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E-super edge bimagic graceful labeling of graphs

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ABSTRACT

For a graph $G=(V,E)$ an injective mapping $f:V\cup E\rightarrow\{1,2,3,\ldots,p+q\}$ is an edge bimagic graceful labeling if there are two constants k_1 and k_2 such that for every edge $uv\in E$, $wt(uv)=|f(u)+f(v)-f(uv)|=k_1,k_2$. An edge bimagic graceful labeling f is called an E-Super edge bimagic graceful labeling if $f(E)=\{1,2,3,\ldots,q\}$ as well as $f(V)=\{q+1,q+2,\ldots,q+p\}$. A graph G is called an E-Super edge bimagic graceful if there exists an E-Super edge Bimagic Graceful Labeling (ESEBGL). In this paper we introduce E-Super edge bimagic graceful labeling of some graphs.

Keywords— *Bi Magic Labelin, Edge Bimagic Labeling, E Super Edge Bimagic Labeling*

1. INTRODUCTION

In this paper, we consider only finite simple undirected graphs with order *p* and size *q*. For graph theoretic notation we follow [4], [5]. A labeling of a graph *G* is a mapping that carries a set of graph elements, usually integers. Many kinds of labeling have been studied and an excellent survey of graph labeling can be found in [2].

In 1966[6],A. Rosa introduced a new graph labeling called β- labeling in which the vertices are labeled with distinct numbers chosen from 0 to *m*, where *m* is the number of edges, such that each edge is labeled with the absolute difference of the labels of its end vertices and it is unique in the graph. A few years later, S.W. Golomb [3] renamed β-labeling as graceful labeling as it is known today.

In 2004, [1] the notion of edge bimagic labeling was introduced by Baskar Babujee. A Graph *G* is called an edge bimagic if the bijection $g:V\cup E\rightarrow\{1,2,\ldots,p+q\}$ such that $g(u)+g(v)+g(uv)$ is either c_1 or c_2, $\forall uv\in E$. An edge bimagic graph is called *E*-Super edge bimagic if the set of edge labels are $\{1,2,\ldots,q\}$.

In this paper, we study the *E*-Super edge bimagic graceful labeling of some graphs.

2. E-SUPER EDGE BIMAGIC GRACEFUL LABELING OF GRAPHS

2.1 Theorem 1

The Friendship graph $F_n, n\geq 3$ has an *E-Super* edge bimagic graceful labeling.

Proof

Let the Friendship graph F_n be with $2n+1$ vertices and $3n$ edges and let

$$V(F_n)=\{u_i\mid 1\leq i\leq 2n\}\cup\{u\}\text{ And }E(F_n)=\{uu_i\mid 1\leq i\leq 2n\}\cup\{u_{2i-1}u_{2i}\mid 1\leq i\leq n\}.$$

Now, we define the bijection as $f:V\cup E\rightarrow\{1,2,\ldots,5n+1\}$ follows.

Case (i)

For odd n, $n \geq 3$.

$$f(u) = 3n + 1$$
$$f(u_{2i-1}) = 3n + 1 + i \text{ , } 1 \leq i \leq n$$
$$f(u_{2i}) = 4n + 1 + i \text{ , } 1 \leq i \leq n$$
$$f(uu_{2i-1}) = i \text{ , } 1 \leq i \leq n$$
$$f(uu_{2i}) = n + i \text{ , } 1 \leq i \leq n$$
$$f(u_{2i-1}u_{2i}) = \begin{cases} 2n + 2i \text{ , } 1 \leq i \leq \frac{n-1}{2} \\ n + 2i \text{ , } \frac{n+1}{2} \leq i \leq n \end{cases}$$

Now, the weights are,

For $1 \leq i \leq n$,

$$\begin{aligned} wt(uu_{2i-1}) &= |f(u) + f(u_{2i-1}) - f(uu_{2i-1})| \\ &= |3n + 1 + 3n + 1 + i - i| \\ &= 6n + 2 \end{aligned}$$

