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Transformation of number system

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ABSTRACT

A number is just a combination of some symbols expressed as numerals or digits in any number system, the number of terms in the combination depends upon the base of a number system (or number of digits/numerals in the number system). Any combination can be represented as $(Q)_m$. Where, Q → Combination (or integer) and $_m$ → base of a number system (or number of digits in the system). $(11000000)_2 \rightarrow (300)_8 \rightarrow (192)_{10} \rightarrow (C0)_{16}$. As you can see that a combination 192 in base 10 system have different number of terms, based on the number system it lies in and as we increase the digits in the number system, the number of terms in the combination decrease but its value remains same. The large combinations in our base 10 system, can be a small combination in a base m system having very large value of m. In base 10 system a combination arrives know as infinity and this combination arrives because we have only 10 digits, if we convert this combination in base m system where $m \rightarrow \infty$, then this combination can become finite in that system.

Keywords— *Flexible number system, Base m system*

1. INTRODUCTION

The value of any combination (or number) depends on the two factors, first is the number of digits in the number system it lies in and second is how these digits are arranged. As $11_{10} > 11_2$ and $21_{10} > 12_{10}$. In first, the value of combinations differs because of their different number systems (due to their different base value). It is same as like measuring a distance in different units. Imagine that the distance between your home and office is 5 km, now if you measure it in mm then it will be 50,00,000 mm.
In second, the values of combinations differ because of the arrangement of digits.

The mathematical calculations and complexities depend on the base of the number system (m) as steps of calculations increases with increase in the number of terms in a combination and having less value of m results more number of terms in the combination. As multiplication of $(10111111)_2$ with $(11000000)_2$ will involve much more steps and also consume more time in comparison if we multiply $(191)_{10}$ with $(192)_{10}$ but, both give same results in different number systems.

So on increasing the base value (or digits) of the number system, steps involved in the calculations can be reduced. We have 4 types of number system which are binary, octal, decimal and hexa-decimal system but we can modify our number system to such systems having a large value of m.

2. RESEARCH METHODOLOGY

Any combination in any numeric system can be represented as

$$Q_m = x_n m^n + x_{n-1} m^{n-1} + x_{n-2} m^{n-2} \ldots\ldots + x_1 m^1 + x_0 m^0$$

Where,

- Q is a combination (integer)
- x is the digit (0 – 9 for base 10 system, 0 and 1 for binary system)
- m is the base value (10 for base 10 system, 2 for binary system)

Let's look it with an example;
$122_{10} = 1\times10^2 + 2\times10^1 + 2\times10^0$
$1100_2 = 1\times2^3 + 1\times2^2 + 0\times2^1 + 0\times2^0$

Likewise, fractional numbers are represented as

$$Q_m = x_1 m^{-1} + x_2 m^{-2} + \ldots\ldots\ldots\ldots + x_n m^{-n}$$

Where,

- Q is a fraction
- x is the digit (0 – 9 for base 10 system, 0 and 1 for binary system)
- m is the base value (10 for base 10 system, 2 for binary system)

Let's understand it with an example

$$0.375_{10} = 3\times10^{-1} + 7\times10^{-2} + 5\times10^{-3}$$
$$0.011_2 = 0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3}$$

2.1 Conversion from base 10 to base m system

For converting a combination from one base system to another, certain mechanisms are required, we assumed base 10 system as reference system because we are familiar with it, so conversion from base 10 system to base m system will be

$$(Q)_{10} \rightarrow (X)_m$$

And

$$(X)_m = Q_nR_nR_{n-1}R_{n-2} \ldots\ldots\ldots R_2R_1$$

Where,

- X → Combination in base m system
- Q → Combination in base 10 System

Q can be written as

$$Q = mQ_1 + R_1$$
$$Q_1 = mQ_2 + R_2$$
$$\downarrow$$
$$Q_n = mQ_{n+1} + R_{n+1}$$

