ABSTRACT

The Permutation Series and Combination Series will ensure the readers to understand proper calculation of Permutation and Combination and will also help them to grow their knowledge on the topic. It is very helpful topic for the students to strengthen their roots in mathematics. The magic that Mathematics have in it will be exposed to the readers to understand their Mathematics thoroughly and will also help them to enlarge their thinking capabilities. Hence, this topic is very interesting as it provides a link between A.P, G.P with Permutation and Combination. It will help to calculate many selections and others in a very small time.

Keywords—Logic, Series, Algebra

1. INTRODUCTION

Till date we have just used permutations and combinations in arrangements, derangements, selection and others. But here, Permutation and Combination is described in a new form called Permutation series and Combination Series.

2. IMPORTANCE OF PERMUTATION SERIES AND COMBINATION SERIES

Just like A.P, G.P, H.P Permutation series and Combination Series we also help us to find out the next term, previous term, sum of Permutations and Combinations up to m terms. It will also strengthen Permutations and Combinations in the minds of the peoples.

This will make a easy way to find larger terms in a very short time interval. Hence, the Permutation series and Combination series is largely important.

3. PERMUTATION SERIES

The formula to find Permutation of n things when r things is taken at a time is:

\[ n^p_r = \frac{n!}{(n-r)!} \]  

(1)

but Permutation series deals with more than 1 permutation keeping n fixed.

Let us take an example:

\[ n^p_r + n^p_{r+1} + n^p_{r+2} + …………………………………………………………+n^p_t \]

In Permutation and Combination Series the last highest term is \( n^p_n \) or \( c^n_n \) as n cannot be less than r.

3.1 To find the next term when any term is given(except the first term)

Let \( t_i = n^p_r \) and \( t_{i+1} = n^p_{r+1} \)

\[
T_i/t_{i+1} = [n!/(n-r)!]/[n!/(n-r-1)!] \\
T_i/t_{i+1} = [n!(n-r-1)!]/[n!/(n-r)!] \\
T_i/t_{i+1} = 1/(n-r) \\
T_{i+1}/t_i = n-r
\]

Hence,

\[ T_{i+1} = (n-r)t_i \]  

(2)

3.2 To find the previous term when any term is given (except the first term).

\[ T_i = n^p_r \] and \( t_{i-1} = n^p_{r-1} \)

\[
T_i/t_{i-1} = [n!/(n-r)!]/[n!/(n-r+1)!] \\
T_i/t_{i-1} = [n!/(n-r)!]/[n!/(n-r+1)!] \\
T_i/t_{i-1} = n/(n-r+1)/n/(n-r)
\]

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3.3 To find any term when a term is given and common difference is known (except the first term).
Suppose, \( t_{a1} = n^r \) and \( t_{a2} = n^{r+1} \)

\[ d = r_2 - r_1 \]

\[ r_1 = r_2 + 1 \]

\[ t_{a1}/t_{a2} = n!/(n-r_1)!/n!/(n-r_2)! \]

\[ t_{a1}/t_{a2} = [n!*(n-r_2)!]/[n!*(n-r_1)!] \]

Hence,

\[ t_{a2} = \frac{[n!*(n-r_1)!]}{[(n-r_1)!]*n!}*t_{a1} \] (4)

Where \( r_1 \) can be any number less than \( n \) and \( d \) is the common difference between \( r_1, r_2, r_3, \) and so on.

3.4 To find \( m^{th} \) term in a Permutation Series
Suppose, a Permutation Series be

\[ n^r + n^{r+1} + n^{r+2} + \ldots + n^m \]

Solution: To find the third term.

First let write the details

First term \( a_r = 1 \)

Common difference = \( d = 2 - 1 = 3 - 2 = 1 \)

Hence, Third term:

\[ T_3 = n^r \]

Hence,

The formula to find \( m^{th} \) term in a Permutation Series

\[ T_m = a_r + (m-1)d \] (5)

Where, \( a_r \) is the first term, \( d \) is the common difference.

