INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact factor: 4.295
(Volume 5, Issue 5)
Available online at: www.ijariit.com

A study on properties of M quasi – class (Q) operator

V. Revathi
revathikrishna79@gmail.com
Sri Ramakrishna Engineering College, Coimbatore, Tamil Nadu

P. Maheswari Naik
maheswarinaik21@gmail.com
Sri Ramakrishna Engineering College, Coimbatore, Tamil Nadu

ABSTRACT

In our study, we have introduced the new class of operator M quasi - class (Q) operator acting on a complex Hilbert space H. An operator $T \in$ M quasi - class (Q) if $T\left(T^{*2}T^2\right) = M\left(T^*T\right)^2 T$ where T^* is the adjoint of the operator T and M is a bounded operator. We investigate some basic properties of the operator.

Keywords— *Hilbert Space, Normal, class (Q), Quasi - class (Q), M quasi - class (Q)*

1. INTRODUCTION

All the way through this paper B (H) be the Banach algebra of all bounded linear operators acting on complex Hilbert space h. An operator T is unitary if $TT^* = T^*T = I$ T is isometry if $T^*T = I$, T is normal if $T^*T = TT^*$, T is quasi-normal if $TT^*T = T^*T^2$. An operator $T \in B(H)$ is called class *(Q)* if $T^{*2}T^2 = \left(T^*T\right)^2$, T is quasi - class *(Q)* if $T\left(T^{*2}T^2\right) = \left(T^*T\right)^2 T$. Let $T = U + iV$, where $U = \mathrm{Re}\,(T) = \dfrac{T+T^*}{2}$ and $V = \mathrm{Im}(T) = \dfrac{T-T^*}{2i}$ are the real and imaginary parts of *T*. We shall write $B^2 = \left(T^*T\right)^2$ and $C^2 = T^{*2}T^2$, where B and C are non - negative definite. The Class *(Q)* operator was introduced at first by A. A. S. Jibril in 2010 and has given some properties of this operator, but this concept was generalized by researchers such as S. Panayappan and N. Sivamani new type of class *(Q)* operator is said to be n-power class *(Q)* operator. In this research, we introduce another class of class *(Q)* operator which is M quasi - class *(Q)* operator and has studied some basic properties of the operator.

2. PRELIMINARIES

2.1 Class (Q) operator

An operator T on H is called class (Q) if $T^{*2}T^2 = \left(T^*T\right)^2$.

2.2 Quasi - class (Q) operator

An operator T on H is called quasi - class (Q) if $T\left(T^{*2}T^2\right) = \left(T^*T\right)^2 T$.

2.3 *M* quasi - class (Q) operator

An operator $T \in B(H)$ is called M quasi - class (Q) if $T\left(T^{*2}T^2\right) = M\left(T^*T\right)^2 T$ where M is a bounded operator from a complex Hilbert Space to itself.

3. MAIN RESULT

3.1 Theorem

A power of M quasi - class (Q) is also an M quasi - class (Q) operator.

3.2 Proof

Let T be an M quasi - class (Q) operator.
We prove the assertion by the mathematical Induction.

T is M quasi - class (Q), the result is true for $m=1$.
That is,

$$T\left(T^{*2}T^{2}\right)=M\left(T^{*}T\right)^{2}T \tag{1}$$

Let us prove the result $m=n$.
That is,

$$\left(T\left(T^{*2}T^{2}\right)\right)^{n}=\left(M\left(T^{*}T\right)^{2}T\right)^{n} \tag{2}$$

Let us prove the result $m=n+1$.
That is,

$$\left[T\left(T^{*2}T^{2}\right)\right]^{n+1}=\left[M\left(T^{*}T\right)^{2}T\right]^{n+1}$$

i.e., $$\left[T\left(T^{*2}T^{2}\right)\right]^{n+1}=\left[MT\left(T^{*2}T^{2}\right)\right]^{n}\left[MT\left(T^{*2}T^{2}\right)\right]$$

$$=\left[M\left(T^{*}T\right)^{2}T\right]^{n}\left[M\left(T^{*}T\right)^{2}T\right] \text{ by (1) and (2)}$$

$$\left[T\left(T^{*2}T^{2}\right)\right]^{n+1}=\left[M\left(T^{*}T\right)^{2}T\right]^{n+1}$$

Thus the result is true for $m=n+1$. Therefore T^{n} is also M quasi - class (Q) for each n.

