



## Hawking Radiation and its Implications as the Information Paradox

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### ABSTRACT

*In this project, a precursory knowledge of black holes, their formation and a blackholes event horizon will be given. Hawking Radiation will be introduced, along with its discovery and proof of its existence. Quantum Field Theory is required for understanding the proof and thus a small description of Quantum Field Theory is given. The Bogoliubov Transformations required for explaining Hawking's proof will be outlined and it implications will be pointed out. The way to physically perceive the phenomenon of Hawking Radiation is also discusses. The Black Hole Information Paradox is also introduced and explained using the no-hair theorem, its proposed resolutions are outlined and finally its implications are briefly described.*

**Keywords:** Black Holes, Hawking Radiation implications on Paradox

### 1. LIFE OF STARS AND FORMATION OF BLACK HOLES

The Universe is full of matter. Clouds of gases of light elements like helium and hydrogen float around the universe in huge masses and varying densities. After reaching a threshold density, the net gravitational force of the cloud on each particle overcomes the momentum of each individual particle causing all the particles of the gas clouds to be drawn to the centre of gravity of the gas cloud system. As all the particles of the gas cloud come together the force due to gravity on each particle increases and causes nuclear fission. This nuclear fission releases energy in an outwards motion, and consequentially counteracts the inward pull of the massive gravitational force. This is how a star is formed.

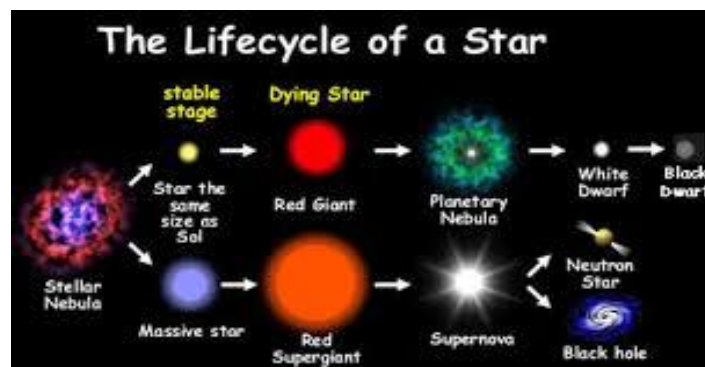


Fig. 1: Life of a Star

Initially, the force produced by the nuclear fission reactions are so strong that they overcome the gravitational force and the volume of the start increases to a peak, this stage in the star's life is called a red-giant. After this point the nuclear reaction are not enough to sustain the growth and the nuclei undergoing fission reactions in the core are not able to produce enough energy to counteract the gravitational force. The effect of the gravitational force exceeds the effect of the nuclear fission and the volume of the star starts to decrease and collapse into itself. Depending on the mass of the star, it can die in two ways. For stars less than 8-10 Solar Masses, it will become a neutron star. For stars of more than 10 Solar Masses, it forms a black hole.

$$\text{Gravity, } G = \frac{gF = M_1 M_2}{r^2},$$

where,

M1= the mass of one nucleus in the core of the star.

M2= the mass of the star,

$$\text{If } M_2 < 10 \text{ SM, then } G < 1.9 \times 10^{31} \text{ kgf.}$$

$1.9 \times 10^{31}$  kgf is approximately the threshold amount of force required to cause the volume keep continuously decreasing, till it reaches 0 eventually. At this point density,  $D = m/v$ , where volume ( $v$ ) is 0. Therefore, the density is infinity. This is called a singularity, and this singularity is called a black hole.

## 2. EVENT HORIZON OF A BLACK HOLE

The gravitational field of a black hole is so strong that nothing in space-time can escape its gravitational field after a distance  $R$ . Beyond the circle inscribed by the radius  $R$ , matter and radiation of any form is impossible due to the immense strength of the gravitational field. This is called the event horizon of the black hole.