Where,

$$Q_1 = \left[\frac{Q}{m}\right]$$

Where, $\left[\frac{Q}{m}\right]$ ← Integer value of $(\frac{Q}{m})$

$$Q_2 = [\frac{Q_1}{m}]$$
$$\downarrow$$
$$Q_n = [\frac{Q_{n-1}}{m}]$$

Note- Q_{n+1} must be equals to 0

$$R_1 = m\times\{\frac{Q}{m}\}$$

Where $[\{\frac{Q}{m}\}$ ← fraction value of $(\frac{Q}{m})]$

$$R_2 = m\times\{\frac{Q_1}{m}\}$$
$$\downarrow$$
$$R_n = m\times\{\frac{Q_{n-1}}{m}\}$$

2.1.1 Conversion from base 10 system to base 6 system

$$192_{10} \rightarrow (X)_6$$

So,

$$(Q)_{10} \rightarrow (X)_m$$
$$(X)_m = Q_nR_nR_{n-1}R_{n-2} \ldots\ldots\ldots R_2R_1$$
$$Q_1 = [\frac{192}{6}] = [32.0] = 32$$
$$Q_2 = [\frac{32}{6}] = [5.3333] = 5$$
$$Q_3 = [0.833] = 0 \text{ (only for verification that } Q_{n+1} = 0)$$
$$R_1 = 6\times0 = 0$$
$$R_2 = 6\times(0.33333) = 2$$
$$(X)_6 = Q_2R_2R_1$$
$$(X)_6 = (520)_6$$

2.2 Number of terms in the new combination

$$(X)_m = Q_nR_nR_{n-1}R_{n-2} \ldots\ldots\ldots R_2R_1$$

In above equation, n+1 will be the number of terms, represented by n_m. The relation between m and n for a particular combination $(X)_m$ converted from Q_{10} is as

$$m^n \leq Q_{10}$$
$$n_m = n + 1$$

Note - If m > Q, then it means that Q is one of the unit digit of that system which implies that $n_m = 1$

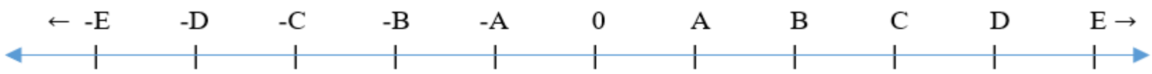
2.3 Modelling of base 16 system (or 16 Decimal system)

This system is totally different from hexa-decimal system even if both have 16 digits, because in hexadecimal system, mathematical numerals are used along with alphabets to reduce calculation and also because we are familiar with numerals but we have avoided numerals in this system because they can make confusion in defining new rules, but for convenience we marked particular numbers in the subscript to our digits. In this system, the digits contain 0 along with alphabets up-to O. Then these digits repeat itself to form a working number system, studying of this type of system looks complicated because we are not enough familiar with alphabets to use them to replace numerals, but it is just a prototype. In future we have to find an appropriate replacement of these alphabets because we have only 26 alphabets and we may need 100 or 1000 or more digits in a system. The below table contains all combinations up-to two digit of this 16 decimal system.

0	A_1	B_2	C_3	D_4	E_5	F_6	G_7	H_8	I_9	J_{10}	K_{11}	L_{12}	M_{13}	N_{14}	O_{15}
A0	AA	AB	AC	AD	AE	AF	AG	AH	AI	AJ	AK	AL	AM	AN	AO
B0	BA	BB	BC	BD	BE	BF	BG	BH	BI	BJ	BK	BL	BM	BN	BO
C0	CA	CB	CC	CD	CE	CF	CG	CH	CI	CJ	CK	CL	CM	CN	CO
D0	DA	DB	DC	DD	DE	DF	DG	DH	DI	DJ	DK	DL	DM	DN	DO
E0	EA	EB	EC	ED	EE	EF	EG	EH	EI	EJ	EK	EL	EM	EN	EO
F0	FA	FB	FC	FD	FE	FF	FG	FH	FI	FJ	FK	FL	FM	FN	FO
G0	GA	GB	GC	GD	GE	GF	GG	GH	GI	GJ	GK	GL	GM	GN	GO
H0	HA	HB	HC	HD	HE	HF	HG	HH	HI	HJ	HK	HL	HM	HN	HO
I0	IA	IB	IC	ID	IE	IF	IG	IH	II	IJ	IK	IL	IM	IN	IO
J0	JA	JB	JC	JD	JE	JF	JG	JH	JI	JJ	JK	JL	JM	JN	JO
K0	KA	KB	KC	KD	KE	KF	KG	KH	KI	KJ	KK	KL	KM	KN	KO
L0	LA	LB	LC	LD	LE	LF	LG	LH	LI	LJ	LK	LL	LM	LN	LO
M0	MA	MB	MC	MD	ME	MF	MG	MH	MI	MJ	MK	ML	MM	MN	MO
N0	NA	NB	NC	ND	NE	NF	NG	NH	NI	NJ	NK	NL	NM	NN	NO
O0	OA	OB	OC	OD	OE	OF	OG	OH	OI	OJ	OK	OL	OM	ON	OO

