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## Symmetric and asymmetric mixed convection heat transfer through vertical channel with porous medium with different oxide nanofluids

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### ABSTRACT

*The problem of mixed convection fluid flow and heat transfer flow through a porous medium, over an infinite vertical plate is studied numerically. The effect of nanofluid properties and concentration and Prandtl numbers are considered in the present study. The governing non-linear partial differential equations of this phenomenon are transformed into non-linear algebraic system utilizing finite difference method. Numerical results for the velocity, temperature and as well as the skin friction, heat transfer is obtained and reported in tabular form and graphically for different values of physical parameters of the problem. The solutions for velocity and temperature are obtained. The effects of Grash of number, Prandtl number of nanofluid and porous parameter on velocity, and temperature, are presented graphically.*

**Keywords**— Mixed Convection, Porous Medium, Prandtl Number, Nanofluid, Symmetric Parameter, Asymmetric Parameter, Brinkman Number

### 1. INTRODUCTION

The flow in ducts with induced buoyancy has wide attention mainly for their engineering application in several electronics control devices. The free convection and mixed convection flows are generally adopted in passive or semi-passive thermal control systems. The fluid flow and heat transfer in inclined and vertical ducts has been the subject of several papers projected their work analytically in the latter decades by number of researches Aung, and Worku [1], Cheng, et.al[2], Hamadah, Wirtz[3], Lavine,[4] Barletta, Zanchini, [5] In addition, analytical solutions are often an opportunity to study the internal reliability of the mathematical models and of the approximations adopted, as well as to enlarge new theoretical results. Boussinesq approximation in duct flows has been proposed. The theoretical investigations on fully developed mixed convection in vertical or inclined ducts are often devoted to a description of the changes on the velocity profiles induced by buoyancy as well as to the determination of the conditions for the onset of flow reversal. The effect of viscous dissipation in the fluid behaviour is negligible is the general assumption in the several published works. This assumption holds whenever the fluid has a sufficiently high thermal conductivity, a sufficiently small Prandtl number and sufficiently high wall heat fluxes are present. Many researchers have been devoted to the analysis of the interplay between the effect of viscous dissipation and the effect of buoyancy with theoretical investigations. Many theoretical studies present either analytical or numerical solutions of the local momentum balance equations under the Boussinesq approximation. The viscous dissipation term from the mathematical point of view is a nonlinear term which present in the energy local energy equation. The nonlinear terms are caused due to both to inertia and to viscous heating are usually neglected when studying fully developed mixed convection flows in vertical channels. As a result, the investigation of such flows allows for a simple analytical determination of the velocity and temperature profiles when viscous heating is present, the solutions to be determined are less simple and analytical methods based either on perturbation expansions or on non-linear extensions of the Frobenius method are needed. Perturbation solutions are obtained by the researchers Becket (1980,1984)[7],[8], Barletta(1999)[9][10], Ingham, Pop (et.al) (1990)[11] with reference to channel flows. The channels with different heating orientation can be considered for the enhancement of heat transfer in different manners. Depending ahead the flow path of the main stream, the buoyancy flow can cause unlike kinds of stream reversals which will revise the complete flow characteristic and enhance the heat transfer in different protocol. The laminar mixed convection in vertical or inclined ducts has been commonly premeditated in the literature. Early exact solutions for the differential equations describing mixed convection in vertical channels and pipes were given by Ostrach(1954)[13] and Morton(1966)[14], Ostrach (1954)[13] used a linearly changing wall temperature profile for vertical flow. Morton (1966) [14] predicted parallel flow for small Ra numbers in downward heated flow. Quentiere(1973)[15] presented approximate analytical solutions for mixed convection between parallel plates. Rogers (1993)[16]