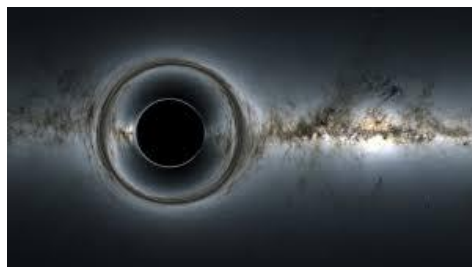


Fig. 2: Event Horizon of Black Hole

At the brink of the event horizon, because of the effect of gravity on time, an observer *of* the event horizon would experience a faster passage of time compared to the object moving towards the event horizon. The observer will experience a slower passage of time as compared to an observer outside the given system. This is according to the phenomenon stated by Einstein's Theory of Relativity called Time Dilation. For example, consider two astronauts, A and B. Let A be the astronaut going into the blackhole, and let B be the astronaut who is observing the system. Since there is a higher gravitational potential at A's position, the passage of his time will be slower compared to the passage of time according to the rest of the universe outside the influence of the gravity. This implies that since the passage of time in A's timeframe is slower to the rest of the universe, he will observe the universe's life pass by in 'fast-forward' and B(observer who's timeframe is outside influence of immense gravity) will see A in 'slow-motion'.

**Time dilation** is a difference in the elapsed **time** measured by two clocks, either due to them having a velocity relative to each other, or by there being a gravitational potential difference between their locations.

Fig. 3: Definition of Time Dilation

## 3. HAWKING RADIATION AND ITS DISCOVERY

The mass of a black hole is logically a constant. It would be approximately equal to the mass of the star it arose from at the exact moment before forming a black hole. Since the sun's mass was large enough that the gravitational field it produced caused its collapse, technically nothing should be able to escape the grasp of the gravitational field, not even matter or energy in the form of radiation. This raises the question, what is hawking radiation and what is it radiating?

In 1974, astrophysicist and cosmologist, Stephen Hawking wrote a paper, which outlined the existence of a possible radiation from blackholes, therefore meaning that net mass-energy quantity of a black hole is decreasing. The most popular description of Hawking Radiation goes like this:

- It is popularly accepted that spacetime is seething with activity, with constant generations and annihilations of virtual matter-antimatter pairs, briefly borrowing and releasing energy to space time itself.
- When such annihilations happen near a the event horizon, due to the immense gravity of black holes, it is possible for one of these virtual particles to fall past the event horizon.
- When one of these virtual particles fall into the black hole, it departs some of its energy to the other, therefore allowing it to escape, and so a form of 'negative-energy' gets added to the black hole.
- This negative energy is a way of showing the energy compensation for the loss of energy experience when the particle enters the event horizon. The adding of negative energy is basically subtracting energy from the black hole.
- This supposedly happens by the black hole's theorized loss in mass.
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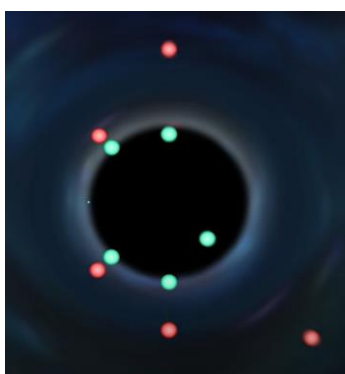


Fig. 4: Visual representation of Virtual Particles and their interactions with Black Holes

### 3.1 Quantum Field Theory

Before getting into the details of Professor Stephen Hawking's proof, let us first look of some basics of Quantum Field Theory.

- Space is full of quantum fields. It can vibrate with many frequencies, just like a guitar string.
- A particle is depicted as a mode on the guitar string, just like the many notes on a guitar string
- Just like a note on a guitar string, a particle tends to be comprised of many vibrational modes.
- These modes still exist in the absence of real particles but keep fluctuating due to quantum uncertainty. These fluctuations are construed as virtual particles.
- The modes of quantum fields can have negative frequencies meaning that they can travel backwards in time and usually correspond to antiparticles.