4. PROPERTIES OF *M* QUASI - CLASS (*Q*) OPERATOR

The following theorem gives some Properties of M quasi - class (Q) Operators.

4.1 Theorem

Let $T \in B(H)$ is an operator if C commutes with U & V and $C^{2}T=\mathrm{M}C^{2}T$ then T is M quasi - class (Q), where $B^{2}=\left(T^{*}T\right)^{2}$, $C^{2}=T^{*2}T^{2}$, $U=\mathrm{Re}(T)=\dfrac{T+T^{*}}{2}$ $V=\mathrm{Im}(T)=\dfrac{T-T^{*}}{2i}$.

Proof. Since $CU=UC$, $BV=VB$

So $C^{2}U=UC^{2}$, $B^{2}V=VB^{2}$

Then $C^{2}T+C^{2}(T)^{*}=TC^{2}+(T)^{*}C^{2}$

$$C^{2}T-C^{2}(T)^{*}=TC^{2}-(T)^{*}C^{2}$$

This gives $TC^{2}=C^{2}T$

$$\therefore T(T^{*2}T^{2})=\left(\left(T^{*}(T^{*}T)T\right)T\right.$$

$$=\left(T^{*}T\right)^{2}T$$

by using the condition $TB^{2}=M\,B^{2}T$

we get $T(T^{*2}T^{2})=M\left(T^{*}\left(T^{*}T\right)T\right)T$

$$T(T^{*2}T^{2})=M\left(T^{*}T\right)^{2}T$$

Then T is M quasi - class (Q) operator.

4.2 Theorem

It $T \in B(H)$ is a class (Q) operator such that $C^{2}U=\dfrac{1}{\mathrm{M}}UC^{2}$, $C^{2}V=\dfrac{1}{\mathrm{M}}VC^{2}$ then T is M quasi - class (Q) operator.

Proof.

$$C^2U = \frac{1}{M}UC^2 \ , \ C^2V = \frac{1}{M}VC^2$$

Then we have

$$C^2(U + iV) = \frac{1}{M}(U + iV)C^2$$

And

$$C^2T = \frac{1}{M}TC^2$$

Therefore

$$\left(T^*\left(T^*T\right)T\right)T = \frac{1}{M}T\left(T^*\left(T^*T\right)T\right)$$

$$T\left(T^*\left(T^*T\right)T\right) = M\left(T^*\left(T^*T\right)T\right)T$$

$$= M\left(T^*T\right)^2 T \qquad \text{(since } T \text{ is a class(Q) operator)}$$

Then T is M quasi - class (Q) operator.

4.3 Theorem

Let T_1, T_2 be two M quasi - class (Q) operators from H to H such that $T_1T_2^{*2} = T_2T_1^{*2} = T_1^{*2}T_2^2 = T_2^{*2}T_1^2 = 0$. Then $T_1 + T_2$ is M quasi - class (Q) operator.

Proof.

Consider T_1, T_2 be two M quasi - class (Q) operators.

$$(T_1 + T_2)\left[(T_1 + T_2)^{*2}(T_1 + T_2)^2\right] = (T_1 + T_2)\left[\left(T_2^{*2} + T_1^{*2}\right)\left(T_1^2 + T_2^2\right)\right]$$

$$= (T_1 + T_2)\left[T_2^{*2}T_1^2 + T_2^{*2}T_2^2 + T_1^{*2}T_1^2 + T_1^{*2}T_2^2\right]$$

$$= (T_1 + T_2)\left[T_2^{*2}T_2^2 + T_1^{*2}T_1^2\right] \qquad \text{since } T_2^{*2}T_1^2 = T_1^{*2}T_2^2 = 0$$

$$= (T_1 + T_2)\left[T_2^{*2}T_2^2 + T_1^{*2}T_1^2\right]$$

$$= T_1T_1^{*2}T_1^2 + T_2T_2^{*2}T_2^2 \qquad \text{since } T_1T_2^{*2}T_2^2 = T_2T_1^{*2}T_1^2 = 0$$

$$= M\left(T_1^{*2}T_1^2\right)T_1 + M\left(T_2^{*2}T_2^2\right)T_2$$

$$= M\left(T_1^*T_1\right)^2T_1 + M\left(T_2^*T_2\right)^2T_2$$

Hence $T_1 + T_2$ is M quasi - class (Q) operator.