and Yao(1987)[17],(1983)[18],(1987)[19] studied mixed convection by the use of stability analyses. Broad effort has been done to resolve the flow field equation for mixed convection flows by mathematical methods. The common consequences were establish for heat transfer effects for mixed convection flows by Lawrence (1966)[20] and Zeldin and Schmidt(1972) [21].The numerical studies started being able to better resolve the velocity profiles of vertical pipe flows with mixed convection as shown in studies by Ingram(1988,1988)[22],[23], Aung(1986)[24] Cebeci (1982)[25], Heggs (1990) [26], Wang(1994) [27] and Hebchi and Acharya(1986)[28]. Modern numerical studies by Evans (1997) [29] and Chang (1993) [30] have focused on the oscillatory stream problem performance that be able to occur in mixed convection flows. Barletta (2005),[31] studied on the being of parallel flow for mixed convection in an inclined duct. Barletta and Zanchini (2001) studied mixed convection with variable viscosity in an inclined channel with approved wall temperatures. Boulama and Galanis (1962) [32] studied analytical result for fully developed mixed convection between parallel vertical plates with heat and mass transfer. Lavine (1989) [33] studied fully developed opposed mixed convection between inclined parallel plates. S. N. Gaikwad, Kamble S. S.,[34] studied the combined convection on vertical parallel plates with symmetry conditions with porous medium. The main objective of this paper is to study the effect of porous parameter and effect of the Prandtl number symmetric parameter and Grashof number on velocity, temperature, rate of heat and mass transfer characteristics of the problem with the flow through porous media with different oxide nanofluids .The set of governing equations are transformed into first order equations and solved with MATLAB software. The different parameters such as  $Gr, Pr, Q, k, F, Br$  are adopted and the flow and heat transfer characteristics are evaluated and the results are expressed graphically with reasonable conclusions.

## 2. MATHEMATICAL MODELING

The problem consists of two parallel infinite parallel plates separated by distance  $y = h$  The plates are held at  $y = 0$  and  $y = h$  on which the temperature is  $T_0 + \delta x$  and  $T_1 + \delta x$  respectively and here  $x$  is the distance measured vertically upwards as shown in the diagram. Two dimensional fully developed flow is considered between the vertical walls considered. The velocity of the flow at everywhere parallel to the  $x$  axis and function of  $y$  alone. Under the assumptions and invoking the Boussinesq approximations the governing equations of the motion of the fluid given by [34]

$$\mu \left( \frac{d^2 u}{dy^2} - \frac{u}{k} \right) = \frac{dp}{dx} + \rho g - \rho g \beta (T - T_\infty) \quad (1)$$

$$k \left( \frac{d^2 T}{dx^2} + \frac{d^2 T}{dy^2} \right) = \rho C_p u \frac{dT}{dx} - \mu \left( \frac{du}{dy} \right)^2 \quad (2)$$

Where  $\mu$  is the viscosity of fluid,  $u$  is the velocity,  $k$  is the thermal conductivity,  $\rho$  is density of fluid,  $g$  is gravitational force,  $\beta$  is the coefficient of thermal expansions,  $T$  is temperature, and  $T_\infty$  is reference temperature.

The boundary conditions are

$$u = 0, T = T_0 + \delta x \text{ on } y = 0 \quad (3)$$

$$u = 0, T = T_1 + \delta x \text{ on } y = h \quad (4)$$

Using the following dimensionless quantities

$$U = \frac{v}{h} u, T = T_0 + \delta x + \delta h \theta, Y = \frac{y}{h} \quad (5)$$

The equations (1) and (2) become

$$\frac{d^2 U}{d^2 Y} + \text{sgn}(\delta) Gr \theta = Q + \Upsilon U \quad (6)$$

$$\frac{d^2 \theta}{d^2 Y} = Pr U + \text{sgn}(\delta) Br \left( \frac{dU}{dY} \right)^2 \quad (7)$$