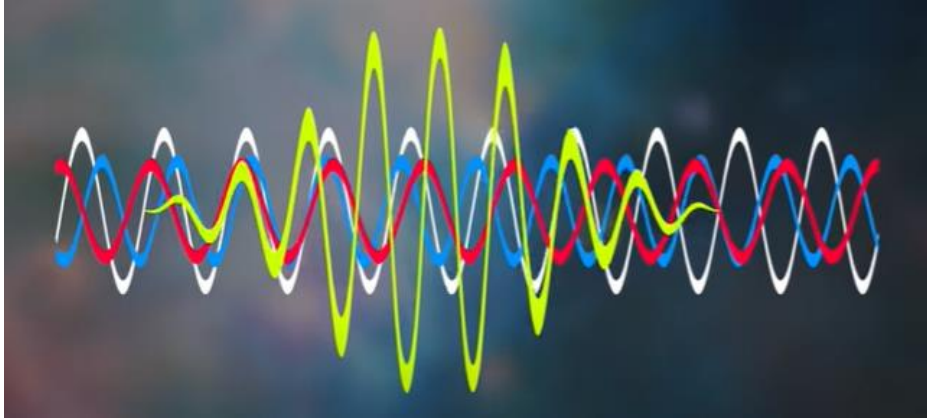


Fig. 5: Fluctuations in Quantum Fields and their comprising Modes

### 3.2 Hawking's Proof

In a quantum state, a quantum field experiences a balance in positive and negative modal frequencies. This can be understood crudely as a balance between virtual matter and antimatter particles. These virtual particles virtually annihilate, as mentioned before. All of this holds up true in flat space-time but may change a bit when introducing a curve in space-time, or an extremely strong gravitational field.

Curvature in spacetime creates an imbalance in the virtual field modes by introducing horizons. Horizons cut off access to certain field modes, hence creating an imbalance in the field modes that defines the vacuum. The immense gravity of black holes would create a huge spacetime curvature, which would in turn wreak havoc on any test quantum field he would have to analyze. To further calculate the implications of this complication he would have to understand the effect of a large gravitational field on a quantum field, which meant the unification of quantum field theory and general relativity. This did not exist and still does not exist. His brilliant work around is as follows:

- He defined a null geodesic, a lightspeed line indicating the position through time from far in the past to far in the future. He assumed it to pass through an instant right before the formation of a black hole.
- Hawking imagined a simple quantum field traversing this path, a field that was in quantum state before the formation of the event horizon.
- He found that the close encounter with the black hole disturbed the vibrational modes that was fundamental to the field.
- When calculating the modal frequencies after the formation of the event horizon he found that the resultant frequencies in flat space looked like the modal frequencies of real particles.
- Therefore, a distant future observer would see radiation coming out of the black hole.

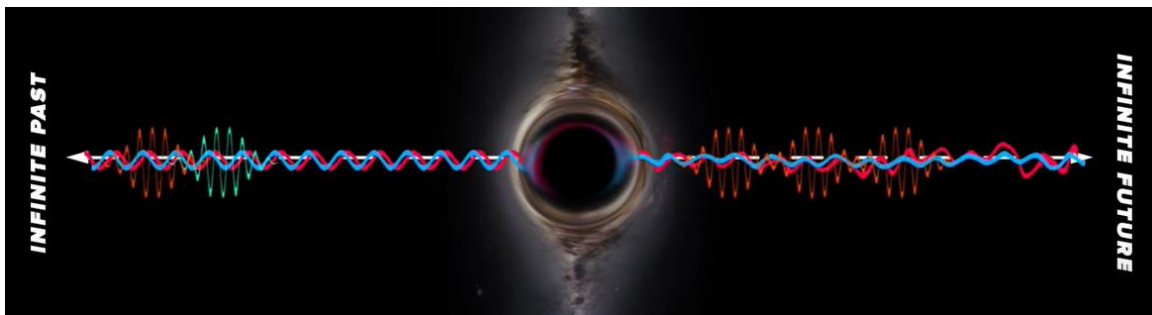


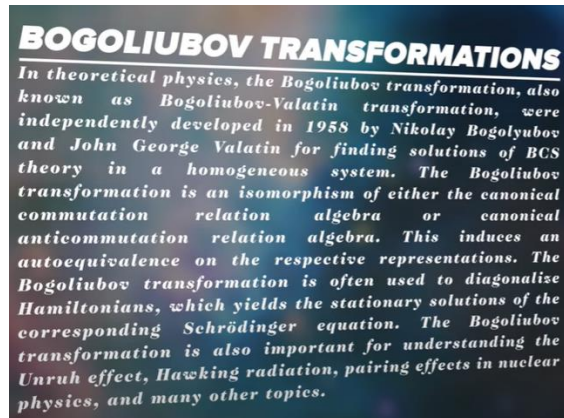
Fig. 6: Visual Representation of Hawking's Proof

He brilliantly demonstrated how the quantum field would behave when describing real and virtual particles in the presence of a blackhole, by measuring the fields in the distant past and future, where calculations were possible, but the effect of the black hole, still visible.

However, to understand the implications of the spacetime curvature caused by a blackhole on the quantum fields, he would require an uneasy marriage of general relativity and quantum mechanics. Without the existence of a quantum field theory, he used what was known as the 'Bogoliubov Transformations'.

