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Fuzzy Z-Pre-continuous functions in Fuzzy Topological Spaces

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ABSTRACT

In this paper we introduce the classical form of Z-continuous functions under the fuzzy setting in the name of "Fuzzy Z-Pre-continuous functions in Fuzzy Topological Spaces". A weaker form of fuzzy Z-continuous function, called fuzzy Z-pre-continuous function is introduced and its properties and characterizations are discussed.

Keywords**—** *Fuzzy set, Topological space, Z-Continuous, Z-pre continuous, Fuzzy Z-Continuous, Fuzzy Z- precontinuous*

1. INTRODUCTION

Fuzzy set theory and Fuzzy topology are approached as generalization of ordinary set theory and ordinary topology. Fuzzy topology is one branch of mathematics, which combines the ordered structure with topological structure. Topological relation between spatial objects is used in geographic information system with positional and attributes information. Fuzzy sets provide a useful tool to describe uncertainty of topological properties of figures and surfaces.

After the discovery of fuzzy sets by Zadeh in 1965, much attention has been paid to generalize the basic concepts of the topological theoryin fuzzy setting. In 1968, C.L. Chang [1] initiated a study on *Fuzzy topological spaces* and defined its properties. Fuzzy sets on the universe X will be denoted by greek letters μ, γ, ζ etc., Fuzzy point will be denoted by x_t, y_v etc.,

The concept of fuzzy pre-open and fuzzy semi open sets was studied by S. Thakur and S. Singh [7]. In 1991, Bin Shahan [8] have defined and studied the concept of fuzzy pre-continuity. Navalagi [4] introduced the prezero sets and coprezero sets of a space with the help of pre-continuous functions. A weaker form of continuity called Z-continuous was introduced by Singal and Nimse [5] in1997.

2. PRELIMINARIES

Definition 2.1

A family T $\subseteq I^X$ of fuzzy set is called a fuzzy topology for Xif it satisfies the following three axioms:

1. 0, 1 ∈ T
2. If λ , μ ∈ T , then $\lambda \wedge \mu$ ∈ T
3. If λ_i ∈T ∀i ∈I, then $\vee\lambda_i$ ∈ T

The pair (X, T) is a *fuzzy topological space*. Every member of T is called *fuzzy open sets*. The complements of fuzzy open sets are called *fuzzy closed sets*.

Definition 2.2

A fuzzy set in X is called a fuzzy point if and only if it takes the value0 ∀y ∈X except one, say x ∈X. If its value at x is $\lambda(0<\lambda\leq 1)$ then the fuzzy point is denoted by p_x where the point x is called its support.

Definition 2.3

Let (X,T) and (Y,S) be two topological spaces. Any function f:(X,T)→(Y,S)is said to be fuzzy continuous if$f^{-1}(\lambda)$ is fuzzy open in (X,T) for each fuzzy open set λ in (Y,S).

Definition 2.4
For a fuzzy set λ in a fuzzy topological space (X, T) the **fuzzy interior** of λ denoted by FInt(λ) is the union of all fuzzy open sets contained in λ.

$$\text{FInt}(\lambda) = \vee \{ \lambda : \lambda \geq \mu \ , \mu \text{ - fuzzy open in X} \}.$$

The **fuzzy closure** of λ denoted by FCl(λ) is the intersection of all fuzzy closed sets containing inλ.

$$\text{FCl}(\lambda) = \wedge \{ \lambda : \mu \geq \lambda \ , \mu \text{ - fuzzy closed in X} \}$$

Definition 2.5
Let p be a fuzzy point in (X, T). A fuzzy set λ is called a **fuzzy neighbourhood** of p if and only if there exist an open fuzzy set μ such that p$\in\mu\leq\lambda$. If λ is open, it is called a *fuzzy open neighbourhood* of pin(X,T).

Definition 2.6
A function f: X → Y is said to be **pre-continuous** at a point x ∈ X if for every open set V of Y containing f (x) there exists a pre-open set U in X, such that x ∈ U and f (U) ⊆ V. The function is said to be *pre-continuous* if it is pre-continuous at each x∈ X.

Definition 2.7
A function f: X → Y is said to be **Z-continuous** if and only if the inverse image of every cozero set of Y is open in X.