Where  $\text{sgn}(\delta) = \text{sign}(\delta)$ ,

$$Q = \frac{h^3}{\mu v} \left( \frac{dp}{dx} + \rho g \right), \text{ (Grashof number) } Gr = \frac{g \beta \Delta T \delta h^3}{v^2}, \text{ (Porous parameter) } F = \frac{d^2}{k}$$

$$\text{(Prandtl number) } Pr = \frac{\mu C_p}{k}, \text{ (Eckert number) } E_c = \frac{v^2}{C_p \Delta A d^2}, \text{ (Brinkman) } Br = E_c Pr$$

The viscous heating is characterized by the Eckert number ( $E_c$ ) and it is very small. This can be seen in its proper perspective that is only at very high rates of shearing wall viscous dissipation be a significant factor. Accordingly, the sign of  $\delta$  as negative so that there is a heating from below, the equations (6) and (7) are transformed into

$$\frac{d^2 U}{dY^2} - Gr \theta - FU - Q = 0 \quad (8)$$

$$\frac{d^2 \theta}{dY^2} - Pr U + Br (U')^2 = 0 \quad (9)$$

The boundary conditions are transformed into

$$U = 0, \theta = 0; \text{ at } Y = 0 \quad (10)$$

$$U = 0, \theta = \sigma; \text{ at } Y = 1 \quad (11)$$

Where  $k = \frac{(T_1 - T_0)}{\delta h}$  which decides the symmetric parameter

## 3. THERMOPHYSICAL PROPERTIES OF NANOFLUID

Metal and metallic oxide nanoparticles when dispersed in small quantities in a base liquid such as water, ethylene glycol, etc are observed to possess higher values of thermal conductivity compared to base liquid. Experiments conducted by Choi[35] with Carbon Nano Tubes (CNT) in engine oil at 1.0% volume concentration, obtained thermal conductivity enhancement of 160% of

the base liquid value. The determination of thermo-physical properties of metal nanoparticles like Cu and metal oxides such as Al<sub>2</sub>O<sub>3</sub>, TiO<sub>2</sub>, CuO, SiO<sub>2</sub>, etc. in different base fluids and the parameters influencing them is undertaken by various investigators. The well-known theoretical models of Maxwell [36] and Hamilton and Crosser [37] predicted lower values of thermal conductivity compared to experimental observations at higher temperatures. Hence experimental determination of thermal conductivity of various nanofluids is undertaken as the theoretical models are still pursued. It is fairly established that the thermal conductivity and viscosity of nanofluids is influenced by concentration and temperature. The influence of particle size on the properties has not been considered in the development of regression equations by these investigators. The thermal conductivity and viscosity of various nanofluids are determined experimentally at different particle sizes, temperatures and concentration by many. Experiments for the estimation of nanofluid convective heat transfer coefficients are undertaken with particle size in the range of 20 to 170nm, temperature range of 20-70°C, volume concentration of less than 4.0% with Al<sub>2</sub>O<sub>3</sub>, Cu, CuO, SiC, TiO<sub>2</sub>, ZrO<sub>2</sub>, etc nanoparticles dispersed in water. The viscosity and thermal conductivity data of metal and their oxide nanoparticles dispersed in water and available in literature is used in the development of regression equations. The nanofluid density and specific heat are determined using the mixture relations given by

### 3.1. Density, $\rho_{nf}$

Applying the principle of mass conservation of the two species in a finite control volume of the nano fluid, the nanofluid density can be obtained from the relation

$$\rho_{nf} = \phi\rho_p + (1 - \phi)\rho_w \quad (12)$$

where  $\phi$  is the volumetric fraction of nano particles in the base fluid.