For, $1 \leq i \leq n$

$$\begin{aligned} wt(uu_{2i}) &= |f(u) + f(u_{2i}) - f(uu_{2i})| \\ &= |3n + 1 + 4n + 1 + i - n - i| \\ &= 6n + 2 \end{aligned}$$

For $1 \leq i \leq \frac{n-1}{2}$,

$$\begin{aligned} wt(u_{2i-1}u_{2i}) &= |f(u_{2i-1}) + f(u_{2i}) - f(u_{2i-1}u_{2i})| \\ &= |3n + 1 + i + 4n + 1 + i - 2n - 2i| \\ &= 5n + 2 \end{aligned}$$

For $\frac{n+1}{2} \leq i \leq n$,

$$\begin{aligned} wt(u_{2i-1}u_{2i}) &= |f(u_{2i-1}) + f(u_{2i}) - f(u_{2i-1}u_{2i})| \\ &= |3n + 1 + i + 4n + 1 + i - n - 2i| \\ &= 6n + 2 \end{aligned}$$

Let the magic constants $6n + 2$ and $5n + 2$ be k_1 and k_2 respectively.

Case (ii)

For even n, $n \geq 4$.

$$f(u) = 3n + 1$$
$$f(u_{2i-1}) = 3n + 1 + i \text{ , } 1 \leq i \leq n$$
$$f(u_{2i}) = 4n + 1 + i \text{ , } 1 \leq i \leq n$$
$$f(uu_{2i-1}) = i \text{ , } 1 \leq i \leq n$$
$$f(uu_{2i}) = n + i \text{ , } 1 \leq i \leq n$$
$$f(u_{2i-1}u_{2i}) = \begin{cases} 2n - 1 + 2i \text{ , } 1 \leq i \leq \frac{n}{2} \\ n + 2i \text{ , } \frac{n+2}{2} \leq i \leq n \end{cases}$$

Now, the weights are,

For $1 \le i \le n$,

$$\begin{aligned} wt(uu_{2i-1}) &= |f(u) + f(u_{2i-1}) - f(uu_{2i-1})| \\ &= |3n+1+3n+1+i-i| \\ &= 6n+2 \end{aligned}$$

For, $1 \le i \le n$,

$$\begin{aligned} wt(uu_{2i}) &= |f(u) + f(u_{2i}) - f(uu_{2i})| \\ &= |3n+1+4n+1+i-n-i| \\ &= 6n+2 \end{aligned}$$

For $1 \le i \le \frac{n}{2}$,

$$\begin{aligned} wt(u_{2i-1}u_{2i}) &= |f(u_{2i-1}) + f(u_{2i}) - f(u_{2i-1}u_{2i})| \\ &= |3n+1+i+4n+1+i-2n+1-2i| \\ &= 5n+3 \end{aligned}$$

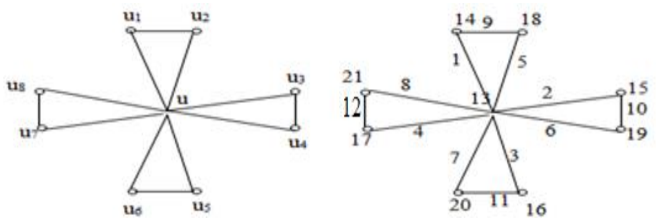
For $\frac{n+2}{2} \le i \le n$,

$$\begin{aligned} wt(u_{2i-1}u_{2i}) &= |f(u_{2i-1}) + f(u_{2i}) - f(u_{2i-1}u_{2i})| \\ &= |3n+1+i+4n+1+i-n-2i| \\ &= 6n+2 \end{aligned}$$

Let the magic constants $5n+3$ and $6n+2$ be k_1 and k_2 respectively.

Hence, the Friendship graph F_n has *E-Super* edge bimagic graceful labeling.

Example: *E*-Super edge bimagic graceful labeling of F_4 is given in the following figure.



