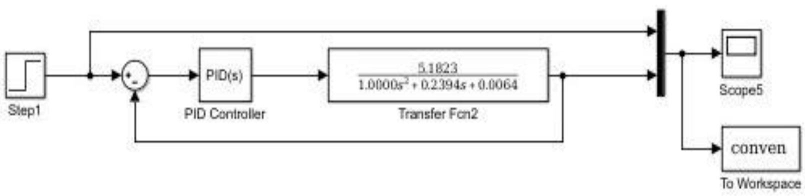


**Fig. 8: PID controller Simulink model**

 

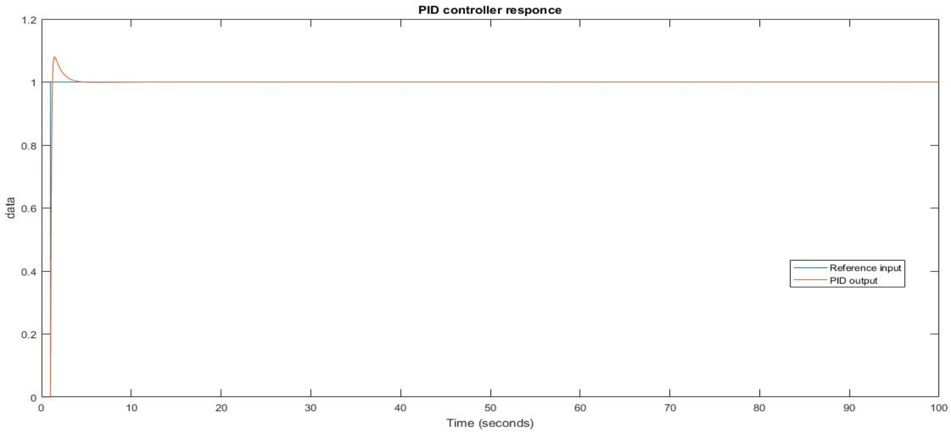


**Fig. 9: PID controller response**

The sliding mode controller simulator model for three tank system plants is shown in Fig (10) below. Figure ( 11) displays the simulation results for controllers with K=1200, $\delta$ = 0.1 and $\lambda$ = 2. SMC efficiency has been found to be effective compared to traditional controllers.

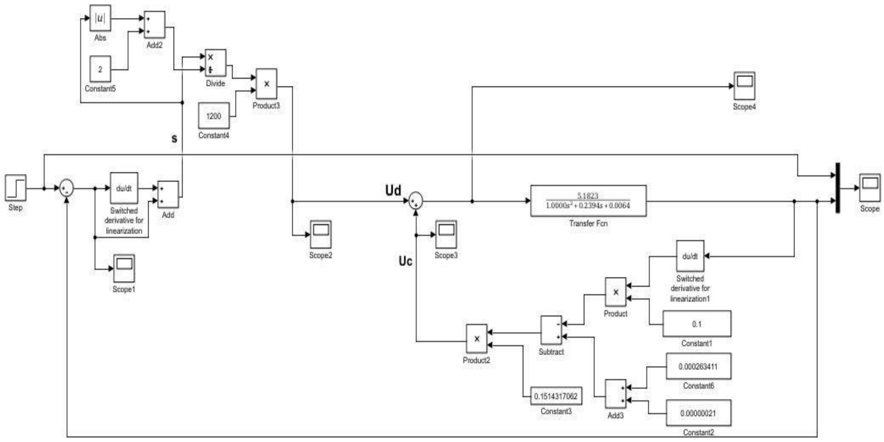


**Fig. 10: sliding mode controller Simulink model**

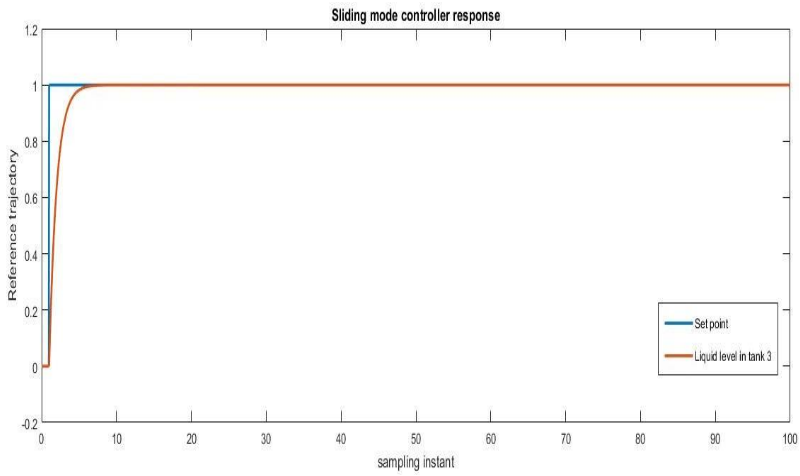


**Fig. 11: Sliding mode controller response**

## 5. CONCLUSION

For different adjustment parameter values, the sliding mode controller's response is obtained. By changing different parameter values of the sliding mode controller, the effect is evaluated, providing a better response without overshooting and reduced settling time. The response of the PID controller is contrasted with the response of SMC, and SMC shows zero overshoot and better response contrasted to the PID controller.

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