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A theoretical paper on the impact of a moving elevator on a person's apparent weight

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ABSTRACT

The following research paper provides a theoretical study on the weight, experienced by a person, travelling in an elevator, known as the apparent weight and its dependence on time. This apparent weight may be different from the actual weight, considering the fact that it is caused due to the net upward force from the ground of the elevator onto the man. It is due to this phenomenon that we sometimes feel heavier and sometimes lighter, when travelling in an elevator. To find this dependence of apparent weight on time, we will also be determining the ratio of the apparent weight and the actual weight, and the fractional change of weight. Following this, various graphical representations will be presented, showing the dependence of the velocity and the acceleration of the elevator and the ration of the apparent and the actual weight with time. To evaluate the overall comfort, a new variable, namely the Comfort Level (CL) will be introduced. As a whole, this paper, aims to evaluate the overall mathematics behind the change in apparent weight during a journey in an elevator, going far beyond the qualitative study done in schools and textbooks.

***Keywords**: Weight, Apparent Weight, Actual Weight, Velocity, Acceleration, Elevator*

1. RESEARCH QUESTION

How does the movement of an elevator affect a person's apparent weight?

2. INTRODUCTION

The weight of an object is a measurement of the pull of gravity by the surface on it. It is a result of the gravitational attraction between the mass of the object and the mass of the planet on which it is placed. In terms of Newton's Second Law, it is equal to the product of the mass of the object and the magnitude of the gravitational field that attracts it. For a body with mass, m and gravitational field strength, g , the weight, W of the body is given by;

$$W=mg$$

The unit of the gravitational field strength is, N Kg^{-1}. This gravitational field strength is also called as the acceleration due to gravity, or the acceleration due to free fall. On the surface of Earth, the value of the gravitational field strength, g, is 9.81 ms^{-2}. This force is always directed vertically downwards. Though, the value of g changes with respect to the location of the body. For instance, the value of the gravitational field strength on the surface of Earth is 9.81 ms^{-2}. At a height of 1000 km above the Earth's surface, this value changes to 7.33 ms^{-2}, and above 2000 km above the Earth's surface, its value changes to 5.68 ms^{-2}.

When an object is placed on the surface, the only force that acts on it, when it is at rest, is the weight of the object, which as discussed earlier, is always directed upwards. The magnitude of this weight is given by; W=mg. Thus, in order to move the object vertically upwards, above the surface, a force greater than the weight of the object has to applied, in order to beat the downward force by the weight and thus, moving the object upwards.

The phenomenon of a person feeling heavier when travelling in an elevator upwards, and feeling lighter when travelling in an elevator downwards, is all due to the basic principle that a person, when judges himself, feels his weight to be the apparent weight, rather than the actual weight, which is the result of a person's force due to his mass (weight) and the force due to the acceleration of the body (F=ma), which is the force by the surface of the elevator on the person inside the elevator. Thus, the force experienced by a person travelling in an elevator can be given by;

$$W = mg + ma$$

Hence, when travelling upwards, the gravitational field strength, g, is positive, and hence the apparent weight is more the actual weight, and so the person feels heavier. Similarly, while travelling downwards, the value of the gravitational field strength, g, is negative, and hence the apparent weight is less than the actual weight, and so the person feels lighter.

In a hypothetical situation, when a person is travelling in an elevator with an acceleration, which is equal to the acceleration die to gravity;

$$W = mg + m(-g) = 0$$

Thus, in this situation, the person feels weightless. Though the actual weight has not changed, the apparent weight has changed drastically, which leads to the person inside the elevator to perceive this weight as 0. This is due to the fact that our perception of weight depends on the net upward force, exerted by the floor on us.