3.5 To find sum of \( m \) terms
Suppose, a series be

\[ n^r + n^{r+1} + n^{r+2} + \ldots + n^m \]

Find the sum of first 3 terms:

Solution: sum of first 3 terms

\[ S_m = n!/[1/(n-1)!+1/(n-2)!+1/(n-3)!] \]

Hence,

In general, to find sum of \( m \) terms the formula

\[ S_m = n!/[1/(n-r)!+1/(n-(r+d))!+\ldots+1/{n-r-(m-1)d}!] \] (6)

Where \( S_m \) means sum of the series.

4. COMBINATION SERIES
The formula to find combination of \( n \) things when \( r \) things is taken at a time is:

\[ ^nC_r = n!/r!*(n-r)! \] (7)

Just like Permutation series, Combination Series will have highest terms as \( ^nC_r \) as \( r \) cannot be more than \( n \) keeping \( n \) constant. For example:

\[ ^nC_1 + ^nC_2 + ^nC_3 + \ldots + ^nC_n \]

4.1 To find the next term when any term is given (except the first term).
Let \( t_{m} = c_r \) and \( t_{n+1} = c_{r+1} \)

\[ T_{r+1}/t_{r+1} = n!/[r!(n-r-1)!] \]

\[ T_{r+1}/t_{r+1} = [n^r*(r+1)!]/[r!*(n-r)!] \]

\[ T_{r+1}/t_{r+1} = [(r+1)!]/[(n-r)!] \]

Hence,

\[ T_{r+1} = [(n-r)/(r+1)]*t_r \] (8)

4.2 To find the previous term when any term is given (except the first term)
Let \( t_{m} = c_r \) and \( t_{n+1} = c_{r-1} \)

\[ T_r/t_r = n!/[r!(n-r+1)!] \]

\[ T_r/t_r = [n^r*(n+1)!]/[r!*(n-r)!] \]

\[ T_r/t_r = [n^r*(n+1)!]/[r!*(n-r+1)!] \]

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Just like Permutation Series, in Combination Series, to find the $m^{th}$ term we use the formula:

$$T_m = \binom{n}{a}(m-1)!d!$$  \hspace{1cm} (11)

Where $d$ is common difference.

4.5 To find sum of $m$ terms

Just like Permutation Series, if $S_m$ be the sum of $m$ terms of a permutation series and first term is $\binom{n}{a}$ where $a=r$ then the formula:

$$S_m=n\left(1/(r!(n-r)!)+(r+d)/(r(n-r)!)-(r-d)/(r(n-r-d)!)\right)+...........................1/((r+(m-1)d)!((n-r-(m-1)d)!))$$  \hspace{1cm} (12)

4.6 Some special combination series with their formula for sum:

- $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots +\binom{n}{n} = 2^n$
- $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots = (2^{n/2})^2$\hspace{1cm}$\cos(n\pi/4)$
- $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\ldots = (2^{n/2})^2$\hspace{1cm}$\sin(n\pi/4)$
- $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\ldots + (-1)^r \binom{n}{r} = 0$\hspace{1cm}$r!$ is any number less than $n$.
- $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots +(-1)^r \binom{n}{r} = 0$
- $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\ldots + (n+1)\binom{n}{r} = (n+2)2^n$\hspace{1cm}$c_r$ is any constant.
- $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots + \binom{n}{r} = 2^n$\hspace{1cm}$c_r$ is any constant.
- $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots = (2n)!/(n!)(n/2)!$\hspace{1cm}$n$ is even
- $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\ldots = (2n)!/(n!)(n/2)!$\hspace{1cm}$n$ is odd

$$\sum_{m=1}^{\infty} (-1)^m \binom{n}{m}(1/2+3/2^2+7/2^3+15/2^4+\ldots + m\text{ terms}) = (2^{n-1})/(2^m(2^{n-1}))$$  \hspace{1cm} (13)

5. REFERENCES
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BIOGRAPHY

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