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Expression for the relative change of height when the distance between the viewer and the body changes; and its cosmological applications

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ABSTRACT

This paper; in brief gives the expression for calculating the height in the frame of reference of the viewer when the distance between the viewer and body changes and how we can use the expression to calculate the diameter and distances in the cosmos. Also, it shows why we cannot use trigonometric ratios when we do such an observation, where we have to calculate the height of a body when the viewer is moving.

Keywords— *Relative height, Angle of sight, Distance, Original height*

1. INTRODUCTION

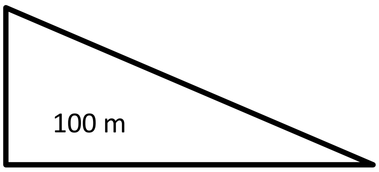
As we know one situation can have many relative manifestations. What we see is actually just a mirage, the actions happening in front of us are just the reality we experience, and they can be different for someone else. Imagine you are looking at a tower which is magnificent, and you are standing just below it, how does it look? Now imagine you have moved away from the tower, now look at the tower, how does it look now?

While you are doing all this imagine your friend is looking at you from a distance from a fixed point, throughout the time. What do you see?

As you might have seen is that the height of the building decreases as we move away from it. Let's see what we get mathematically. Your friend while standing behind tries to calculate the height of the building using trigonometric ratios.

Case 1: when you are close to the building

Let's see how your friend sees when you are moving away from the tower.



Let's assume your friend sees you at a distance of 100 meters from the building and the angle of sight is 45°.

Therefore, $\tan\theta = \frac{height\ of\ building}{distance\ of\ building\ from\ viewer}$

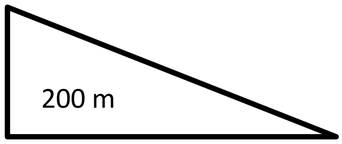
$$\tan 45° = \frac{x}{100}$$

$$x = 100$$

That's why the height of the tower is 100m.

Case 2: when you have moved away from the building.

Now, you have moved behind as we had imagined and your friend sees you going behind.



As you have moved behind, the distance has increased and also the angle has decreased. Let's assume the distance is now 200 meters and the angle of elevation is30°.

Therefore, $\tan\theta = \frac{height\ of\ building}{distance\ of\ building\ from\ viewer}$

$$\tan 30° = \frac{X}{200}$$

$$X = 115.47005$$

As we had imagined, we should get the second height smaller than the first one that is x>X, but we get the opposite that is X>x, which is wrong according to our imagination.

2. WHY DID WE NOT GET THE REQUIRED RESULT USING TRIGONOMETRIC RATIOS?

We didn't get the required result from the trigonometric ratios because in trigonometric ratios there is always a third person involved, who is seeing everything happen and is present in the background (which was your friend in our example). So when we get the height of the tower we get it from his (friend) frame of reference, not ours (viewer).

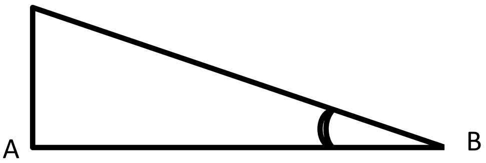
3. THE RELATION BETWEEN DISTANCE FROM THE BODY AND ANGLE OF SIGHT

The angle of sight is the angle with which the viewer sees the top of the body. This angle can either be the angle of elevation or angle of depression.

Now imagine again, that you are looking at the tower and you start moving behind. What happens to your head when you start moving away from the tower? You can see that your head (in order to keep looking at the top of the tower) naturally comes downward which means, that the angle of sight decreases. But now, if you imagine that you are moving towards the tower, what happens to your head? You can see that your head naturally moves upwards, which means the angle of sight increases. So we get the relation,

$$angle\ of\ sight\ \propto \frac{1}{distance}$$

In trigonometry or in triangles,



The angle shown above remains constant even when the distance AB changes, up to a certain limit. But in real life when we move this angle keeps changing.

First conclusion: If we want to calculate the height of the body relative to the viewer when the viewer is moving then we cannot use trigonometric ratios.

4. HOW DO WE CALCULATE THE HEIGHT OF THE BODY IN THE FRAME OF REFERENCE OF THE VIEWER?

For this, we need to derive an expression. Again let's imagine we are looking at the tower and we should think of the factors that are affecting the apparent height of the building.

So by imagining we get one thing for sure that the distance from the body is a factor, the farther we go the smaller the body looks. The second one is the angle of sight, more the angle taller the body appears. The third is the original height of the building. The original height is a factor because; if you see man and giraffe from an equal distance, which one appears bigger? It is the giraffe; hence original height is a factor. The apparent height depends on the distance from the body, the angle of sight and the original height of the body. So the mathematical relation is

$$realtive\ height\ \propto \theta$$

$$realtive\ height\ \propto \frac{1}{d}$$

$$realtive\ height\ \propto h$$

$$\therefore\ \ \Delta h \propto \frac{\theta h}{d}$$

On removing the proportionality,

$$\boldsymbol{\Delta h = k\frac{\theta h}{d}},$$

Where k is the proportionality constant.

Where,
Δh = apparent height
θ = angle of depression/elevation in radian
d = distance from the body

Using this expression we can calculate the height in the frame of reference of the viewer.