### 3.2. Specific Heat, $C_{p_{nf}}$

The thermal conservation of energy of the two species in a finite control volume will yield the overall

$$C_{p_{nf}} = \frac{(1-\phi)(c_p)_w + \phi(\rho c_p)_p}{(1-\phi)\rho_w + \phi\rho_p} \quad (13)$$

The bulk material properties listed in are used in the development of regression equations for density and specific heat. Vajjha and Das [38] considered specific heat ratio to be dependent on concentration and bulk temperature in the development of regression equation. Hence equations applicable for metal and their oxide nanoparticles dispersed in water are

$$\rho_{nf} = \rho_w(0.9973 + 0.03479\phi + 0.0000619T_b) \quad (14)$$

$$C_{p_{nf}} = C_{p_w}(1.036 - 0.0298\phi - 0.001037T_b) \quad (15)$$

with an average deviation of less than 3.7% where  $\phi$  is in percent and  $T_b$  in °C. Nguyen et al. [39] conducted experiments for the determination of viscosity of Al<sub>2</sub>O<sub>3</sub> and CuO nanofluids in water at different concentrations and particle sizes in the ambient temperatures of 22 and 25°C. Experiments revealed the viscosity of Al<sub>2</sub>O<sub>3</sub> with particle sizes of 36 and 47nm and that of CuO with 29nm size predicted close values for volume concentration less than 4%. The dependence of viscosity ratio on concentration and particle diameter by Sarit et al. [40],[42] and specific heat ratio on concentration and bulk temperature by Vajjha and Das [38] has been considered in the development of regression equations. Based on these observations, it is can be stated that viscosity and thermal conductivity of nanofluids are influenced by volume concentration, temperature and particle size. Hence, the available experimental data in literature for a maximum temperature of  $T_{max}=70^{\circ}$  C and particle size of  $d_{max} = 50$  nm is used to develop regression equation valid for various metals and their oxide nanoparticles dispersed in water.

### 3.3 Thermal Conductivity and viscosity

$$k_{nf} = k_w(0.9808 + 0.0142\phi + 0.003883T_b - 0.00068d_p) \quad (16)$$

$$\mu_{nf} = \mu_w(0.9042 + 0.1245\phi + 0.0043d_p - 0.0001206T_b) \quad (17)$$

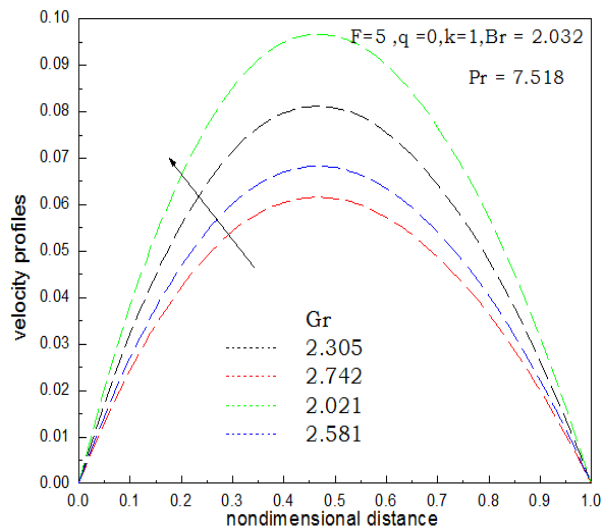
For  $0 \leq \phi \leq 3.7$ ,  $20 \leq T_b \leq 70$ ,  $20 \leq d_p \leq 150$  with a maximum deviation of less than 10% where  $\phi$  is in percent,  $T_b$  in °C and  $d_p$  in nm. Yurong He et al. [41] observed the shear viscosity to increase and thermal conductivity to decrease with particle size.