In this article, the dependence of time had been explored with relation to a person's perceived/apparent weight. Theoretically, when an elevator moves from one floor to the other, its velocity increases exponentially in the beginning. At the point of half distance, its velocity is maximum. After that it decreases exponentially. Thus, by these empirical observations, a sine function has been chosen to show the dependence of time to the elevators' velocity. This sine function is then used to find the net weight or the apparent weight. Using this, the fractional change of weight is calculated. The fractional change of weight is given by the formula:

$$FCW = \frac{apparent\ weight - actual\ weight}{actual\ weight} \qquad (1)$$

Following this, a graphical representation will be presented, showing the dependence of time with respect to the velocity, acceleration, and the Fractional Change of weight.

Through this paper, the concepts of apparent weight, specifically in an accelerating elevator, will be explored at great depths, with thorough mathematical rigor rather than the just the qualitative exploration, which is usually done on most of the schools and textbooks.

3. THEORETICAL ANALYSIS

Consider an elevator, with a man standing inside it, with mass, m. The force acting on the man, when the elevator is at rest, is the weight, which is in the downward direction, given by the formula;

$$W = mg$$

Now, if the elevator moves in the upward direction with an acceleration, a, the net force acting on the man is given by the difference between the two forces. Since the force due to the acceleration of the elevator is more, the net force acting on the man is;

$$F(net) = F\ (due\ to\ elevator) - F(\ due\ to\ gravity)$$

Here, F(due to gravity) is mg. But, since the direction of gravity is in the downward direction, the force due to gravity is; $-mg$
Thus,

$$\Rightarrow F = ma - (-mg)$$

$$\Rightarrow F = ma + mg \qquad (2)$$

This is the magnitude of the force experienced by the man inside the elevator. This is, thus, the apparent weight perceived by the man. When the elevator is allowed to fall freely, the acceleration of the elevator is the same as that of gravity, i.e. $a = -g$. This leads to the net force becoming zero, and hence the person feels weightless, or perceives himself to be weightless.

From equation (1), the acceleration, a, is the magnitude of vector $\frac{d\vec{v}}{dt}$ and F, is the magnitude of the vector, $\vec{F}$. Thus, equation (1) can be written in the following vector form:

$$\vec{F} = m.\frac{d\vec{v}}{dt} + mg\,\hat{k} \qquad (3)$$

Here, since the direction of gravity is always vertical, we can put the unit vector for the vertical direction, $\hat{k}$, with mg.

From equation (2), the dependence of time on velocity can be controlled by the function $f(t)$. Keeping $f(t) > 0$, for all values of t, we have to introduce another constant, which will control the velocity to be positive or negative. For this paper, we will consider the constant to be ϕ. Since its role is solely to control the direction of the velocity vector, it will have the value of either -1, or $+1$. If the elevator moves in the upward direction, ϕ will be $+1$, and If the elevator moves in the downward direction, ϕ will be -1. Since, the velocity vector is always vertical for an elevator, we will also add $\hat{k}$, to the equation of $\vec{v}$.

$$\text{Thus, } \vec{v} = \phi f(t)\hat{k} \quad (4)$$

Substituting equation (2) into equation (3), we get;

$$\vec{F} = m.\frac{d\,(\phi f(t)\hat{k})}{dt} + mg\,\hat{k} \quad (5)$$

Taking m and $\hat{k}$, we get;

$$\vec{F} = m\left(\phi\frac{d\,f(t)}{dt} + g\right)\hat{k} \quad (6)$$

Since the direction of the force is also in the vertical direction,

$$\vec{F} = F * \hat{k} \quad (7)$$

Substituting Equation (6) into Equation (5), and eliminating, $\hat{k}$, we get;

$$F = m\left(\phi\frac{d\,f(t)}{dt} + g\right) \quad (8)$$

From equation (1);

$$FCW = \frac{apparent\ weight - actual\ weight}{actual\ weight} \quad (9)$$

Since the Apparent weight is the same as F from equation (8), and Actual weight is mg, the Fractional Change of Weight can now be expressed as;

$$FCW = \frac{F - mg}{mg} \quad (10)$$

$$\Rightarrow FCW = \frac{m\left(\phi\frac{d\,f(t)}{dt} + g\right) - mg}{mg}$$

Taking m common in the numerator;