### 3.3 Bogoliubov Transformations

- They can be used for finding the effect of curved spacetime on quantum fields, by drawing functional connections between values in two regions of flat spacetime.
- In this scenario Hawking used it to describe a ‘mixing’ of positive and negative modes that are caused due to space time curvature in the quantum field geodesic he assumed previously.



**Fig. 7: Definition of Bogoliubov Transformations.**

The physical interpretations of the mixing of positive and negative modal frequencies vary and aren't tied down to one solution. Since Hawking's calculations were purely in theory, nothing can be cemented unless proved, and as the trend follows, technology is far behind theoretical science.

### 4. IMPLICATIONS OF THE BOGOLIUBOV TRANSFORMATIONS:

- Hawking's own interpretations of his calculations using the Bogoliubov transformations entails the scattering of certain modal frequencies, will others seem to be lost to the gravity experienced beyond the event horizon. Some are nudged off their timeline by the gravitational field of the forming event horizon and some are lost to it.
- Meanwhile, some particles remain unscathed without scattering. The quantum state must be reconstructed from the remaining modes that were not lost to the newborn event horizon. Upon construction modes are formed which resemble modes of real particles, as mentioned before. The missing modes might be able to give us information of the resultant hawking radiation.
- Blackholes tend to scatter the modes whose wavelength is of roughly the same order of its own radius (Schwarzschild Radius). The radiation that emerges is distorted in the same wavelength range as the radius and so it forms ‘wave-packets’ or particles that have De Broglie wavelengths about as large as the event horizon radius. This implied that the more massive the black hole, the larger the wavelength of the radiation.
- Stephen Hawking graphed the results of the frequency-Schwarzschild Radius and noticed something spectacular. The graph resembled thermal radiation. Black holes would have an apparent heat glow that was dependent on their mass. Through further calculations he further deduced that it should be directly proportional to the surface are of the surface area of the event horizon.

**HAWKING TEMPERATURE**  

$$T = \frac{\hbar c^3}{8\pi G M k_B} = \frac{6.17 \times 10^{-8} K}{\text{mass of black hole (in Suns)}}$$

**Fig. 8: Relation derived by Hawking.**

- This meant that large black holes would radiate extremely slowly, and as the blackholes shrunk its rate of radiation would increase and ultimately, the smallest blackholes would radiate explosively.
- The premise of matter and antimatter pairs being pulled apart by the event horizon is backed by Stephen Hawking's calculations. His math describes the splitting or mixing of pure positive and negative frequency modes. Therefore, we can interpret the mixing to be the ‘translation’ of the quantum information of the virtual particles to real particles and for those modes that escape, there exists a corresponding set of modes, linked by quantum entanglement that are trapped behind the event horizon. This may be drawn in parallel to the particles that were ‘swallowed’ by the event horizon.
- Although the radiation will have De Broglie wavelength the size of Schwarzschild radius, there is huge quantum uncertainty in pinpointing where on the event horizons surface area the radiation radiates from, making it more of a global radiation rather than local.



Fig. 9: Implications of Bogoliubov Transformations

### 5. PERCIEIVING HAWKING RADIATION

The radiations produced globally, meaning simultaneously from the entire surface area rather than one position, is only visible to far-off observers. It will be extremely, tediously slow, and take almost forever to observe.

Hawking radiation would most likely be in the form of highly concentrated gamma radiations, so it would be invisible to the human eye. The only way humans could observe the ‘explosion of a blackhole’ was if it evaporated in a nearby solar system. To an observer in free fall through the event horizon, the radiation would not be visible. However, an astronaut with a hypothetical ‘jetpack’ hovering above the event horizon at a certain distance, one can observe particulate radiation. This radiation is called Unruh Radiation.