Definition 2.8
Let (X, T) be a fuzzy topological space. Any $\mu \in I^X$is called a **fuzzy zero set** if there exists a function f: T → I such that $\mu=\vee_{i\in J}\{\lambda_i\in \boldsymbol{I}^X{:}\boldsymbol{f}(1-\lambda_i)=0,1-\lambda_i\in T,\lambda_i\neq1\}$.The complement of a fuzzy zero set is said to be a **fuzzy cozero set**.

Definition 2.9
Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy pre zero set** if there exists a fuzzy pre-continuous function f:T→$\boldsymbol{I}$ such that $\lambda=\vee_{i\in J}\{ \mu_i\in\boldsymbol{I}^X$: each μ_i is fuzzy preclosed, $\mu_i \neq 1\}$ and is denoted by FPZ(f).

The complement of a fuzzy pre zero set is called a **fuzzy coprezero set**.

Definition 2.10
Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$is called a **fuzzy Z-open set** if for each x $\in\lambda$, there exists a fuzzy cozero set μ of X such that x $\in\mu \leq \lambda$.

The complement of a fuzzy Z-open set is called **fuzzy Z-closed set**.

Definition 2.11
Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$of a fuzzy topological space X is called a **fuzzy preopen set** if A ≤fInt(fCl(A)).

The complement of a fuzzy pre-open set of (X, T) is called a **fuzzy copreopen set**.

Note 2.1
The family of all fuzzy pre-open (fuzzy pre-closed) sets of (X, T)is denoted by FPO(X) (resp.FPC(X)).

Definition 2.12
Let (X, T) be a fuzzy topological space. Any $\lambda \in I^X$ is called a **fuzzy pre-neighbourhood** of x $\in\lambda$ (or f pre-nbd) if there exists aμ ∈ FPO(X) such that x $\in\mu \leq \lambda$.

Remark 2.1
Every fuzzy pre neighborhood μ of x in (X, T) is a fuzzy preopen set in (X,T).

Definition 2.13
Let (X, T) and (Y, S) be two topological spaces. A function f:(X,T) →(Y,S) is said to be **fuzzy pre-continuous** if for every fuzzy open set γ of (Y, S) containing $f(x_t)$ there exists a fuzzy preopen set μin (X,T)such that$x_t\in\mu$and$f(\mu)\leq\gamma$.

Definition 2.14
Let (X, T) and (Y, S) be any two topological spaces. A function f: (X, T) →(Y, S) is said to be **fuzzy Z-continuous** if the inverse image of every fuzzy cozero set of (Y, S) is fuzzy open in (X, T).

Proposition 2.1
Every fuzzy zero set is a fuzzy prezero set.

Proof:

Let λ be a fuzzy zero set of X.

Then there exists a fuzzy continuous function f: T→I such that:

$$\lambda=\vee_{i\in J}\{\mu_i:\mu_i\in T' f(\mu_i)=0,\mu_i f=1\}.$$

Since every fuzzy open set is fuzzy preopen, we have $\mu_i\in T'$ is fuzzypreopen.

Thus there exists a fuzzy continuous function f : T →I such that $\lambda=\vee_{i\in J}\{\mu_i:f(\mu_i)=0,$ each μ_i isfuzzy pre-closed and $\mu_i\neq1\}$.

Hence every fuzzy zero set is a fuzzy prezero set.

Remark 2.2

Every fuzzy cozero set is a fuzzy coprezero set.

3. FUZZY Z−PRECONTINUOUS FUNCTIONS

Definition 3.1

Let (X,T) and (Y,S) be any two topological spaces. A function f:(X,T)→(Y,S) is said to be **fuzzy Z-precontinuous** if for every fuzzy cozero set γ of(Y,S)containing $f(x_t)$, there exists a fuzzy preopen set μ in (X,T) such that $x_t\in\mu$ and $f(\mu)\leq\gamma$.

Theorem 3.1

Let (X, T) and (Y,S) be any two topological spaces. For a function f:(X,T)→(Y,S),the following statements are equivalent:

(a) f is fuzzy Z-precontinuous.
(b) The inverse image of every fuzzy cozero set of (Y, S) is fuzzy preopen in (X, T).
(c) The inverse image of every fuzzy zero set of (Y, S) is fuzzy preclosed in (X, T).

Proof:

(a) ⇒(b)

Let γ be any fuzzy cozero set in (Y, S).