5. THE UNITS OF 'K'

The unit of k will be derived from the above expression. The units of the above values-

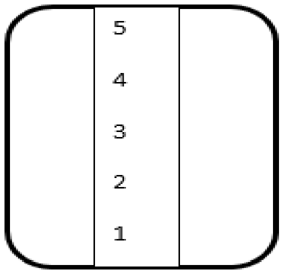
$$m = k\frac{rad * m}{m}$$

$$\therefore\ \ k = m/rad$$

The unit of k will be m/rad.

6. HOW TO CALCULATE THE VALUE OF K?

We need to create a special apparatus to calculate the value of k. We need to create a rectangular sheet of any transparent material with less refractive properties so we get a clear image. This transparent sheet should have the scale printed on it, so we can see through the sheet and we can measure its height relative to us and also we need to measure the angle at which we are looking at the body.



This is how the instrument should be made.

So we have all the values and then just substitute it back into the formula and this is how we can calculate the value of k.

7. WHY DOES THE HEIGHT OF THE BODY VARY RELATIVE TO THE VIEWER WHEN THE VIEWER IS IN MOTION?

When the viewer moves with respect to the tower, then the distance between the viewer and the body changes and as a result the angle of sight changes which results in a change in height of the building relative to the viewer. When the viewer moves away from the body, then the distance between them increases, therefore the angle of sight decreases. And as we know that relative height is directly proportional to the angle of sight and inversely proportional to the distance, as distance increases and angle decreases, and the original height of the body remains constant. We get the result that the height of the body seems to decrease relative to the viewer. But when the viewer moves towards the tower, the opposite happens with the viewer. The distance between them decreases and the angle of sight increases, which results in the body looking bigger.

8. PROOF OF THE FORMULA

We have used the fact that the size of the sun looks different from each planet, so have assumed the size of the sun as seen from earth to be standard for comparison. Also, the angle of viewing the sun is taken as the angle of the axis of rotation of that planet. Also, the height of the body is taken as the sun's diameter because that's the height of the sun.
Here the sun is our body and we are viewing the sun from different planets, this is similar to viewing a tower but here the tower is the sun.

Here for the proof,
d = distance from sun
h = diameter of sun
θ = angle of axis of rotation of the planet
Δh = size of sun seen from that planet
For Earth,
d = $149.6 * 10^9$ m
h = $1.391 * 10^9$ m
θ = 0.410 rad

$\Delta h = x$

The equation we get by substituting the values in the formula. We get,

$$x = 0.410 * \frac{1.391*10^9}{149.6*10^9}k \quad \sim ①$$

For Mars,

d = $22.79 * 10^{10}$ m

h = $1.391 * 10^9$ m

θ = 0.436 rad

$\Delta h = \frac{2}{3}x$

The equation we get by substituting the values in the formula. We get,

$$\frac{2}{3}x = 0.436 * \frac{1.391*10^9}{227.9*10^9}k \quad \sim ②$$

On substituting equation ① in equation ②, we get,

$$\frac{2}{3}\left(0.410 * \frac{1.391 * 10^9}{149.6 * 10^9}k\right) = 0.436 * \frac{1.391 * 10^9}{227.9 * 10^9}k$$

By cancelling all equal terms we get,

$$\frac{2}{3}\left(\frac{0.410}{149.6}\right) = \frac{0.436}{227.9}$$

∴ LHS =$1.82709 * 10^{-3}$

∴ RHS = $1.91312 * 10^{-3}$

∴ $RHS - LHS = 8.603 * 10^{-5}$

∴ LHS ≈ RHS

For Neptune (Triton),

d = $4.495 * 10^{12}$ m

h = $1.391 * 10^9$ m

θ = 0.523599 rad

$\Delta h = \frac{x}{30}$

The equation we get by substituting the values in the formula. We get,

$$\frac{x}{30} = 0.52 * \frac{1.391*10^9}{4.495*10^{12}}k \quad \sim ③$$

On substituting equation ① in equation ③, we get,

$$\frac{1}{30}\left(0.410 * \frac{1.391 * 10^9}{149.6 * 10^9}k\right) = 0.52 * \frac{1.391 * 10^9}{4.495 * 10^{12}}k$$

RHS = $1.60917 * 10^{-4}$

LHS = $1.27074 * 10^{-4}$

$RHS - LHS = 3.3843 * 10^{-5}$

∴ $RHS \approx LHS$

Hence by the above examples, we get the formula is valid.

9. WHY DIDN'T WE GET RHS AND LHS EXACTLY EQUAL?

We didn't get the RHS and LHS exactly equal because-

- While substituting in the formula we didn't consider the whole value of θ
- The size of the sun from each planet as used above is just an approximation and also the above values account for atmospheric refraction and hence there is a slight error
- The distance between the planets is just an approximation.

10. POSSIBLE APPLICATIONS

- The formula above can be used to find the distance between bodies in the cosmos; also if they are luminous, we can calculate the intensity of light coming from the star and its brightness. We can estimate the brightness by the relation-

$$intensity \propto \frac{1}{distance^2}$$

- Also if we know the distance between Earth and that body, then we can calculate the diameter of that body.
- Also, this formula can be used to improve the GPS technology, if we could imply this formula in an actual satellite then the satellite could also identify the dimensions of bodies.

11. SUMMARY

Trigonometric ratios cannot be used when the height relative to the viewer is required. Also when we are calculating height using the trigonometric ratios, the fact should be considered that it is the third person and not relative to the viewer.

We can use $\boldsymbol{\Delta h = k\frac{\theta h}{d}}$ for calculating the height relative to the viewer. Also, it has a wide range of applications in cosmology and communication.

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