## 4. NUMERICAL METHOD

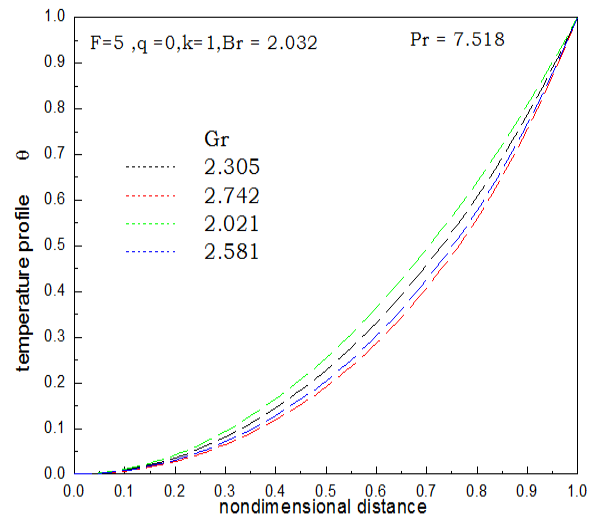
The nondimensional nonlinear differential equations (8) and (9) are solved with the boundary conditions equations (10) and (11) with commercial MATLAB software by using finite difference method. Initially the equations are converted into first order differential equations and apply the explicit finite difference method. The properties of nanofluids are predicted with help of equations available equations (12),(13),(14),(15),(16),(17) for the different oxide fluid Al<sub>2</sub>O<sub>3</sub>, SiO<sub>2</sub>, ZnO, CuO. The properties which are evaluated for the present study Pr=7.518, 6.474 8.478 6.674; Gr=2.305, 2.742, 2.021, 2.58; Br= 2.032, 1.902, 2.1795, 1.940; Ec=0.2703, 0.2572, 0.2869, 0.2388, and other parameters symmetric and unsymmetrical values are k = 0.0, 0.5, 0.75, 1; Q=0, 5, 10, 15; F = 5, 10, 15, 20. are considered present study.

## 5. RESULTS AND DISCUSSIONS

The fluid flow and temperature distribution of the results are represented with graphically and discussed as follows. Fig 1.0 & fig.2.0 representation of variation of velocity profiles and temperature profiles with different values of Gr [ 2.305, 2.74, 2.021, 2.581] at  $F=5$   $Q=0$   $Br = 2.032$   $Pr = 7.518$ . The velocity profiles are reducing with increasing Gr values because of the buoyancy effect. Similarly, the from the Fig.2.0 the temperature profiles are high gradient at  $Gr = 2.74$  due to the buoyancy effect. The temperature profiles are increases in its gradient from  $Gr = 2.021$  to  $Gr = 2.74$ .



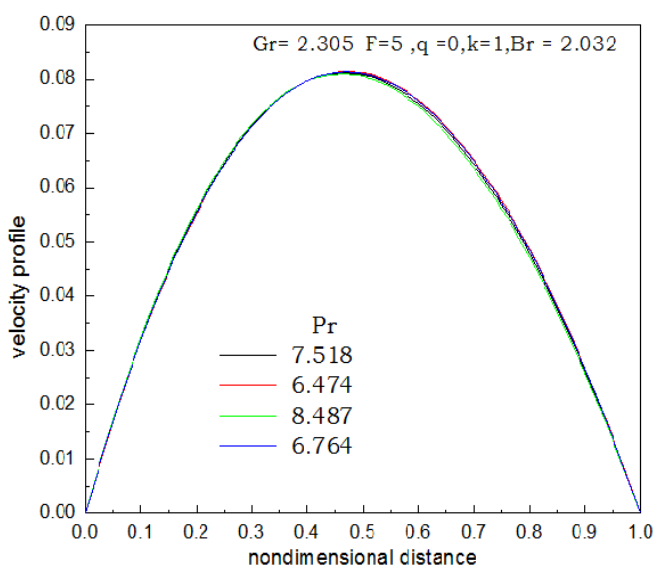
**Fig. 1: Representation of velocity profiles with non-dimensional distance with different values of Gr**  
 $F = 5, q = 0, k = 1, Br = 2.032, Pr = 7.518$



