INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact factor: 6.078
(Volume 6, Issue 2)
Available online at: www.ijariit.com

Irrational Numbers in the Physical Universe, and its effects on length, time and mass: Introduction to Physical Irrational Numbers

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ABSTRACT

If we want to extend the length of a body (without stretching it), what can we do? We can simply add an element of desired length and extend that body. Keeping this in mind that to extend the length of the body we can add elements. Now, let's take a body of length pi, which is 3.1415 m (first 4 places). Now let's write this distance as sum of elements: 3 m + 0.1 m + 0.04 m + 0.001 m + 0.0005 m. So, as we go ahead the decimal places, we can see that the length of the elements which we are adding are getting smaller and smaller, and as we know in our physical universe, the smallest particle to exist is a quark, hence it will be our smallest element which we can add. And as there are no particles beyond quark hence the number will end at moment the diameter of a quark is reached, as the diameter of a quark is 10^{-18} m, hence all irrational numbers will terminate at the eighteenth place. This is true only for the bodies in the physical world. These numbers behave like constants or rational numbers, but they are fundamentally irrational. Similarly, we will find the termination of irrational numbers when they represent mass and time.

Keywords— *Pi, Irrational Numbers, Quark, Physical Universe, Termination*

1. INTRODUCTION

The numbers which cannot be completely represented as a ratio of two numbers are called irrational numbers. Basically, these are the numbers which cannot be written as a simple fraction. Their decimal value neither terminates nor becomes periodic. These irrational numbers are very useful in mathematics and physics which affect our day to day life. From calculating the orbit of a satellite, to figuring out the time period of a pendulum, irrational numbers are everywhere. Some examples are pi, golden ratio, Euler's constant and $\sqrt{2}$.

Mathematics is limitless, but physics is not. Hence there are certain concepts which are quiet well applicable in mathematics but not that well applicable in the physical world, one such concept is irrationality. The physical world is constrained within the laws of physics. Hence these mathematical concepts apply a bit differently in the physical world. Firstly, how irrational numbers are obtained in the physical universe:

1. By using trigonometry, at certain angles we get irrational values, and if we apply trigonometric ratios to real life situations. For example, getting distance between 2 bodies. Example: Let a body be of height 1 meter, and angle of elevation to the top be 30°, then the distance will be

$$\tan\theta = \frac{height\ of\ building}{distance\ of\ building\ from\ viewer}$$
$$\tan 30° = \frac{1}{distance\ of\ building\ from\ viewer}$$
$$distance\ of\ building\ from\ viewer = 100\sqrt{3}\ m$$

Hence distance we get is irrational

2. By constructing a circle of radius of 0.5 m.

$$circumference\ of\ circle = 2\pi r$$
$$circumference\ of\ circle = 2\pi\frac{1}{2}$$
$$\therefore\ circumference\ of\ circle = \pi\ \ meters$$

Here pi is an irrational number.

How do these irrational numbers behave in the physical world?

2. IRRATIONAL NUMBERS REPRESENTING LENGTH

As we know we can write any number as sum of other numbers, this is applicable in the physical universe also. For example, to extend a 5 cm element to 7 cm (without stretching the original length), we can add 2 cm element to it, and a 7 cm element is obtained.

Now, let us find out how can an irrational number be written as the sum of elements? Let that irrational number be π, and hence the length of that element will be π meter. Therefore, it will be written as:

$$\pi = 3.14159........$$
$$\therefore \pi = 3 + 0.1 + 0.04 + 0.001 + 0.0005 + 0.00009......$$

Pi is an irrational number hence its decimals will go on till infinity. We cannot write all the digits as they are very long, so only few digits are taken for explanation.

Hence, for making an element of length pi (3.14159 m); we will add smaller elements to the 3 m body as shown above. Notice that each element which is added is smaller than the previous one. In mathematics, smaller and smaller elements can keep adding up; but in physics there are limitations. As elements keep adding up, their length decreases. At one point the length of these elements would be smaller than a molecule, then smaller than an atom, then smaller than proton and then finally a quark. And as quarks are elementary particles, there is nothing smaller than a quark. Hence the quark will be the smallest particle which can be added. We can draw 2 conclusions from the above statements, one, that there are infinitely smaller particles than the quark, second that pi and other irrational should terminate at the value which is equal to the diameter (length) of the quark. The first point is not possible because we know there are not infinitely small particles beyond the quark, according to the standard model.