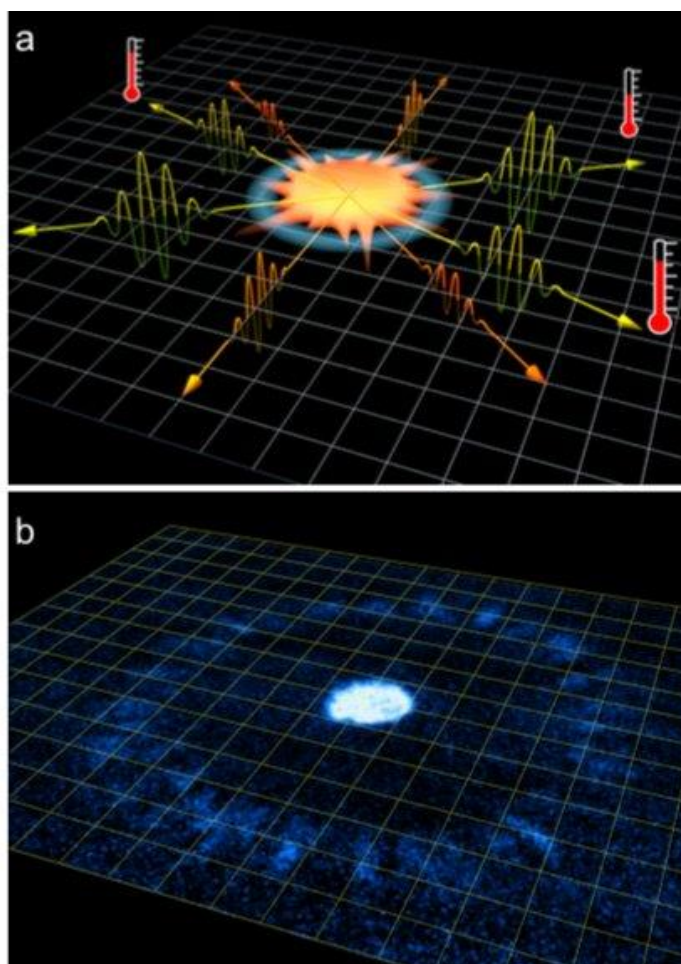


Fig. 10: Quantum representation of Unruh Effect.

### 6. THE BLACK HOLE INFORMATION PARADOX

Black Holes are engines of mass destruction, seemingly destroying any form of mass that traverses its event horizon. While it cannot destroy mass and energy, we know that the mass that is subject to its gravitational field will eventually add to its mass. However, we know also know that this mass can escape through Stephen Hawking’s eponymously named radiation. The discovery of Hawking Radiation, while a beautiful and brilliant discovery, has implausible implications. It refutes one of the most fundamental laws of quantum physics- ‘Conservation of Information’. Every particle has its own unique set of information, that cannot be destroyed. This information is what defines the particle. Information in this sense is quantifiable by characteristics such as momentum, spin, position, angular momentum etc. Through the knowledge of this information we can calculate the future state and past state of an event using deterministic physics.

Newton's force equations, Maxwell's electromagnetic equations, and even Schrödinger's Equation in Quantum mechanics are all deterministic in nature, meaning that if you know the state of a system at one point in time you can essentially calculate the change in state over a period of time.

- According to the no hair theorem properties of the particles making up a blackholes are indistinguishable by virtue of their composite particles.
- This is analogous to two bald men in a room. Two bald men, who's backs are towards the observer, are only distinguishable in the shape of their head.
- Upon closer inspection, a zoomed in frame of the two test subjects are indistinguishable as they are bald, and the shape is imperceptible through this 'zoomed-in' frame of reference.
- This contradicts most laws of science as composite particles should be enough to identify a subject just like elements and compounds are identified by their composite atoms and just like solutions and mixtures are identified by their composite molecules.
- However, Black Holes follow the no-hair theorem, as the composite particles of two test black holes are indistinguishable.

According two quantum mechanics, two particles must always be distinguishable in some aspect as their quantum information must be different. So, it may seem as though the no-hair theorem is contradictory to quantum mechanics.

However, while we are unable to perceive the individuality of the particles within the event horizon of the black holes, that does not mean it is not there, i.e. each particle within the event horizon is unique, but we cannot perceive its uniqueness.

What is to note is that Hawking radiation does not depend on the nature of the black holes. The radiation, mostly photons, are not a direct function of the particles within the blackhole. This is the information paradox. If the black hole is able to delete and rewrite information then how is this information conserved?