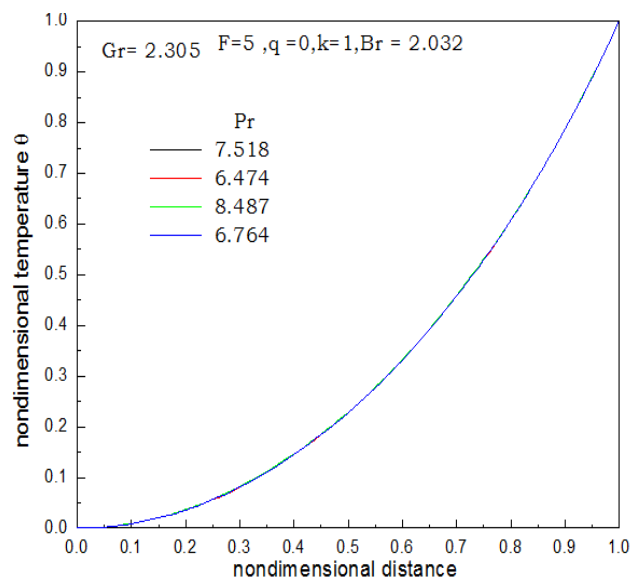
**Fig. 2: Representation of velocity profiles with non-dimensional distance with different values of Gr**  
 $F = 5, q = 0, k = 1, Br = 2.032, Pr = 7.518$

From the Fig3.0 & Fig4.0 represents the velocity and temperature profiles with different Prandtl numbers at  $Gr = 2.305, F=5, Q = 0, Br = 2.032$ . From the diagram there is no much variation in velocity and temperature profiles. Fig 5.0 & 6.0 is drawn for the variation in the flow field and temperature field at different porous parameter  $F = 5, 10, 15, 20$  at the  $Gr = 2.32, Pr = 7.518, Br = 2.032, k = 1$ . The velocity fields are decreases with increasing in the porous parameter from 5 to 20 and at high porous parameter the velocity profiles are very less because of the reducing in the velocity flow. The fig 6.0 describes the temperature fields with variation in porous parameter. From the fig the temperature is merge and no significance effect on temperature fields with porous parameter. Fig 7.0 & fig 8.0 reports the effect of velocity and temperature files in vertical plates for various values of  $Q$  varying from the 0, 2, 5, 10 keeping all the values are constant.

From the fig7.0 the velocity of the fluid is in linear motion  $Q = 0$ , and the profile of the velocity is gradually changing from linear to parabolic with increasing  $Q$ . The fig 8.0 represents the variation of temperature with nondimensional distance. The temperature profile is linear at  $Q = 0$  and its profile is changing from linear to sinusoidal with increasing the  $Q$  and it is completely sinusoidal at higher  $Q$  values. Fig 9 and 10 represents the variation of flow field and temperature files with variation of symmetry parameter  $k$ , the values of  $k$  is varying from 0 to 1. When  $k$  is equal to 1 the both plates are at same temperature is consider as symmetric condition and for other values of the plates are at temperatures is considered as the unsymmetrical condition. The values of  $k = 0$  only one plate is heating and other plates is at not adiabatic. Fig 9.0 represents the velocity profile for different values of  $k$  0, 0.5, 0.75, 1 the velocity profiles are reverses in the nature and varied from 0 to -0.16 in parabolic nature and the temperature field is increases with increasing the value of  $k$ . The fig 10 represents the variation of temperature field with the symmetric parameter. the temperature field is increases with increasing in the  $k$  values.



**Fig. 3: Variation of velocity profile with nondimensional with Prandtl number at**  
 $Gr = 2.305, F = 5, Q = 0, k = 1, Br = 2.032$



**Fig. 4: variation of temperature profile with non-dimensional with Prandtl number at**  
 $Gr = 2.305, F = 5, Q = 0, k = 1, Br = 2.032$