Hence now the expansion is as follows:

$3\,m$ +
$1*10^{-1}\,m$ +
$4*10^{-2}\,m$ +
$1*10^{-3}\,m$ +
$5*10^{-4}\,m$ +
$9*10^{-5}\,m$ +
$2*10^{-6}\,m$ +
$6*10^{-7}\,m$ +
$5*10^{-8}\,m$ +
$3*10^{-9}\,m$ + (Size of a molecule)
$5*10^{-10}\,m$ + (Size of an atom)
$8*10^{-11}\,m$ +
$9*10^{-12}\,m$ +
$7*10^{-13}\,m$ +
$9*10^{-14}\,m$ + (Size of a nucleus)
$3*10^{-15}\,m$ + (Size of a proton)
$2*10^{-16}\,m$ +
$3*10^{-17}\,m$ +
$8*10^{-18}\,m$ (Size of a quark)

And hence, the value of pi is terminated at the eighteenth place. Similarly, if we consider different irrational numbers in the above example, we get that all irrational numbers will also terminate at the 18th place. We can conclude that: All irrational numbers when represent length or distance, then will terminate at the 18th place of the decimal expansion.

$$\therefore \pi = 3.141592653589793238$$

Hence, we can also write other irrational numbers, in the new form, in which irrational numbers terminate at the 18th place. Example:

$$\sqrt{2} = 1.414221356237309504$$
$$\sqrt{3} = 1.732050807568877935$$
$$\sqrt{5} = 2.236067977499789696$$

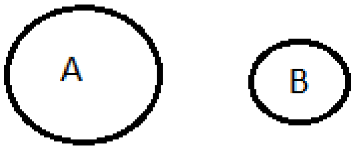
One possible question can arise why didn't we consider plank length as the smallest element? The answer to that is, there is no physical particle of plank length, and if a particle doesn't exist physically then how can we add it to the length in the physical universe, and hence we consider the last element as the quark.

3. IRRATIONAL NUMBERS REPRESENTING TIME

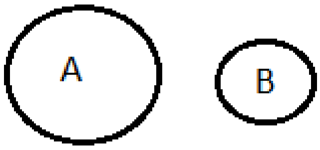
For time, consider a collision is going to happen exactly at pi seconds (considering pi for the example). So for this explanation consider that we are watching a video of this collision, hence we can forward the video and also play it at a slower speed. As mentioned earlier the bodies will colloid exactly at pi seconds. So, as the video plays; the bodies start coming closer and closer. As we can control the video and play it at a slower speed, hence we can see the events which would be happening beyond the first two decimal places. Diagrams showing the movement of the bodies

1. At 1 second

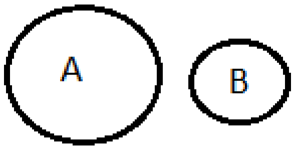
2. At 3 seconds



3. At 3.14 seconds



4. At 3.1415 seconds



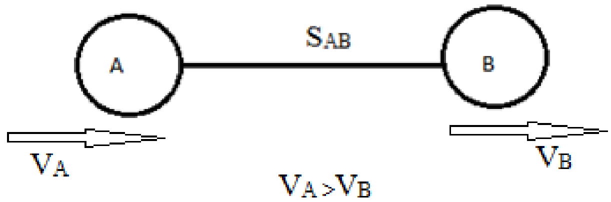
As we can see in the above example as the time increases, the distance between the bodies is decreasing. Hence, at one time they will come in contact and colloid. But that's not the case here. As the bodies are set to colloid exactly at pi seconds, hence the bodies will keep coming close to each other until the time pi is achieved. As the value of pi cannot be achieved hence the bodies will keep coming close to each other. The smallest possible distance between 2 bodies is the plank length. Hence as the plank length is achieved the bodies go no further, which means this is the final distance between them.

The collision we have discussed till now can happen in two ways:

(a) The 2 bodies move is the same direction and the body which is behind (relative to the first body) has greater velocity than the body in front.
(b) The 2 bodies move towards each other such that their directions are opposite.

Case 1: The 2 bodies move in the same direction for collision.

For termination of irrational numbers representing time we will derive an expression, which will give us the time at which the distance between the bodies is plank length. Let 2 bodies be on a trajectory of a collision, and let the distance between them be S_{AB}. And are moving in the same direction, body A is travelling with velocity V_A and body B is travelling with velocity V_B in same direction. Also $V_A > V_B$



Let's say these bodies colloid at T seconds. To make calculation easier we will consider one body stationary and calculate velocity with respect to other body.

$$\therefore final\ velocity = V_A - V_B$$

This final velocity which we have calculated is also called relative velocity. As velocities are vector quantities, they can be subtracted in the way we have done. As we know,

$$velocity = \frac{distance}{time}$$

$$\therefore V_A - V_B = \frac{S_{AB}}{T}$$

$$T = \frac{S_{AB}}{V_A - V_B}$$

And as we need time of the collision, here we will take S_{AB} as plank length, because that's the minimum distance. Therefore, we will substitute the value of plank length in place of S_{AB}.

$$\therefore \boldsymbol{T} = \frac{\mathbf{1.6 * 10^{-35}}}{\boldsymbol{V_A - V_B}}$$

Hence on substituting the 2 velocities we can get time of collision.