For each $x_t\in f^{-1}(\gamma)$ we have by (a),

There exists a fuzzy preopen set $\mu\in I^X$ such that $x_t\in\mu$ and $f(\mu)\leq\gamma$.

$\Rightarrow x_t\in\mu\leq f^{-1}(\gamma)$.

Thus $f^{-1}(\gamma)$is a fuzzy preneighbourhood of each of its points. Therefore,$f^{-1}(\gamma)$is fuzzy preopen in(X,T).

(b) ⇒(c)

Let ζ be any fuzzy zero set in(Y, S). Then 1 - ζ is a fuzzy cozero set in (Y,S).

Consider, $1-f^{-1}(\zeta)=f^{-1}(1)-f^{-1}(\zeta)$ and by (b), $f^{-1}(1-\zeta)=1-f^{-1}(\zeta)$ is fuzzy preopen in (X,T).

Therefore, $f^{-1}(\zeta)$is fuzzy preclosed in(X,T).

(c) ⇒(a)

Let $x_t\in$(X, T).

Let γ beany fuzzy cozero set in(Y,S) containing $f(x_t)$

Then $f(x_t)\not\in 1-\gamma$ and $1-\gamma$ isafuzzyzeroset.

Since, $f^{-1}(1-\gamma)=f^{-1}(1)-f^{-1}(\gamma)$.

By (c), $f^{-1}(1-\gamma)=1-f^{-1}(\gamma)$ is a fuzzy preclosed set.

Since $f(x_t)\not\in 1-\gamma$ and $1-\gamma$ wehave, $x_t\not\in f^{-1}(1-\gamma)$ and so $x_t\not\in 1-f^{-1}(\gamma)$

Thus $f^{-1}(\gamma)$ is a fuzzy preopenset containing $x_t$and $f(f^{-1}(\gamma))\leq\gamma$.

Therefore, f is fuzzy Z-precontinuous.

Theorem 3.2

Let (X,T) and (Y,S) beany 2 topological spaces. For a function f:(X,T)→(Y,S), the following statements are equivalent:

(a) f is fuzzy Z-precontinuous.
(b) The inverse image of every fuzzy zero set of (Y, S) is fuzzy prezero in (X, T).
(c) The inverse image of every fuzzy cozero set of (Y, S) is fuzzy coprezero in (X, T).

Proof:

(a) ⇒(b)

Let f be fuzzy Z-precontinuous.

Let μ be any fuzzy zero set in(Y, S).

Then there exists a fuzzy continuous function g : S → I such that, $\mu=\vee_{i\in J}\{\lambda_i\in \boldsymbol{I}^Y: g(\lambda_i)=0,\lambda_i\in S',\lambda_i\neq1\}$.

We claim that $g\circ f$: (X, T) →$\boldsymbol{I}$ is fuzzy precontinuous.

Let γ be any fuzzy closed set in I with $\gamma\neq1$.

Then γ is a fuzzy zero set.

As g is fuzzy continuous, $g^{-1}(\gamma)$ is a fuzzy zero set in (Y, S).
Since f is fuzzy Z-precontinuous, $(g \circ f)^{-1}(\gamma) = f^{-1}(g^{-1}(\gamma))$ isfuzzy preclosed in (X, T).
Now, as $g \circ f$ is fuzzy precontinuous,
$f^{-1}(\mu) = f^{-1}(g^{-1}\{0\}) = (g \circ f)^{-1}\{0\}$ is a fuzzy prezero set in (X, T).
Hence the inverse image of every fuzzy zero set of (Y, S) is fuzzy prezero in (X, T).

(b) ⇒(c)
Let μ be any fuzzy cozero set in (Y, S).
Thus 1 - μ is a fuzzy zero set in (Y, S).
$\Rightarrow f^{-1}(1-\mu)$is a fuzzy prezero set in(X,T).

Consider,$1-f^{-1}(\mu)=f^{-1}(1)-f^{-1}(\mu)$.
$\Rightarrow 1-f^{-1}(\mu)=f^{-1}(1-\mu)$ is fuzzy prezero in(X,T).
Therefore,$f^{-1}(\mu)$is fuzzy coprezero in(X,T).