2.2 Theorem 2

The Fan graph $F_{1,n}, n \ge 3$ has an *E-Super* edge bimagic graceful labeling.

Proof

Let the Fan graph $F_{1,n}$ be with $n+1$ vertices and $2n-1$ edges and let

$V(F_{1,n}) = \{u_i \mid 1 \le i \le n\} \cup \{u\}$ And $E(F_{1,n}) = \{u_i u_{i+1} \mid 1 \le i \le n-1\} \cup \{uu_i \mid 1 \le i \le n\}$

Now, we define the bijection as $f : V \cup E \to \{1,2,\ldots,3n\}$ as follows.

Case (i):

For odd *n*, $n \ge 3$.

$$f(u) = 2n$$

$$f(u_i) = 2n+i, \; 1 \le i \le n$$

$$f(uu_i) = i, \; 1 \le i \le n$$

$$f(u_i u_{i+1}) = \begin{cases} n-1+2i, & 1 \le i \le \frac{n-1}{2} \\ 2i+1, & \frac{n+1}{2} \le i \le n-1 \end{cases}$$

Now, the weights are,

For, $1 \le i \le n$

$$\begin{aligned} wt(uu_i) &= |f(u) + f(u_i) - f(uu_i)| \\ &= |2n + 2n + i - i| \\ &= 4n \end{aligned}$$

For, $1 \le i \le \frac{n-1}{2}$

$$\begin{aligned} wt(u_i u_{i+1}) &= |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})| \\ &= |2n + i + 2n + i + 1 - n + 1 - 2i| \\ &= 3n + 2 \end{aligned}$$

For, $\frac{n+1}{2} \le i \le n-1$

$$\begin{aligned} wt(u_i u_{i+1}) &= |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})| \\ &= |2n + i + 2n + i + 1 - 2i - 1| \\ &= 4n \end{aligned}$$

Let the magic constants $4n$ and $3n+2$ be k_1 and k_2 respectively.

Case (ii)

For even *n*, $n \ge 4$.

$$\begin{aligned} f(u) &= 2n \\ f(u_i) &= 2n + i \ , 1 \le i \le n \\ f(uu_i) &= i, \ 1 \le i \le n \\ f(u_i u_{i+1}) &= \begin{cases} n + 2i \ , & 1 \le i \le \frac{n-2}{2} \\ 2i + 1 \ , & \frac{n}{2} \le i \le n-1 \end{cases} \end{aligned}$$

Now, the weights are,

For, $1 \le i \le n$

$$\begin{aligned} wt(uu_i) &= |f(u) + f(u_i) - f(uu_i)| \\ &= |2n + 2n + i - i| \\ &= 4n \end{aligned}$$

For, $1 \le i \le \frac{n-2}{2}$

$$\begin{aligned} wt(u_i u_{i+1}) &= |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})| \\ &= |2n + i + 2n + i + 1 - n - 2i| \\ &= 3n + 1 \end{aligned}$$

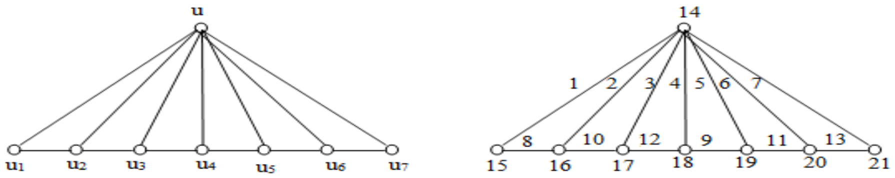
For, $\frac{n}{2} \le i \le n-1$

$$\begin{aligned} wt(u_i u_{i+1}) &= |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})| \\ &= |2n + i + 2n + i + 1 - 2i - 1| \\ &= 4n \end{aligned}$$

Let the magic constants $4n$ and $3n+1$ be k_1 and k_2 respectively.