The answer we get from the above equation is in the form of ten to the power some value. But as we know, we can represent the power in various ways, for example $1.69 * 10^{-2}$ can be written as $16.9 * 10^{-3}$ also $169 * 10^{-4}$ etc. So which value should we choose? For each value we choose we get different places of termination. Hence, we need to choose a representation in which all decimal places are accounted. Hence, we will choose the termination in which we have removed the decimal and accordingly changed the power. We choose this method because this accounts for all decimal points. For example, $1.231 * 10^{-3}$ will be written as $1231 * 10^{-6}$ and $2.37 * 10^{-1}$will be written as $237 * 10^{-3}$. And now will take the power as the value of termination, and ignore the minus sign.

Also, if we get any value of time less than $5.39 * 10^{-44}$, then we will ignore that number and terminate the irrational number at the 44th. This is because time also has a limit that is plank time and this is the minimum time interval possible. Hence for all values below plank time would be terminated at 44th place.

Example 1:
Let's consider 2 bodies colloid at $\sqrt{2}$ seconds, one body moves with 10 m/s and the other body moves with the velocity 8 m/s.

For finding the termination of irrational time, firstly we will find the change in velocity

$$\Delta V = V_A - V_B = 10 - 8 = 2\ m/s$$

Substituting in the formula

$$T = \frac{1.6 * 10^{-35}}{2} = 0.8 * 10^{-35}$$

Converting the solution into proper form

$$8 * 10^{-36}\ s$$

As the bodies achieve plank length at the above time, hence this time interval will be the irrational time's last digit. Hence, we will see the power of the solution and at that value we will terminate the number; which in this case is the 36th place. We can ignore the rest of the term of the answer, and only consider the power.

$$\therefore T = \sqrt{2} = 1..414213562373095048801688724209698078$$

Example 2:
Let's consider 2 bodies are on a path of colliding and colloid at pi seconds. One body moves at 19 m/s and the other at 2260 m/s.

For finding the termination of irrational time, firstly we will find the change in velocity

$$\Delta V = V_A - V_B = 2260 - 19 = 2241\ m/s$$

substituting this value in the formula

$$T = \frac{1.6 * 10^{-35}}{2241} = 0.000713966979 * 10^{-35}$$

$$\therefore T = 7.13966979 * 10^{-39}$$

Converting the solution into proper form,

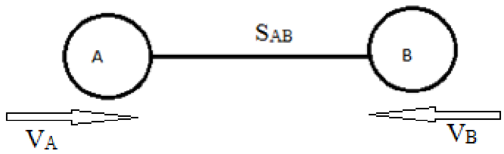
$$\therefore T = 713966979 * 10^{-47}$$

As we can see here the power, we get is less than the plank time, the power is less than 44. Hence in this case we will terminate pi at its 44th place.

$$\therefore \pi = 3.14159265358979323846\ 26433832795028841971 6939$$

Case 2: The 2 bodies move in opposite direction for collision.

Let us consider a similar case, where there are 2 bodies which move in opposite directions. Let one body be A moving with V_A and another body B moving at V_B towards each other.



Like we did before, we will find the relative velocity. The only difference will be that the velocity of one of the bodies will be negative as it's in the opposite direction. For now, let the right direction be positive and the left side as negative direction. Hence, the relative velocity is

$$relative\ velocity = V_A - (-V_B)$$

$$\therefore relative\ velocity = V_A + V_B$$

As we know,

$$velocity = \frac{distance}{time}$$

$$\therefore V_A + V_B = \frac{S_{AB}}{T}$$

$$T = \frac{S_{AB}}{V_A + V_B}$$

The minimum distance here also is plank length.

$$\therefore T = \frac{1.6 * 10^{-35}}{V_A + V_B}$$

Rests of the rules about writing the value of 'T' are same as mentioned above. Hence for finding the place of termination of irrational numbers when they move in opposite directions is,

$$\therefore \boldsymbol{T} = \frac{\mathbf{1.6 * 10^{-35}}}{\boldsymbol{V_A + V_B}}.$$

Example 1: Consider 2 bodies are moving towards each other with the velocity 10 m/s and 15 m/s respectively. They colloid at $\sqrt{5}$ seconds.

For finding the termination of irrational time, firstly we will find the final relative velocity

$$relative\ velocity = 10 + 15$$

$$\therefore relative\ velocity = 25\ m/s$$

Substituting the value in the formula we get,

$$\therefore T = \frac{1.6 * 10^{-35}}{25}$$

$$\therefore T = 6.4 * 10^{-37}\, s$$

Converting the solution into the proper form

$$T = 64 * 10^{-38}\, s$$

As the power is 38, hence time will terminate at the 38th position, that is √5.