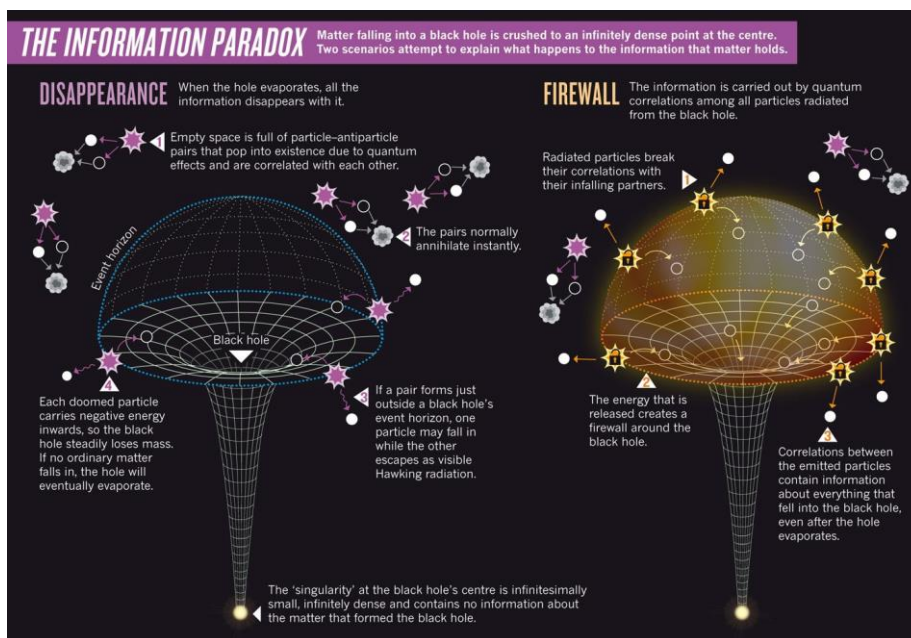


Fig. 11: Black Hole Information Paradox

## 7. RESOLUTION OF THE PARADOX

True physics does not allow for the existence of paradox. If the current understanding of Quantum Mechanics and General Relativity, is correct then Hawking radiation should exist and with it everything he proved. This must be resolved through a deeper understanding of General Relativity and Quantum Theory.

Some of the most widely accepted solutions to the paradox are as follows:

### 7.1 The Einsein-Carten Theory

- This theory is one of the first proposed resolutions to the information paradox and is one of the more outlandish ones.
- It is a modification of the Theory of General Relativity and predicts that a rotating black hole can give birth to a whole new universe, and the quantum information that goes into a black hole, through a wormhole, can go reach the new universe.
- The quantum information remains preserved in the new universe and is hence conserved, it is only inaccessible to us.
- This solution to the information paradox is attributed to Freeman Dyson, but was championed by Hawking himself.
- This theory could not be supported however as it gave no solution to non-rotating blackholes, and the math to calculate the creation of a new universe was too complex to prove.

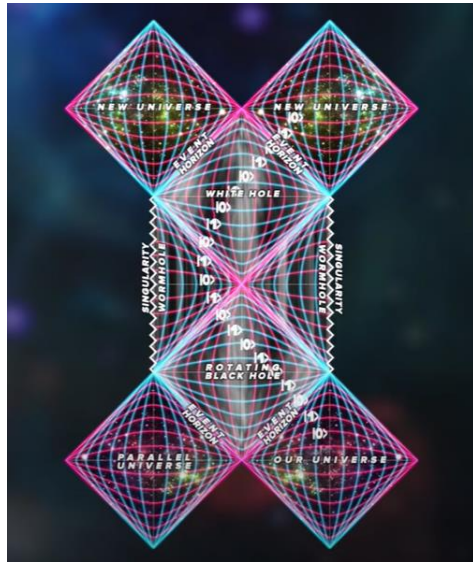


Fig. 12: Penrose Diagram of Einstein-Cartan Theory

### 7.2 Horizon Circumference Information Preservation Theory

- This is the main competitor to the previously mentioned theory.
- According to this theory, the quantum information of anything that falls into the black hole does not really cross the event horizon but is rather ‘encoded’ into the circumference of the event horizon.
- This destroys the need for a new universe.
- To the outside observer, the information never left the universe and is in turn conserved, for an inside observer the information is smeared in the horizon for all time, making essentially invisible, but still there.