Hence, the Fan graph $F_{1,n}, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Example: *E*-Super edge bimagic graceful labeling of $F_{1,7}$ is given in the following figure.



2.3 Theorem 3

The graph $C_n + K_1, n \geq 3$ has an *E*-Super edge bimagic graceful labelin .

Proof

Let the graph $C_n + K_1$ be with $n+1$ vertices and $2n$ edges and let $V(C_n + K_1) = \{u_i \mid 1 \leq i \leq n\} \cup \{u\}$ and

$$E(C_n + K_1) = \{u_i u_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_1 u_n\} \cup \{uu_i \mid 1 \leq i \leq n\}.$$

Now, we define the bijection as $f : V \cup E \to \{1, 2, \ldots, 3n+1\}$ as follows.

Case (i):

For odd, $n \geq 3$.

$$f(u) = 3n+1$$
$$f(u_i) = 2n+i,\ 1 \leq i \leq n$$
$$f(uu_i) = n+i, 1 \leq i \leq n$$
$$f(u_i u_{i+1}) = 2i, 1 \leq i \leq \frac{n-1}{2}$$
$$f\left(u_{\frac{n-1}{2}+i} u_{\frac{n-1}{2}+i+1}\right) = 2i-1,\ 1 \leq i \leq \frac{n-1}{2}$$
$$f(u_n u_1) = n$$

Now, the weights are,

$$\begin{aligned} wt(uu_i) &= |f(u) + f(u_i) - f(uu_i)| \\ &= |3n+1+2n+i-n-i| \\ &= 4n+1 \end{aligned}$$

For, $1 \leq i \leq \frac{n-1}{2}$

$$\begin{aligned} wt(u_i u_{i+1}) &= |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})| \\ &= |2n+i+2n+i+1-2i| \\ &= 4n+1 \end{aligned}$$

For, $1 \leq i \leq \frac{n-1}{2}$

$$\begin{aligned} wt\left(u_{\frac{n-1}{2}+i} u_{\frac{n-1}{2}+i+1}\right) &= \left| f\left(u_{\frac{n-1}{2}+i}\right) + f\left(u_{\frac{n-1}{2}+i+1}\right) - f\left(u_{\frac{n-1}{2}+i} u_{\frac{n-1}{2}+i+1}\right) \right| \\ &= \left| 2n + \frac{n-1}{2} + i + 2n + \frac{n-1}{2} + i + 1 - 2i + 1 \right| \\ &= 5n+1 \end{aligned}$$

$$wt(u_n u_1) = |f(u_n) + f(u_1) - f(u_n u_1)|$$
$$= |2n + n + 2n + 1 - n|$$
$$= 4n + 1$$

Let the magic constants $4n+1$ and $5n+1$ be k_1 and k_2 respectively.

Case (ii)
For even $n \geq 4$.

$$f(u) = 3n+1$$
$$f(u_i) = 2n+i,\ 1 \leq i \leq n$$
$$f(uu_i) = n+i, 1 \leq i \leq n$$
$$f(u_i u_{i+1}) = 2i, 1 \leq i \leq \frac{n}{2}$$
$$f\left(u_{\frac{n}{2}+i} u_{\frac{n}{2}+i+1}\right) = 2i+1,\ 1 \leq i \leq \frac{n-2}{2}$$
$$f(u_n u_1) = 1$$

Now, the weights are,

$$wt(uu_i) = |f(u) + f(u_i) - f(uu_i)|$$
$$= |3n+1+2n+i-n-i|$$
$$= 4n+1$$

For, $1 \leq i \leq \frac{n}{2}$

$$wt(u_i u_{i+1}) = |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})|$$
$$= |2n+i+2n+i+1-2i|$$
$$= 4n+1$$