$$\Rightarrow FCW = \frac{m\left(\phi\frac{d\,f(t)}{dt} + g - g\right)}{mg}$$

Dividing g on the numerator and the denominator;

$$\Rightarrow FCW = \frac{\phi\frac{d\,f(t)}{dt} + g - g}{g}$$

$$\Rightarrow FCW = \frac{\phi}{g} * \frac{d\,f(t)}{dt} \quad (11)$$

Here, the sign of FCW depends on the sign of $\frac{\phi}{g}$ and on the sign of $\frac{d\,f(t)}{dt}$. When this Fractional Change of Weight is positive, the weight felt by the person inside the elevator increases. This is when, both, $\frac{\phi}{g}$ and $\frac{d\,f(t)}{dt}$ have the same sign. Similarly, when $\frac{\phi}{g}$ and $\frac{d\,f(t)}{dt}$ have the opposite same signs, FCW is negative, and the weight felt by the person inside the elevator decreases.

As discussed earlier, when an elevator moves from one floor to the other, its velocity increases exponentially in the beginning. At the point of half distance, its velocity is maximum. After that it decreases exponentially. Thus, by these empirical observations, a sine function has been chosen to show the dependence of time to the elevators' velocity. This sine function will then be used to find the net weight or the apparent weight.

Here, the sine function, showing the dependence of time to the elevators' velocity, will thus be a graph of $f(t)$ (by Equation (4)). Let t_0 be the time that it takes for the elevator moves from one point with zero speed to the next point of zero speed. Thus, we can propose the following form of $f(t)$;

$$f(t) = K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \quad (12)$$

From the conditions for equation (4); $f(t) > 0$. Thus, $K > 0$ and $n > 0$. Also, since we are looking at the value of $f(t)$ only from time, 0 to t_0, for $f(t)$;

$$\text{Domain: } 0 \le t \le t_0$$

$$\text{Range: } 0 \leq f(t) \leq K$$

By equation (4), we know that, $\vec{v} = \phi f(t)\hat{k}$. Thus, substituting equation (12) in equation (4), we get;

$$\vec{v} = \phi K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \hat{k} \quad (13)$$

We know that, acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Thus, in order to find the acceleration, we differentiate, $\vec{v}$ from equation (13), with respect to t:

$$\vec{a} = \frac{d}{dt}\left(\phi K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \hat{k}\right)$$

Taking out the constants;

$$\vec{a} = \phi K\left(\frac{d}{dt}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n\right)\hat{k}$$

Differentiating using Chain Rule;

$$\vec{a} = \phi K n\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\}\frac{\pi}{t_0}\hat{k}$$

Rearranging;

$$\vec{a} = \phi\frac{nK\pi}{t_0}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\}\hat{k} \qquad (14)$$

From equation (13), the maximum value of ϕ and $\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n$ is 1. So, the maximum velocity which the elevator can attain is:- $1 * K * 1^n$. Hence, Maximum Velocity = K

In a sine graph, the maximum velocity occurs, exactly between its two zeroes, that is at:- $\frac{t_0}{2}$.

From equation (6); $\vec{F} = m\left(\phi\frac{d\,f(t)}{dt} + g\right)\hat{k}$; and equation (12); $f(t) = K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n$; Finding the force exerted by the elevator on the man;

$$\vec{F} = m\left(\phi\frac{d\left(K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n\right)}{dt} + g\right)\hat{k}$$

Separating the constants and differentiating using Chain Rule, we get,

$$\vec{F} = m\left(\phi\frac{Kn\pi}{t_0}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} + g\right)\hat{k} \qquad (15)$$

From Equation 12: $f(t) = K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n$ and Equation 11: $FCW = \frac{\phi}{g} * \frac{d\,f(t)}{dt}$;

$$FCW = \frac{\phi}{g} * \frac{d\left(K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n\right)}{dt} \qquad (16)$$

$$\Rightarrow FCW = \frac{\phi K n\pi}{g t_0} * \left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} \qquad (17)$$

