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RP-101: Formulation of a special class of standard bi-quadratic congruence of composite modulus

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ABSTRACT

In this paper, a very special class of bi-quadratic congruence of composite modulus is formulated. Such congruence has a very large number of solutions. The solutions can be obtained in the least time. Sometimes it became possible to find the solutions orally. Thus the formulation of solutions is the merit of the paper. This formulation made the study of bi-quadratic congruence very interesting and attractive for the readers. The first time a huge number of solutions can be calculated mentally. This is one more merit of the paper.

Keywords— *Bi-Quadratic congruence, Binomial expansion formula, Composite modulus, Formulation*

1. INTRODUCTION

A solvable standard bi-quadratic congruence of composite modulus is a congruence of the type $x^4 \equiv a^4 \ (mod\ m)$; m being a prime or composite integer. The solutions are the values of x that satisfy the congruence. If m is a prime positive integer, the congruence is called a standard bi-quadratic congruence of prime modulus. But if m is a composite integer, then it is called a standard bi-quadratic congruence of composite modulus. Such types of congruence are always solvable.

2. LITERATURE-REVIEW

In the literature of mathematics, nowhere is found the discussion or the formulation of the said congruence. A definition of bi-quadratic residue and three problems are mentioned in [6]. Earlier mathematicians (it seems) were not interested in it. It is found in the internet that only Gauss had thought of it. But couldn't find any way to the solutions. The author's understanding opens the gate of entry to the solutions of the congruence directly. The author already formulated many standard bi-quadratic congruence of prime and composite modulus [1], [2], [3], [4], [5].

3. NEED OF RESEARCH

Even the author found one more such special bi-quadratic congruence of composite modulus, yet remained to formulate. Here, in this paper, the author considered such congruence for formulation and his efforts are presented in this paper. This is the need of the paper.

4. PROBLEM-STATEMENT

Here, the problem is "To formulate the solutions of the congruence $x^4 \equiv a^4 \ (mod\ b.a^m); a, b\ are$ positive integers, $b \neq al,\ m \geq 4$".

5. ANALYSIS AND RESULTS

Consider the congruence $x^4 \equiv a^4 \ (mod\ b.a^m); a, b\ are$ positive integers, $b \neq al.$

For its solutions, let us consider that

$$x \equiv b.a^{m-3}k \pm a \ \ (mod\ b.a^m).$$

Then

$$x^4 \equiv (b.a^{m-3}k \pm a)^4 \ (\text{mod b}.a^m)$$

$$\equiv (\ b.a^{m-3}k)^4 \pm 4.(b.a^{m-3}k)^3.a + 6.(b.a^{m-3}k)^2.a^2 \pm 4.(b.a^{m-3}k).a^3 + a^4$$

$$\equiv a^4 \ (mod\ b.a^m)$$

By binomial expansion formula.

Thus,

$x \equiv b.a^{m-3}k \pm a \ \ (mod\ b.a^m)$ is a solution of the said congruence.

But, if we consider $k = a^3$, then

$$x \equiv b.a^{m-3}k \pm a \ \ (mod\ b.a^m)$$

$$\equiv b.a^{m-3}.a^3 \pm a \ (mod\ b.a^m)$$

$$\equiv b.a^m \pm a \ \ (mod\ b.a^m)$$

$$\equiv 0 \pm a \equiv a \ (mod\ b.a^m)$$

Which is the same solution as for $k = 0$.

Similarly, for higher values of k, the solutions repeats as for $k = 1, 2, 3, \dots$..

Therefore, all the required solutions are given by

$$x \equiv b.a^{m-3}k \pm a \ (mod\ b.a^m); k = 0, 1, 2, \dots \dots \dots, (a^3 - 1).$$

These are $2a^3$ incongruent solutions for all values of k. The congruence has two solutions for every value of k and k has a^3 different values.

In particular, if $b = 1, then$ one get the solutions

$$x \equiv a^{m-3}k \pm a \ (mod\ a^m); k = 0, 1, 2, \dots \dots \dots, (a^3 - 1).$$

5.1 Illustrations

Consider the congruence $x^4 \equiv 81 \ (mod\ 1215\ )$.

It can be written as $x^4 \equiv 3^4 \ (mod\ 5.3^5)$.

It is of the type

$$x^4 \equiv a^4 \ (mod\ b.a^m); \ \ with\ a = 3, m = 5, b = 5.$$

The solutions are

$$x \equiv b.a^{m-3}k \pm a \ (mod\ b.a^m); k = 0, 1, 2, \dots \dots \dots, (a^3 - 1).$$

$$\equiv 5.3^{5-3}k \pm 3 \ (mod\ 5.3^5); k = 0, 1, 2, \dots \dots \dots \dots .., 27 - 1.$$

$$\equiv 45k \pm 3 \ (mod\ 1215); k = 0, 1, 2, 3, \dots \dots .26.$$

$$\equiv 0 \pm 3; 45 \pm 3; 90 \pm 3; \dots \dots \dots \dots; 1170 \pm 3 \ (mod\ 1215)$$

$$\equiv 3, 1212; 42, 48; 87, 93; \dots \dots \dots \dots; 1167, 1173 \ (mod\ 1215).$$

$$\equiv 3, 42, 48, 87, 93, \dots \dots \dots \dots, 1167, 1173, 1212 \ (mod\ 1215).$$

There are 54 solutions of the congruence!!!

Consider the congruence when $b = 1.\ i.e.\ the\ congruence\ x^4 \equiv 2401 \ (mod\ 16807)$.

It is of the type

$$x^4 \equiv a^4 (mod\ a^m)\ with\ a = 7, m = 5.$$

The solutions are

$$x \equiv a^{m-3}k \pm a \ (mod\ a^m); k = 0, 1, 2, \dots \dots \dots, (a^3 - 1).$$

$$\equiv 7^{5-3}k \pm 7 \ (mod\ 7^5); k = 0, 1, 2, \dots \dots \dots, (7^3 - 1).$$

$$\equiv 49k \pm 7 \ (mod\ 16807); k = 0,1, 2, 3, \dots \dots \dots \dots, 342.$$

$$\equiv 0 \pm 7; 49 \pm 7; 98 \pm 7; \dots \dots \dots \dots \dots; 16758 \pm 7 \ (mod\ 16807).$$

$$\equiv 7, 16800; 42, 56; 91, 105; \dots \dots \dots \dots; 16751, 16765 \ (mod\ 16807).$$

$$\equiv 7, 42, 56, 91, 105, \dots \dots \dots \dots \dots ., 16751, 16765, 16800 \ (mod\ 16807).$$

There are 686 solutions of the congruence!!!

6. CONCLUSION

Therefore, it can be concluded that the congruence $x^4 \equiv a^4 \ (mod\ b.a^m)$ has $2a^3$ incongruent solutions

$$x \equiv b.a^{m-3}k \pm a \ (mod\ b.a^m); k = 0, 1, 2, \dots \dots .(a^3 - 1).$$

But if

$b = 1, then\ the\ congruence\ x^4 \equiv a^4\ (mod\ a^m)$ also has $2a^3$ incongruent

Solutions given by

$$x \equiv a^{m-3}k \pm a\ (mod\ a^m); k = 0, 1, 2, \dots \dots . (a^3 - 1).$$

7. MERIT OF THE PAPER

Here, in this paper, a special class of bi-quadratic congruence of composite modulus is formulated. Formulation is the merit of the paper**.**

8. REFERENCES

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