(c) ⇒(a)
Let μ be a fuzzy cozero set in (Y, S).
$\Rightarrow f^{-1}(\mu)$ is a fuzzy coprezero set in (X, T).
$\Rightarrow f^{-1}(\mu)$ is a fuzzy preopen set in (X, T).
∴f is fuzzy Z-precontinuous.

Theorem 3.3
Let (X, T), (Y, S), (W, R) be any three fuzzy topological spaces. Let f:(X,T)→(Y, S) and g : (Y, S) →(W, R) be functions such that f is fuzzy Z-precontinuous and g is fuzzy continuous then $g \circ f$: (X, T) →(W, R) is fuzzy Z-precontinuous.

Proof:
Let γ be a fuzzy zero set in (W, R). Since g is fuzzy continuous,
$g^{-1}(\gamma)$is a fuzzy zero set in(Y,S).Since f is fuzzy Z-precontinuous,
$f^{-1}(g^{-1}(\gamma))=(g \circ f)^{-1}(\gamma)$is a fuzzy preclosed set in (X,T).
Thus $g \circ f$ is fuzzy Z-precontinuous.

Theorem 3.4
Let I be an index set and f_i: (X, T)→(X_i, T), $i \in I$ be a collection of functions. The function f:(X,T)→ Π(X_i,T) be defined by $f(x_t)=(f_i(x_t)), i \in I$. If the function f is fuzzy Z-precontinuous then each f_i:(X,T)→(X_i,T) is fuzzy Z-precontinuous.

Proof:
Let f:(X,T)→Π(X_i,T)be a fuzzy Z-precontinuous function.
Then $f_i= \pi_i \circ f$ where π_i is the projection of X onto a factor space X_i.
Then by **theorem 3.4**, we have $f_i= \pi_i \circ f$ is fuzzy Z-precontinuous.
∴ Each f_i is fuzzy Z-precontinuous.

Definition 3.2
Let (X, T) be a fuzzy topological space and $\lambda \in$ (X, T). A point $x_t \in$ (X, T) is said to be **fuzzy Z-adherent point** of λ if every fuzzy cozero set containing x_t intersects λ. The set of all fuzzy Z-adherent points of λ is denoted by λ_z.

A point $x_t \in$ (X, T) is said to be **fuzzy pre Z-adherent** point of λ if every fuzzy coprezero set containing x_t intersects λ. The set of all fuzzy pre Z-adherent points of λ is denoted by λ_{pz}.

Remark 3.1
$\lambda_{pz} \subset \lambda_z$, since every fuzzy cozero set is a fuzzy coprezero set. A subset λ of a fuzzy topological space (X, T) is fuzzy pre Z-closed if and only if $\lambda = \lambda_{pz}$.

Theorem 3.5
Let (X, T) and (Y, S) be any two topological spaces. A function f: (X, T) → (Y, S) is fuzzy Z-precontinuous if and only if for each $\lambda \in$ (X, T), $f(\lambda_{pz}) \leq (f(\lambda))_z$.

Proof:
Let f be fuzzy Z-precontinuous.
Then the inverse image of every fuzzy Z-closed set of (Y, S) is fuzzy pre Z-closed in (X, T).

Let $\lambda \in$(X, T) then $(f(\lambda))_z$ is fuzzy Z-closed in (Y, S).
Thus $f^{-1}((f(\lambda))_z)$ is fuzzy pre Z-closed in (X, T).

Now $f(\lambda) \leq (f(\lambda))_z$
$\Rightarrow \lambda=f^{-1}(f(\lambda)) \leq f^{-1}((f(\lambda))_z)$
$\Rightarrow \lambda_{pz}=f^{-1}(f(\lambda))_{pz} \leq (f^{-1}(f(\lambda)_z))_{pz}=f^{-1}(f(\lambda))_z$
$\Rightarrow f(\lambda_{pz}) \leq (f(\lambda))_z$.

Conversely,

Let μ be a fuzzy Z-closed set in (Y, S). Then $f(f^{-1}(\mu)_{pz}) \leq (f(f^{-1}(\mu)))_z \leq \mu_z = \mu$

$\Rightarrow f^{-1}(\mu)_{pz} = f^{-1}(\mu)$

$\Rightarrow f^{-1}(\mu)$is fuzzy pre Z-closed. Hence f is fuzzy Z-precontinuous.

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