$\therefore T = \sqrt{5} = 2.2360679774997896964091736687312762354 4$

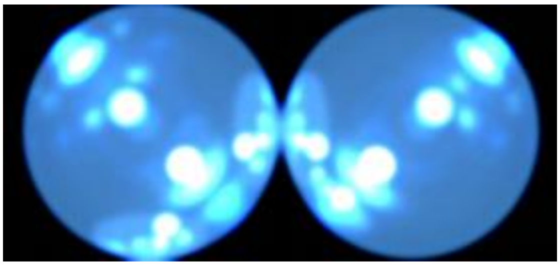


Diagram above shows 2 very small bodies just before they collide. As we can a very small gap between the 2 bodies. This how the collision between 2 bodies will look like, when they are set to colloid at an irrational time. The small gap between the bodies here represents the Planck length. This diagram is just for the sake of explaining, not to scale.

4. IRRATIONAL NUMBERS REPRESENTING MASS

Like we did during termination of length we will do something similar to that for mass. For making a body with a certain mass we can simply add components. So, if we have a body with mass of an irrational number then, we will again add smaller and smaller components. As a result, we will keep adding elements, but as we keep adding elements the mass of these elements will keep decreasing. As we know we can't go on adding elements of lighter and lighter mass hence we have to terminate it at the lightest element possible.

The lightest particle known is a neutrino. Hence this last element we can add is a neutrino. The mass of a neutrino is $18 * 10^{-35}$ Kg. Hence all mass related values will terminate at the 35th decimal place.

4.1 Real life relevance

As mentioned before that all these fundamental properties will terminate at an elementary particle as the physics beyond them doesn't exist. These elementary particles don't exist independently, and decay very soon. So, if we have to actually measure, we won't be able to measure the effect of these particles. We can measure them up to a certain limit, where they can exist as a stable body. This will generally be an atom. When we do calculations, we will consider all the decimal places as mentioned above; but if we actually go to measure out these quantities, we wouldn't get accurate value till the last decimal place.

4.2 Introduction to Physical Irrational Numbers

As we have seen in the above cases that irrational number while representing physical quantities, they terminate at certain places according to the situation. But as we know irrational numbers never terminate. Hence these numbers don't have the characters of irrational numbers; they are also not rational numbers because they are values of under root numbers. These numbers are fundamentally irrational numbers. These numbers require a new set; hence we introduce a new set which accommodates irrational numbers representing physical quantities. This new set is Physical Irrational Numbers. This set includes numbers which are irrational and represent physical quantities and terminate according to the given situation. The numbers which belong to this set are fundamentally irrational.

4.3 Properties of Physical Irrational Numbers

They have properties similar to rational numbers but different from irrational numbers

(a) Squares or powers: The square or powers of these numbers will also terminate according to the physical quantity they represent. If length is squared or raised to power some value then we will terminate the value at the 18th place. If time is squared or raised to power some value then it will terminate at 44th place. If mass is squared or raised to power some value then it will terminate at 35th place.

(b) Product with Rational number: Unlike irrational numbers, their product gives rational number.

(c) Product with Irrational number: It will give an irrational number

4.4 Significance

The values which we get are very small that they can be ignored. But as we advance us would need accurate values form our experiments and we need the accurate values of our constants which will help us further in our calculations. Also when we talk about microscopic objects, even a small decimal can change the whole value and give us the wrong picture of the reality. Also, when we have to consider the extremely large distance, even a small decimal change in the angle can affect the distance (length of arc). Hence it is important to take into account even the smallest decimal values.

5. SUMMARY

Mathematics is limitless and nothing can contain it, but physics is contained by the laws of nature and has limits. When we use irrational numbers in the physical universe, they are bound by the physical laws like there are no particles smaller than a quark, the smallest length possible is the plank length and the lightest particle we can have is a neutrino. We can generalize the place of termination for irrational numbers representing length and mass. For length all irrational numbers will terminate at the 18th place. And for all irrational numbers representing mass will terminate at 35th. But for time we need to substitute the values in the formula $\boldsymbol{T} = \frac{\mathbf{1.6*10^{-35}}}{\boldsymbol{V_A - V_B}}$ or $\boldsymbol{T} = \frac{\mathbf{1.6*10^{-35}}}{\boldsymbol{V_A + V_B}}$ according to the situation, and we get the point of termination and by the rules mentioned in the paper.

6. ACKNOWLEDGEMENTS

This paper is only possible with the constant support and love of my parents, Ritu Saxena and Aaakash Saxena. I would like to thank my friends with whom I got the basic idea about this paper, Akshat Vatal and Farhaan Shaikh. I would like to also thank my teachers Mrs Anupriya Iyer and Yogendra Murugudu who guided me throughout process. At last I would like to thank Sir Albert Einstein for leaving behind such a great legacy.

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