This number system works similar to our base 10 system, only we have used alphabets instead of numerals and we have increased our digits up-to 16, every number system has its own rules of calculations as in our base 10 system 1+1=2 but in this system A+A gives B and B + D = F.

2.3.1 Number Line: The complete set of these combinations comprises of positive combinations and negative combinations and these can be represented by a number line as



On both sides of the centre point of 0 the digits go on increasing

2.3.2 Tables of multiplication: The following tables gives some idea of how multiplication work in this system and these are almost similar as of our base 10 system.

●A×A	= A	●B×A	= B	●C×A	= C	●D×A	= D
●A×B	= B	●B×B	= D	●C×B	= F	●D×B	= H
●A×C	= C	●B×C	= F	●C×C	= I	●D×C	= L
●A×D	= D	●B×D	= H	●C×D	= L	●D×D	= A0
●A×E	= E	●B×E	= J	●C×E	= O	●D×E	= AD
●A×F	= F	●B×F	= L	●C×F	= AB	●D×F	= AH
●A×G	= G	●B×G	= N	●C×G	= AE	●D×G	= AL
●A×H	= H	●B×H	= A0	●C×H	= AH	●D×H	= B0
●A×I	= I	●B×I	= AB	●C×I	= AK	●D×I	= BD
●A×J	= J	●B×J	= AD	●C×J	= AN	●D×J	= BH
●A×K	= K	●B×K	= AF	●C×K	= BA	●D×K	= BL
●A×L	= L	●B×L	= AH	●C×L	= BD	●D×L	= C0
●A×M	= M	●B×M	= AJ	●C×M	= BG	●D×M	= CD
●A×N	= N	●B×N	= AL	●C×N	= BJ	●D×N	= CH
●A×O	= O	●B×O	= AN	●C×O	= BM	●D×O	= CL
●A×A0	= A0	●B×A0	= B0	●C×A0	= C0	●D×A0	= D0

●E×A	= E	●F×A	= F	●G×A	= G	●H×A	= H
●E×B	= J	●F×B	= L	●G×B	= N	●H×B	= A0
●E×C	= O	●F×C	= AB	●G×C	= AE	●H×C	= AH
●E×D	= AD	●F×D	= AH	●G×D	= AL	●H×D	= B0
●E×E	= AI	●F×E	= AN	●G×E	= BC	●H×E	= BH
●E×F	= AN	●F×F	= BD	●G×F	= BJ	●H×F	= C0
●E×G	= BC	●F×G	= BJ	●G×G	= CA	●H×G	= CH
●E×H	= BH	●F×H	= CO	●G×H	= CH	●H×H	= D0
●E×I	= BM	●F×I	= CF	●G×I	= CO	●H×I	= DH
●E×J	= CB	●F×J	= CL	●G×J	= DF	●H×J	= E0
●E×K	= CG	●F×K	= DB	●G×K	= DM	●H×K	= EH
●E×L	= CL	●F×L	= DH	●G×L	= ED	●H×L	= F0

●E×M	= DA	●F×M	= DN	●G×M	= EK	●H×M	= FH
●E×N	= DF	●F×N	= ED	●G×N	= FB	●H×N	= G0
●E×O	= DK	●F×O	= EJ	●G×O	= FI	●H×O	= GH
●E×A0	= E0	●F×A0	= F0	●G×A0	= G0	●H×A0	= H0