For, $1 \leq i \leq \frac{n-2}{2}$

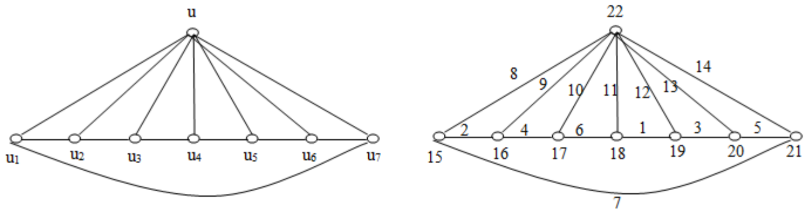
$$wt\left(u_{\frac{n}{2}+i} u_{\frac{n}{2}+i+1}\right) = \left|f\left(u_{\frac{n}{2}+i}\right) + f\left(u_{\frac{n}{2}+i+1}\right) - f\left(u_{\frac{n}{2}+i} u_{\frac{n}{2}+i+1}\right)\right|$$
$$= \left|2n + \frac{n}{2} + i + 2n + \frac{n}{2} + i + 1 - 2i - 1\right|$$
$$= 5n$$

$$wt(u_n u_1) = |f(u_n) + f(u_1) - f(u_n u_1)|$$
$$= |2n+n+2n+1-1|$$
$$= 5n$$

Let the magic constants $4n+1$ and $5n$ be k_1 and k_2 respectively.

Hence, the graph $C_n + K_1, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Example: *E*-Super edge bimagic graceful labeling of $C_7 + K_1$ is given in the following figure.



2.4 Theorem 4

The graph $S_n^2, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Proof:

Let the graph S_n^2 be with $4n+1$ vertices and $4n$ edges and let

$$V(S_n^2) = \{u\} \cup \{u_i \mid 1 \leq i \leq n\} \cup \{v_i \mid 1 \leq i \leq n\} \cup \{u_i^{'} \mid 1 \leq i \leq n\} \cup \{v_i^{'} \mid 1 \leq i \leq n\}$$

$$E(S_n^2) = \{uu_i \mid 1 \leq i \leq n\} \cup \{uu_i^{'} \mid 1 \leq i \leq n\} \cup \{uv_i \mid 1 \leq i \leq n\} \cup \{v_i v'_i \mid 1 \leq i \leq n\}$$

Now, we define the bijection as $f : V \cup E \rightarrow \{1, 2, \ldots, 8n+1\}$ follows.

$$
\begin{aligned}
f(u) &= 4n+1 \\
f(u_i) &= 4n+1+i, \ 1 \leq i \leq n \\
f(v_i) &= 5n+1+i, \ 1 \leq i \leq n \\
f(u_i^{'}) &= 6n+1+i, \ 1 \leq i \leq n \\
f(v_i^{'}) &= 7n+i+1, \ 1 \leq i \leq n \\
f(uu_i) &= i, \ 1 \leq i \leq n \\
f(uv_i) &= n+i, \ 1 \leq i \leq n \\
f(u_i u_i^{'}) &= 2n+2i \ , 1 \leq i \leq n \\
f(v_i v_i^{'}) &= 2n-1+2i, \ 1 \leq i \leq n
\end{aligned}
$$

Now the weights are,
For, $1 \leq i \leq n$

$$
\begin{aligned}
wt(uu_i) &= |f(u) + f(u_i) - f(uu_i)| \\
&= |4n+1+4n+1+i-i| \\
&= 8n+2
\end{aligned}
$$

$$
\begin{aligned}
wt(uv_i) &= |f(u) + f(v_i) - f(uv_i)| \\
&= |4n+1+5n+1+i-n-i| \\
&= 8n+2
\end{aligned}
$$

