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RP-96: Formulation of solutions of a special standard cubic congruence of composite modulus– An integer multiple of power of prime integer

B. M. Roy
roybm62@gmail.com
Jagat Arts, Commerce and Indiraben Hariharbhai Patel Science College, Goregaon, Maharashtra

ABSTRACT

In this paper, a standard cubic congruence of composite modulus - an integer multiple of a power of prime integer, is considered for study and formulation. The author established the formulation of the solutions of the said congruence successfully. It is a bold attempt by the author for the formulation of the congruence. The congruence is not formulated by earlier mathematicians. The established formula is tested and found true. Formulation of the congruence is the merit of the paper.

Keywords*—Binomial cubic expansion, Cubic congruence, Formulation, Prime-power- modulus*

1. INTRODUCTION

In this paper, the author considered a standard cubic congruence of an integer multiple of prime-power modulus for his study. It was not studied by earlier mathematicians.

The first time the author has taken a bold attempt to study and formulate the solutions of the congruence under consideration.

The author has gone through different books on Number Theory and found no discussion or formulation on this standard cubic congruence of an integer multiple of prime-power modulus. Some recent papers of the author on the formulation of standard cubic congruence of composite modulus are found [1], [2], [3], [4], [5].

2. PROBLEM STATEMENT

The problem is:

"To formulate the cubic congruence: $x^3 \equiv a^3 \ (mod\ b.p^n)$

When

$$n > 3 \ \ with\ a = p.$$

3. ANALYSIS AND RESULTS

Consider the congruence: $x^3 \equiv a^3 \ (mod\ b.p^n)$.

CASE-I: If $a = p$, then the congruence under consideration becomes:

$$x^3 \equiv p^3 \ (mod\ b.p^n)\ with\ n > 3.$$

For its formulation, let us consider that: $x \equiv b.p^{n-2}k + p \ (mod\ b.p^n)$.

Then, using binomial cubic expansion

$$x^3 \equiv (b.p^{n-2}k + p\ )^3 \ (\text{mod } b.p^n)$$
$$\equiv (b.p^{n-2}k)^3 + 3.(b.p^{n-2}k)^2.p + 3.bp^{n-2}k.p^2 + p^3 \ (\text{mod } b.p^n)$$
$$\equiv p^3 + \ b.p^{n-2}k\{\ 3p^2 + 3p.p^{n-2}k + (p^{n-2}k)^2\} \ \ (\text{mod } bp^n)$$
$$\equiv p^3 + bp^{n-2}k.\{p^2m\} \ (\text{ mod } bp^n)$$
$$\equiv p^3 + p^nk.m \ (\text{ mod } bp^n)$$
$$\equiv p^3 \ (\text{mod } bp^n).$$

Thus, it can be said that

$$x \equiv bp^{n-2}k + p \pmod{bp^n};\ k = 0, 1, 2, \ldots (p-1), p, \ldots \ldots.., (p^2-1)$$

Gives all the solutions of the congruence.

But if $k = p^2, p^2+1, \ldots \ldots$, it can be easily seen that the solutions of the congruence are the same as for $k = 0, 1 \ldots$

Thus, the congruence has p^2 − solutions for $k = 0, 1, 2, \ldots \ldots, (p^2-1)$.

4. ILLUSTRATIONS

Consider the congruence $x^3 \equiv 125 \pmod{3750}$.

It can be written as $x^3 \equiv 5^3 \pmod{6.5^4}$.

It is of the type: $x^3 \equiv p^3 \pmod{bp^n}$ with $p = 5, b = 6, n = 4$,

having $p^2 = 5^2 = 25$ solutions.

The solutions are then given by

$$x \equiv bp^{n-2}k + p \pmod{bp^n} \text{ with } k = 0,1, 2, \ldots..(p^2-1).$$

$$\text{i.e. } x \equiv 6.5^{4-2}k + 5 \pmod{6.5^4}$$

$$\text{i.e. } x \equiv 150k + 5 \pmod{3750};\ k = 0, 1, 2, 3, 4 \ldots \ldots \ldots 24.$$

$$\text{i.e. } x \equiv 5, 155, 305, 455, 605, 755, 905, 1055, 1205, 1355,$$

$$1505, 1655, 1805, 1955, 2105, 2255, 2405, 2555, 2705,$$

$$2855, 3005, 3155, 3305, 3455, 3605 \pmod{3750}.$$

Consider one more example: $x^3 \equiv 343 \pmod{33614}$.

It can be written as $x^3 \equiv 7^3 \pmod{2.7^5}$.

It is of the type: $x^3 \equiv p^3 \pmod{bp^n}$ with $p = 7, b = 2, n = 5$,

having $p^2 = 7^2 = 49$ incongruent solutions.

The solutions are then given by

$$x \equiv bp^{n-2}k + p \pmod{b.p^n} \text{ with } k = 0,1, 2, \ldots..(p^2-1)$$

$$\text{i.e. } x \equiv 2.7^{5-2}k + 7 \pmod{2.7^5}$$

$$\text{i.e. } x \equiv 686k + 7 \pmod{33614};\ k = 0, 1, 2, 3, 4, \ldots \ldots \ldots.48.$$

$$\text{i.e. } x \equiv 7, 693, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots..32935 \pmod{33614}.$$

Similarly, consider one more example as $x^3 \equiv 27 \pmod{324}$.

It can be written as $x^3 \equiv 3^3 \pmod{4.3^4}$.

It is of the type $x^3 \equiv p^3 \pmod{b.p^n}$ with nine solutions

$$x \equiv 4.3^2k + 3 \equiv 36k + 3 \pmod{324};\ k = 0, \ldots \ldots 8.$$

$$\text{i.e. } x \equiv 3, 39, 75, 111, 147, 183, 219, 255, 291 \pmod{324}.$$

5. CONCLUSION

Therefore, it can be concluded that the standard cubic congruence of composite modulus –an integer multiple of a power of prime:

$$x^3 \equiv p^3 \pmod{bp^n};\ n \geq 4 \text{ is formulated for its solutions.}$$

The solutions are given by

$$x \equiv bp^{n-2}k + p \pmod{bp^n};\ k = 0, 1, 2, \ldots \ldots, (p^2-1).$$

It has exactly p^2-incongruent solutions.

6. MERIT OF THE PAPER

In this paper, a standard cubic congruence of an integer multiple of prime-power modulus is formulated successfully and tested true for its solutions. The first time a formula is established. The formulation is the merit of the paper.

7. REFERENCES

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