●I×A	= I	●J×A	= J	●K×A	= K	●L×A	= L
●I×B	= AB	●J×B	= AD	●K×B	= AF	●L×B	= AH
●I×C	= AK	●J×C	= AN	●K×C	= BA	●L×C	= BD
●I×D	= BD	●J×D	= BH	●K×D	= BL	●L×D	= CO
●I×E	= BM	●J×E	= CB	●K×E	= CG	●L×E	= CL
●I×F	= CF	●J×F	= CL	●K×F	= DB	●L×F	= DH
●I×G	= CO	●J×G	= DF	●K×G	= DM	●L×G	= ED
●I×H	= DH	●J×H	= E0	●K×H	= EH	●L×H	= F0
●I×I	= EA	●J×I	= EJ	●K×I	= FC	●L×I	= FL
●I×J	= EJ	●J×J	= FD	●K×J	= FN	●L×J	= GH
●I×K	= FC	●J×K	= FN	●K×K	= GI	●L×K	= HD
●I×L	= FL	●J×L	= GH	●K×L	= HD	●L×L	= I0
●I×M	= GE	●J×M	= HB	●K×M	= HO	●L×M	= IL
●I×N	= GN	●J×N	= HL	●K×N	= IJ	●L×N	= JH
●I×O	= HG	●J×O	= IF	●K×O	= JE	●L×O	= KD
●I×A0	= I0	●J×A0	= J0	●K×A0	= K0	●L×A0	= L0

●M×A	= M	●N×A	= N	●O×A	= O
●M×B	= AJ	●N×B	= AL	●O×B	= AN
●M×C	= BG	●N×C	= BJ	●O×C	= BM
●M×D	= CD	●N×D	= CH	●O×D	= CL
●M×E	= DA	●N×E	= DF	●O×E	= DK
●M×F	= DN	●N×F	= ED	●O×F	= EJ
●M×G	= EK	●N×G	= FB	●O×G	= FI
●M×H	= FH	●N×H	= G0	●O×H	= GH
●M×I	= GE	●N×I	= GN	●O×I	= HG
●M×J	= HB	●N×J	= HL	●O×J	= IF
●M×K	= HO	●N×K	= IJ	●O×K	= JE
●M×L	= IL	●N×L	= JH	●O×L	= KD
●M×M	= JI	●N×M	= KF	●O×M	= LC
●M×N	= KF	●N×N	= LD	●O×N	= MB
●M×O	= LC	●N×O	= MB	●O×O	= NA
●M×A0	= M0	●N×A0	= N0	●O×A0	= O0

These tables give the direct results of multiplication of unit digits and by using these we can process out two or three digit multiplications as.

B
A F
× F
(F+B) D → HD

So, AF × F = HD

I didn't derive the other arithmetic operations for this number system, but we can perform all operations in this system. When we perform calculations in any of the number system, we use base 10 system at some point to make easiness in calculation processes because we are so habitual of it and we had memorised every quick trick of calculations in it. But every number system have its own rules of calculations and can work independently as like this base 16 system.

3. RESULTS

3.1 Conversion Rule is

$$(Q)_{10} \rightarrow (X)_m$$

Where

$$(X)_m = Q_n R_n R_{n-1} R_{n-2} \ldots\ldots\ldots R_2 R_1$$

$$Q_n = [\frac{Q_{n-1}}{m}]$$

Where, $\left[\frac{Q}{m}\right]$ ← Integer value of $(\frac{Q}{m})$

$$R_n = m \times \{\frac{Q_{n-1}}{m}\}$$

Where $[\{\frac{Q}{m}\}$ ← fraction value of $(\frac{Q}{m})]$

3.2 Number of terms in a combination n_m in a base m system is given by

$$n_m = n + 1$$
$$m^n \leq Q_{10}$$

4. APPLICATIONS

(a) Gives flexibility of converting a large combination (number) of base 10 in a small combination of base m having large value of m.
(b) Reduces the steps involved and time consumed in calculations.
(c) Gives a path to modify a new mathematics having many more digits.

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