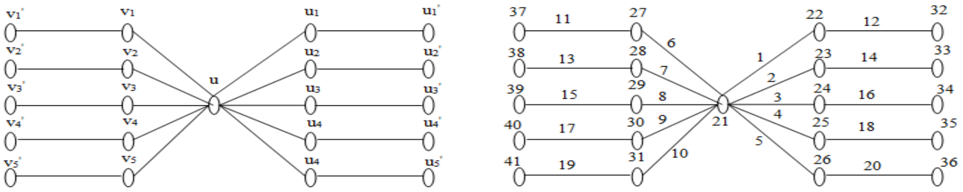
$$
\begin{aligned}
wt(u_i u_i^{'}) &= |f(u_i) + f(u_i^{'}) - f(u_i u_i^{'})| \\
&= |4n+1+i+6n+1+i-2n-2i| \\
&= 8n+2
\end{aligned}
$$

$$
\begin{aligned}
wt(v_i v_i^{'}) &= |f(v_i) + f(v_i^{'}) - f(v_i v_i^{'})| \\
&= |5n+1+i+7n+1+i-2n+1-2i| \\
&= 10n+3
\end{aligned}
$$

Let the magic constants $8n+2$ and $10n+3$ be k_1 and k_2 respectively.

Hence, the graph $S_n^2, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Example: *E*-Super edge bimagic graceful labeling of S_8^2 is given in the following figure.



2.5 Theorem 5

The graph $B_{n,n}^2, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Proof:

Let the graph $B_{n,n}^2$ be with $2n+2$ vertices and $3n$ edges and let

$$V(B_{n,n}^2) = \{u\} \cup \{v\} \cup \{u_i \mid 1 \leq i \leq n\} \cup \{v_i \mid 1 \leq i \leq n\} \text{ And}$$

$$E(B_{n,n}^2) = \{uu_i \mid 1 \leq i \leq n\} \cup \{vu_i \mid 1 \leq i \leq n\} \cup \{vv_i \mid 1 \leq i \leq n\}$$

Now, we define the bijection as $f : V \cup E \rightarrow \{1, 2, \ldots, 5n+2\}$ as follows.

$$\begin{aligned}
f(u) &= 3n+1 \\
f(v) &= 3n+2 \\
f(u_i) &= 3n+2+i,\ 1 \leq i \leq n \\
f(v_i) &= 4n+2+i,\ 1 \leq i \leq n \\
f(uu_i) &= i,\ 1 \leq i \leq n \\
f(vv_i) &= 2n+i,\ 1 \leq i \leq n \\
f(vu_i) &= n+i,\ 1 \leq i \leq n
\end{aligned}$$

Now the weights are,

For, $1 \leq i \leq n$

$$\begin{aligned}
wt(uu_i) &= |f(u) + f(u_i) - f(uu_i)| \\
&= |3n+1+3n+2+i-i| \\
&= 6n+3
\end{aligned}$$

For, $1 \leq i \leq n$

$$\begin{aligned}
wt(vv_i) &= |f(v) + f(v_i) - f(vv_i)| \\
&= |3n+2+4n+2+i-2n-i| \\
&= 5n+4
\end{aligned}$$

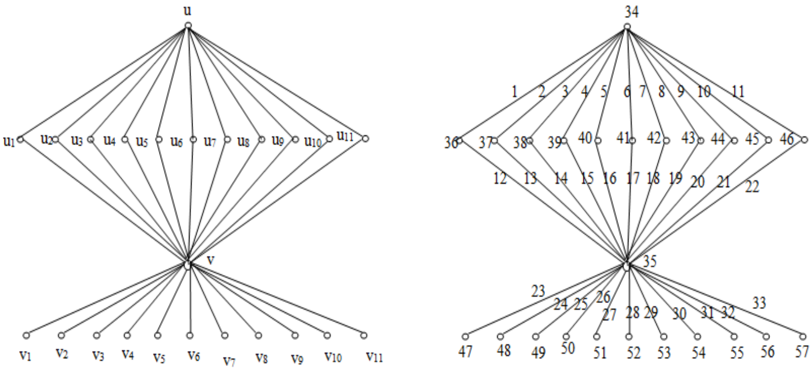
For, $1 \leq i \leq n$

$$\begin{aligned}
wt(vu_i) &= |f(v) + f(u_i) - f(vu_i)| \\
&= |3n+2+3n+2+i-n-i| \\
&= 5n+4
\end{aligned}$$

Let the magic constants $6n+3$ and $5n+4$ be k_1 and k_2 respectively.