4.4 Theorem

Let T_1, T_2 be two M quasi - class (Q) operators from H to H such that $T_1T_2^{*2} = T_2T_1^{*2} = T_1^{*2}T_2^2 = T_2^{*2}T_1^2 = 0$. Then $T_1 - T_2$ is M quasi - class (Q) operator.

4.5 Theorem

Let $T_1 \in M$ quasi - class (Q) operator and $T_2 \in$ quasi - class (Q) operator.

Then $T_1T_2 \in M$ quasi-class (Q) operator if

a) T_1 and T_2 are commutes with each other.

b) $T_1^2T_2^{*2}$ is equal to $T_2^{*2}T_1^2$

Proof.

$T_1 \in M$ quasi - class (Q) operator, and

$T_2 \in$ quasi - class (Q) operator

$$(T_1T_2)\left[(T_1T_2)^{*2}(T_1T_2)^2\right] = (T_1T_2)\left[\left(T_2^{*}T_1^{*}\right)^2(T_1T_2)^2\right]$$

$$= (T_1T_2)\left[\left(T_2^{*2}T_1^{*2}\right)\left(T_1^{2}T_2^{2}\right)\right]$$

$$= (T_1T_2)\left[\left(T_1^{*2}T_2^{*2}\right)\left(T_1^{2}T_2^{2}\right)\right]$$

$$= T_1\left(T_2T_1^{*2}\right)\left(T_2^{*2}T_1^{2}\right)T_2^{2}$$

$$= T_1\left(T_1^{*2}T_2\right)\left(T_1^{2}T_2^{*2}\right)T_2^{2}$$

$$= T_1T_1^{*2}T_2T_1^{2}T_2^{*2}T_2^{2}$$

$$= T_1T_1^{*2}T_1^{2}T_2T_2^{*2}T_2^{2}$$

$$= M\left(T_1^{*2}T_1^{2}\right)T_1M\left(T_2^{*2}T_2^{2}\right)T_2$$

$$= M\left(T_1^{*2}\left(T_1^{2}T_1\right)\left(T_2^{*2}T_2^{2}\right)T_2\right)$$

$$= M\left(T_1^{*2}\left(T_1^{2}T_1\right)\left(T_2^{*2}T_2^{2}\right)T_2\right)$$

$$= M\left(T_1^{*2}T_1^{2}T_2^{*2}T_1T_2^{2}T_2\right)$$

$$= M\left(T_1^{*2}T_2^{*2}T_1^{2}T_2^{2}T_1T_2\right)$$

$$= M\left[(T_2T_1)^{*2}(T_1T_2)^2(T_1T_2)\right]$$

$$= M\left[(T_1T_2)^{*2}(T_1T_2)^2\right](T_1T_2)$$

$$= M\left[(T_1T_2)^{*}(T_1T_2)\right]^2(T_1T_2)$$

Hence the product $T_1 T_2$ is M quasi - class (Q) operator.

4. REFERENCES

[1] A.A.S.Jibril, on operators for which $\left(T^{*2}T^{2}\right) = \left(T^{*}T\right)^{2}$, International Mathematics Forum, 5, 2010, 46, 2255-2262.
[2] A.A.S.Jibril, on n power normal operators, The Arabian Journal for Science and Engineering Volume 33, Number 2A.
[3] S.Panayappan, N.Sivamani, on n power class (Q) operators, Int.Journal of Math. Analysis, 6, No. 31 (2012), 1513-1518.
[4] T.Veluchamy, K.M.Manikandan, on n Power Quasi Normal Operators on the Hilbert Space.