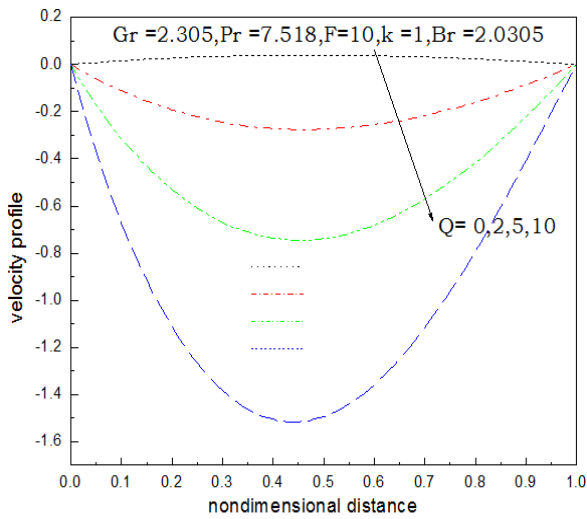


Fig. 5: Variation of velocity profiles with distance for different values of  $F$  at  $Gr = 2.3$   $Pr = 7.518$   $Br = 2.032$   $k = 1$

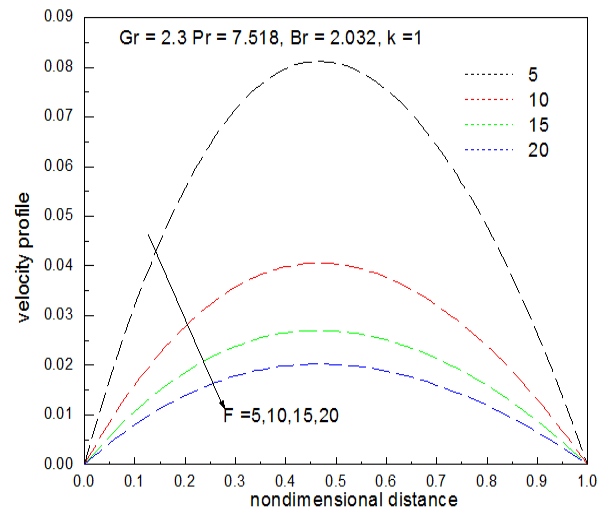


Fig. 6: Variation of temperature profiles with distance for different values of  $F$  at  $Gr = 2.3$   $Pr = 7.518$   $Br = 2.032$   $k = 1$

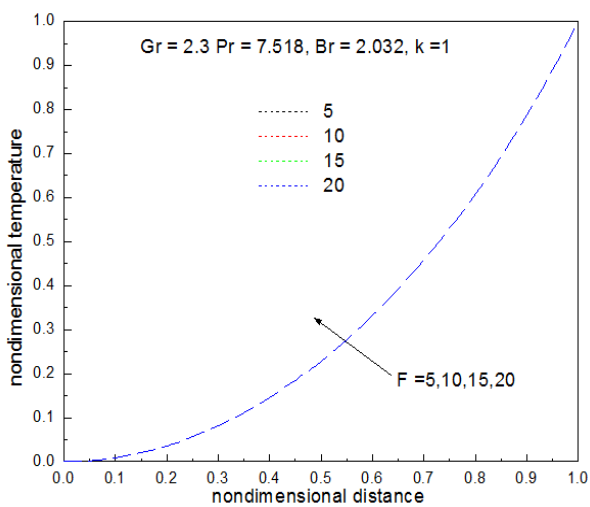


Fig. 7: Variation of velocity with distance for different values of  $Q$  at  $Gr = 2.305$ ,  $pr = 7.518$ ,  $F = 10$ ,  $k = 1$ ,  $Br = 2.305$

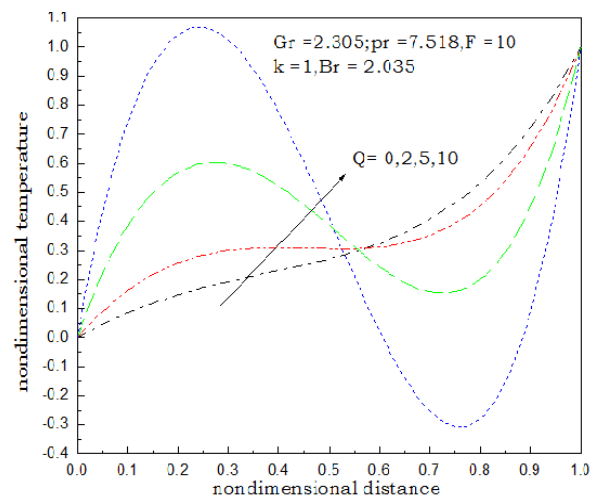


Fig. 8: Variation of temperature with distance for different values of  $Q$  at  $Gr = 2.305$ ,  $pr = 7.518$ ,  $F = 10$ ,  $k = 1$ ,  $Br = 2.305$