Hence, the graph $B_{n,n}^2, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Example: *E*-Super edge bimagic graceful labeling of $B_{11,11}^2$ is given in the following figure.



2.6 Theorem 6

The graph $P_n^2, n \geq 3$ has an *E*-Super edge bimagic graceful labeling.

Proof

Let the graph P_n^2 be with n vertices and $2n-3$ edges and let

$V(P_n^2) = \{u_i \mid 1 \leq i \leq n\}$ And $E(P_n^2) = \{u_i u_{i+1} \mid 1 \leq i \leq n-2\} \cup \{u_i u_{i+2} \mid 1 \leq i \leq n-2\} \cup \{u_{n-1} u_n\}$.

Now, we define the bijection as $f : V \cup E \to \{1, 2, \ldots, 3n-3\}$ as follows:

$$f(u_i) = 2n-3+i, \ 1 \leq i \leq n$$
$$f(u_i u_{i+1}) = 2i, \ 1 \leq i \leq n-2$$
$$f(u_{n-1} u_n) = 1$$
$$f(u_i u_{i+2}) = 2i+1, \ 1 \leq i \leq n-2$$

Now the weights are,
For, $1 \leq i \leq n-2$

$$\begin{aligned} wt(u_i u_{i+1}) &= |f(u_i) + f(u_{i+1}) - f(u_i u_{i+1})| \\ &= |2n-3+i+2n-3+i+1-2i| \\ &= 4n-5 \end{aligned}$$

For, $1 \leq i \leq n-2$

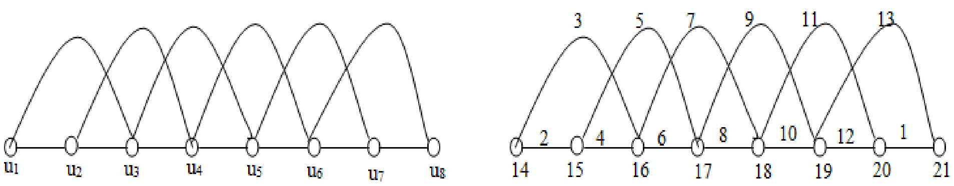
$$\begin{aligned} wt(u_i u_{i+2}) &= |f(u_i) + f(u_{i+2}) - f(u_i u_{i+2})| \\ &= |2n-3+i+2n-3+i+2-2i-1| \\ &= 4n-5 \end{aligned}$$

$$\begin{aligned} wt(u_{n-1} u_n) &= |f(u_{n-1}) + f(u_n) - f(u_{n-1} u_n)| \\ &= |2n-3+n-1+2n-3+n-1| \\ &= 6n-8 \end{aligned}$$

Let the magic constants $4n-5$ and $6n-8$ be k_1 and k_2 respectively.

Hence, the graph $P_n^2, n \geq 3$ has an *E-Super* edge bimagic graceful labeling.

Example: *E*-Super edge bimagic graceful labeling of P_8^2 is given in the following figure



3. REFERENCES

[1] Baskar Babujee,J. Bimagic labeling in path graphs, Journal of Mathematics education, Vol 38, pp. 12-16,2004.
[2] Gallian, J. A. (2017). A dynamic survey of graph labeling, Electron. J. Combin. 16:#DS6.
[3] Golomb, S.W. How to number a graph. In Graph Theory and Computing. Academic press, Newyork, 1972, pp. 23-37.
[4] Harary, F. (1969). Graph Theory, Addison-Wesley.
[5] Marr, A. M and Wallis, W. D. (2013). Magic graphs, Birkhauser-Springer, Second edition, Boston.
[6] Rosa. A. On certain valuations of the vertices of a graph. In theory of Graphs, International Symposium, Rome July 1966(1967) pp. 349-355