Let R be a ratio between the apparent weight and the actual weight of the man inside the elevator. Thus, from equation 8:

$$R = \frac{F}{mg} = \frac{m\left(\phi\frac{d\,f(t)}{dt} + g\right)}{mg} = \frac{\phi}{g} * \frac{d\,f(t)}{dt} + 1 \qquad (18)$$

Substituting Equation 12: $f(t) = K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n$ into Equation 18: $R = \frac{\phi}{g} * \frac{d\,f(t)}{dt} + 1$;

$$R = \frac{\phi}{g} * \frac{d}{dt}\left(K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n\right) + 1$$

$$\Rightarrow R = \frac{\phi}{g} * \frac{nK\pi}{t_0}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} + 1 \quad (19)$$

From Equation 13: $\vec{v} = \phi K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \hat{k}$. Also, since, $Velocity = \frac{displacement}{time}$, let, for an elevator, the displacement, which is always in the vertical direction be y. Thus,

$$\vec{v} = \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \phi K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \quad (20)$$

We can use integration to find y, which is basically the vertical distance between the two floors, between which the elevator takes, t_0.

$$\Rightarrow y = \left|\int_0^{t_0} \phi K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n dt\right| \quad (21)$$

Here, since, $|\phi| = 1$;

$$\Rightarrow y = K\left|\int_0^{t_0}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n dt\right| \quad (22)$$

Integrating and rearranging, we get,

$$K = \frac{y}{t_0} * \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n+1}{2}\right)} \quad (23)$$

Since velocity is the displacement divided by the time, from the above equation, $\frac{y}{t_0}$ can be considered as the average velocity. Substituting the above equation in Equation 13: $\vec{v} = \phi K\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \hat{k}$, we get;

$$\vec{v} = \phi\frac{y}{t_0} * \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n+1}{2}\right)}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^n \hat{k} \quad (24)$$

Substituting equation 23, in equation 17: $FCW = \frac{\phi K n\pi}{g t_0} * \left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\}$, we get;

$$FCW = \frac{\phi n\pi}{g t_0} * \frac{y}{t_0} * \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n+1}{2}\right)} * \left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} \quad (25)$$

From the above equation, taking $\frac{y}{t_0}$ as the average velocity;

$$FCW = \frac{\phi n\pi}{g*y} * v_{avg}^2 * \frac{\sqrt{\pi}\,\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n+1}{2}\right)} * \left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} \quad (26)$$

Thus, FCW – The Fractional Change of Weight is directly proportional to the square of the average velocity. Thus, for a fixed value of v_{avg}, larger 'y', which is the distance between the 2 positions, will lead to a fall in the value of the fractional change of weight, and a decrease in 'y', will lead to the increase in the value of the fractional change of weight. Thus, for longer heights, with an almost constant average velocity, the feeling of being heavier or lighter will be less distinct. In the above equations, 'n', controls the time duration of the accelerated motion. Smaller values of $'n'$ would result in a small duration of accelerated motion. In most elevators, the motion where acceleration takes place, is usually for a very small amount of time at the beginning and at the end. It is during this time period that a change of weight is perceived.

Trying to find out the level of comfort that could be provided in a journey, we will introduce a new variable, Comfort Level (CL). This comfort level will depend on the following factor;

The time duration where the person in the elevator experiences a change in weight as a result of acceleration. More this time duration, less the comfort level, and vice-versa

Thus, we will define the parameter, CL, in the following way:

$$CL = \left\{\frac{\int_0^{t_0} |F-mg|\,dt}{t_0}\right\}^{-1} \quad (27)$$

We have taken the modulus of $F - mg$, considering the fact that after half the time, the value of $F - mg$, will decrease, finally by the exact same amount as the start, and this will result in its value being zero, which we want to avoid.