## 6. CONCLUSIONS

This paper reports numerical study of the mixed convection through vertical channel in presence of porous media with  $Al_2O_3$ ,  $SiO_2$ ,  $ZnO$ ,  $CuO$  different oxide nanofluid with constant Brinkman number. The effect of porous parameter ( $F$ ), the Grashof number ( $Gr$ ), symmetric parameter ( $k$ ), buoyancy parameter ( $Q$ ), Prandtl number ( $Pr$ ) on velocity, temperature, presented graphically. From the above result and discussions, it concludes that in the absence of the viscous dissipation, the velocity increases with an increase of porous parameter, and decreases as the Grashof number increases. Also, with an increase of porous parameter, decreases the temperature. The velocity profiles are decreases with increasing the Prandtl number. The symmetric parameter  $k$  is most effective on the velocity profile and temperature profiles. The velocity profiles and temperature are increases with the values of  $k$ .

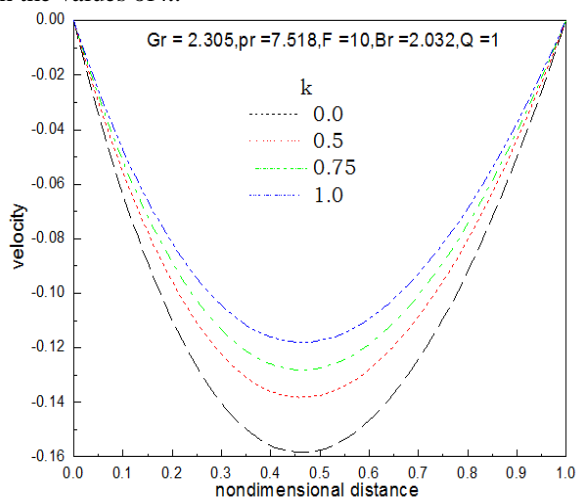


Fig. 9: Variation of velocity profiles with distance for different values of  $k$  at  $Gr = 2.305$ ,  $Pr = 7.518$ ,  $Br = 2.032$ ,  $F = 10$

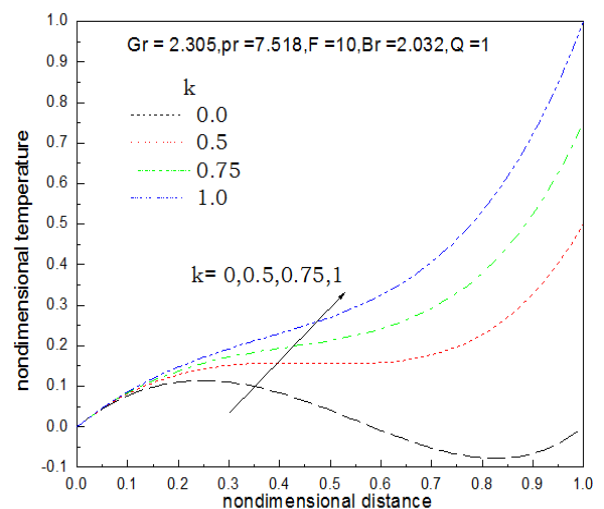


Fig. 10: Variation of velocity profiles with distance for different values of  $k$  at  $Gr = 2.305$ ,  $Pr = 7.518$ ,  $Br = 2.032$ ,  $F = 10$



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