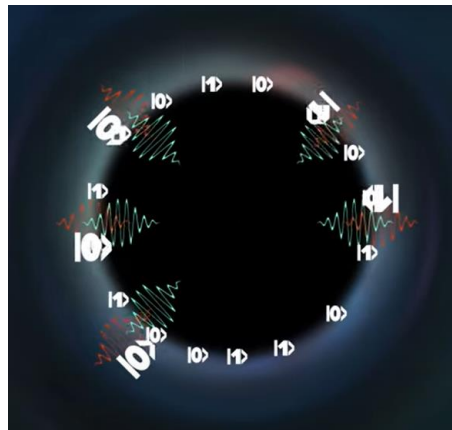


Fig. 13: Circumferential Preservation of Quantum Information

### 7.3 Shortcomings of Solution of Information Paradox

- There is no known mechanism of imprinting information on a given horizon as described by the theory
- To an outside observer, the information gets imprinted onto the circumference of the event horizon, however to an inside observer the object will merely fall in. This implies that the quantum information will be doubled.
- This breaks the ‘No-cloning theory’, as described as part of quantum mechanics, which states that quantum information cannot be reproduced out of nothing.
- The latter was solved through black hole complementarity.

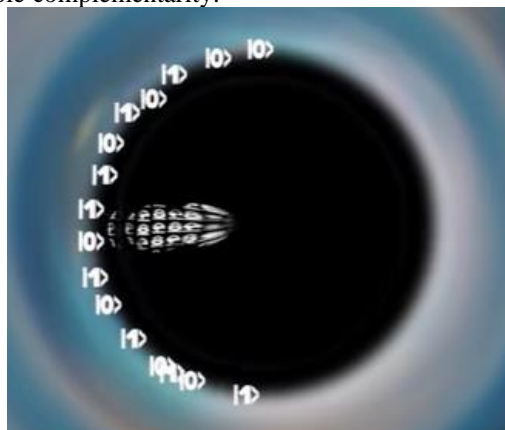


Fig. 14: Accidental cloning of Information

#### 7.4 Black Hole Complementarity

- Physicist Leonard Susskind came up with an elegant argument to this problem.
- He argued that since the matter and its quantum information cannot be observed at the same time, it must be because these two don't exist at the same time at all, and therefore not being duplicated or contradicting quantum mechanics.
- Just like there are some characteristics or quantum observables that we cannot accurately know/measure at the same time, like position and momentum, black hole complementarity suggests that the state of the interior and the exterior of a black hole are not knowable simultaneously in the same way.

### 8. IMPLICATIONS OF THE INFORMATION PARADOX AND ITS SOLUTIONS

The black hole information paradox was a result of hawking radiation. To understand either meant the uneasy unification relativistic physics and quantum mechanics, which has not been achieved till now. The proposed theories of the given solutions are all 'hacks' to work around the major pre-requisite.

Through the solving of the information paradox we can expect major advance in theoretical physics. It was shown how through blackhole complementarity the problem of conservation of quantum information is preserved, however there remains the problem of the unknown mechanism through which quantum information is imprinted.

Physicist Gerard T'Hooft calculated a possible solution to the above problem.

#### 8.1 Gerard T'hooft's Solution

- Upon precision calculations he found that it was possible that instead of freezing at the event horizon it can contribute to distorting the event horizon further.
- This meant that they could now influence the hawking radiation and possibly carrying away their quantum information with them.



Fig. 15: Distortions on the Event Horizon due to Quantum Information

T'Hooft's solution had stunning implications. He realized that the three dimensional gravitational and quantum mechanical description of the state of the interior of the black hole was fully inscribed on the two-dimensional event horizon. This meant that the final unification of relativity and quantum mechanics might mean the interpretation of the entire three-dimensional universe's information on a two-dimensional surface, somewhat like a hologram. Leonard Susskind formalized this idea in the form of string-theory.

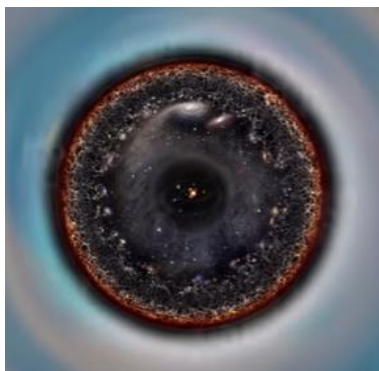


Fig. 16: Universe as the Interpretation of 2-D information

There is whole other theory that information can escape the event horizon through quantum tunneling and was proposed by Stephen Hawking but not enough is known about the phenomenon to give a full-fledged solution to the information paradox. The black hole information paradox is in no way fully solved and all of the aforementioned theories have been tested or proved experimentally. It still remains one of the most unanswerable questions in advanced physics to this date. It is however, truly wonderful that one 1974 paper written by paper led to the formation of a whole new branch in physics- String Theory and Supersymmetry, and how it led to so many new and unthought of possibilities.

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