From equation 18: $R = \frac{F}{mg}$ and equation 27: $CL = \left\{\frac{\int_0^{t_0} |F-mg|\,dt}{t_0}\right\}^{-1}$, we get;

$$CL = \left\{\frac{\int_0^{t_0} |mgR - mg|\,dt}{t_0}\right\}^{-1}$$

$$CL = \left\{\frac{\int_0^{t_0} mg(R-1)\,dt}{t_0}\right\}^{-1} \quad (28)$$

From Figure (5) and Figure (6);

$$CL = \left\{\frac{\int_0^{t_0/2} mg(R-1)\,dt}{t_0/2}\right\}^{-1}$$

Substituting Equation 19: $R = \frac{\phi}{g} * \frac{nK\pi}{t_0}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} + 1$ in the above equation;

$$CL = \left\{\frac{\int_0^{\frac{t_0}{2}} mg\left(\frac{\phi}{g} * \frac{nK\pi}{t_0}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} + 1 - 1\right) dt}{\frac{t_0}{2}}\right\}^{-1}$$

Taking out the constants and considering ϕ as 1;

$$CL = \left\{\frac{2mnK\pi}{t_0^2}\int_0^{t_0/2}\left\{\sin\left(\frac{\pi}{t_0}t\right)\right\}^{n-1}\left\{\cos\left(\frac{\pi}{t_0}t\right)\right\} dt\right\}^{-1} \quad (29)$$

The result of the above integration is; $\frac{t_0}{n\pi}$, thus, CL;

$$CL = \frac{2mnK\pi}{t_0^2} * \frac{t_0}{n\pi} = \frac{t_0}{2ma} = \frac{1}{m*K/(t_0/2)} = \frac{1}{m*V_{maximum}/(t_0/2)} = \frac{1}{m*a_{avg}} \quad (30)$$

Thus, as the average acceleration or the mass decreases, the comfort level increases.

Combining the above equation with equation 23: $K = \frac{y}{t_0} * \frac{\sqrt{\pi}\Gamma\left(\frac{n}{2}+1\right)}{\Gamma\left(\frac{n+1}{2}\right)}$,

$$CL = \frac{t_0^2}{2my\sqrt{\pi}} * \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)} \quad (31)$$

From the above equation, it is clear that the comfort level of travelling in the elevator increases with decreasing, n. This can also be shown from Figure 1, where a small value of n leads to an increase in the time when the elevator is moving with constant velocity. In summary, we can say that as the amount of time where the elevator is moving with constant velocity increases, the comfort level increases.

4. GRAPHICAL REPRESENTATION OF THE THEORETICAL FINDINGS

4.1 Dependence of Time on velocity

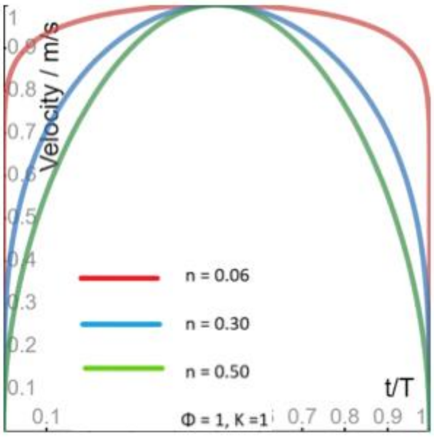

Fig.1

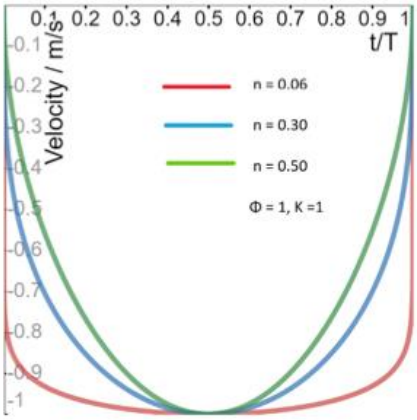

Fig. 2

Figure 1 shows Dependence of Time on velocity for an upward moving elevator. Here, it is clearly very evident that as the value of 'n' decreases, the topmost sharpness of the curve decreases, and velocity is constant for a longer period of time with small accelerations taking place at the start and by the end.

Figure 2 shows Dependence of Time on velocity for a downward moving elevator. The negative value of the velocity is due to the fact that the motion is in the downward direction. Here, it is as the value of 'n' decreases, the topmost sharpness of the curve decreases, and velocity is constant for a longer period of time with small accelerations taking place at the start and by the end.

4.2 Dependence of time on acceleration

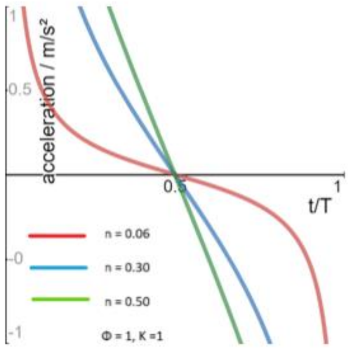

Fig. 3

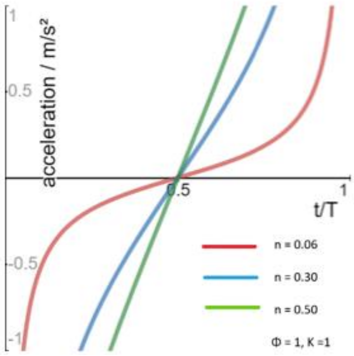

Fig. 4

Figure 3 shows Dependence of time on acceleration for an upward moving elevator. Starting from a certain positive value, it decreases to zero, and then goes negative. The positive values in the beginning shows that the elevator starts off by accelerating, but later, slows down, hence decelerating, hence the negative sign after half the time.

Figure 4 shows Dependence of time on acceleration for a downward moving elevator. Starting from a certain negative value, it increases to zero, and then goes positive. The negative values in the beginning shows that the elevator starts off by decelerating, but later, slow down, hence accelerating, hence the positive sign after half the time.

4.3 Dependence of time on R, the ratio of the apparent weight and the actual weight

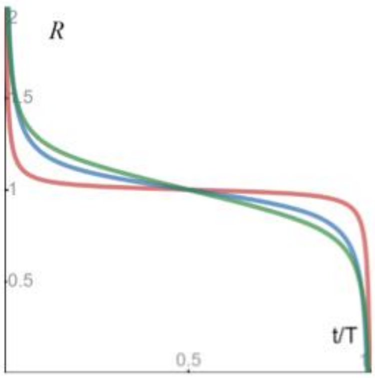

Fig. 5

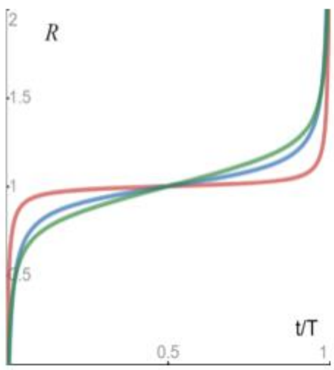

Fig. 6

Figure 5 shows Dependence of time on R, the ratio of the apparent weight and the actual weight for an upward moving elevator. For small values of 'n', the curve remains close to $R = 1$. Initially, the value of R is greater than 0. At $t = \frac{t_0}{2}$; the value of R is zero. Following this, the value of R goes negative. As a result, initially, the person in the elevator feels heavier and after half the time, feels lighter.

Figure 6 shows Dependence of time on R, the ratio of the apparent weight and the actual weight for a downward moving elevator. For small values of 'n', the curve remains close to $R = 1$. Initially, the value of R is lower than 0. At $t = \frac{t_0}{2}$; the value of R is zero. Following this, the value of R goes positive. As a result, initially, the person in the elevator feels lighter and after half the time, feels heavier

5. CONCLUSION

Through the present paper, the purpose of demonstrating the theoretical aspect of apparent weight, during a non-uniform motion, has been accomplished. Here, we had been able to theoretically explore, how the apparent weight depends on the time, along with quantifying, exactly how a journey in the elevator can be made more comfortable. Using this, we found that the comfort level in an elevator can only be increased as the duration of the time, the elevator performs uniform motion, increases. No doubt, this is an extremely difficult task to accomplish in real life. The inclusion of the various technological challenges has not been done in this paper, but can be a scope for further research in this field.

6. REFERENCES

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