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Exponential stability in time-delay systems

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ABSTRACT

In this paper, we defined exponential stability for nonlinear time-delay systems with delayed impulses. We derive the Lyapunov-based sufficient conditions for exponential stability. We show that the nonlinear impulsive time-delay system without impulse input delays is exponentially stable under the conditions. It is shown that the stable nonlinear impulsive time-delay system. It is a magnitude of the delayed impulses is sufficiently small, under the same conditions. The delayed impulses do not destroy the stability of the sizes of the impulse input delays.

Keywords— *Impulsive systems, Delayed impulses, Exponential stability*

1. INTRODUCTION

The impulsive systems are dynamical systems that are typically described by ordinary differential equations with instantaneous state jumps. An impulsive system has important applications in various fields, communication constraints. The impulsive control based on impulsive systems. We can provide an efficient way to deal with plants cannot endure continuous control inputs.

Assumption: 1

The delay $d(t)$ is time-varying satisfies:

$$‘d_1 \leq d(t) \leq d_2; \;\; d(t) \leq \mu$$

Where $0 \leq d_1 < d_2$ and $\mu > 0$ are known constants.

Assumption: 2

The activation function $g_{i(.)}, (1 = 1,2, \dots, n)$ satisfies

$$\sigma_i^- \leq \frac{g_i(S_1) - g_i(S_2)}{S_1 - S_2} \leq \sigma_i^+ \tag{1.1}$$

For any $S_1, S_2 \in R, S_1 \neq S_2$.

Where σ_i^- and σ_i^+ are known constants.

Moreover, we assume that the initial condition of the system (1) has the form

$$u(t) = \varphi(t), t \in [-d_2, 0]$$

Where function $\varphi(t)$ is continuous.

Then, by using the well-known Brouwers fixed-point of the transformed system and u^* is an equilibrium point of a system (1). For the sake of simplicity in the exponential stability of the system. We make the transformation $x(.) = u(.) - u^*$then we have

$$x(t) = -Ax(t) + Bf\big(x(t)\big) + Cf\left(x\big(t - d(t)\big)\right) \tag{1.2}$$

Where $x(t) = [x_1(t), \dots, x_n(t)] \in R^n$the state vector of is transformed system and u^* is an equilibrium point of system. Note that

$$f_j\big(x_j(t)\big) = g_j\big(x_j(t) + u_j^*\big) - g_j\big(u_j^*\big)$$

with $f_j(0) = 0, (j = 1,2, \dots n)$.

From condition, $f_j(.)$ satisfies the following condition:

$$\sigma_j^- \leq \frac{f_j(s)}{s} \leq \sigma_j^+ \forall s \in R, f_j(0) = 0 \tag{1.3}$$

It is obvious that the equilibrium point of the system is exponential stable if and only if the zero solution of the system is exponential stable.

Definition

The equilibrium point of the system is said to be exponentially stable if there exist positive scalars c and k such that

$$\|u(t)-u^*\| \le c\|\phi(\theta)-u^*\|e^{-kt}$$

Furthermore, k is called the exponential convergence rate.

Definition

For a given impulsive time sequence $\{t_k\}$, the trivial solution of system is said to be Exponentially Stable (ES) if there exist positive scalars ρ_0, M, and λ such that

$$\|\phi\|_\tau < \rho_0.$$

Lemma

Consider the system

$$\dot{x}(t) = f(t, x_0), t > t_0, t \ne t_{k,}$$

Satisfying (A_1)-(A_2) Assume that $\{t_k\} \in s_{min}(\beta_0)$ and $(l_0-1)\beta_0 < d \le l_0\beta_0$ for some positive integer l_0.
Let $\varrho = (1+h)^{l_0}e^{k_1 d}$.
Then for any $\phi \in PC\left([-\tau, 0], \mathcal{B}\left({}^{\rho}/_{\varrho}\right)\right)$, $|X(t, t_0, \phi| \le \varrho\|\phi\|_\tau, \quad t \in [t_0-\tau, t_0+d]$.

Proof

Set$X(t) = X(t, t_0, \phi)$.
Since $(l_0-1)\beta_0 < d \le l_0\beta_0$, the maximum number of impulse times on the interval $(t_0, t_0+d]$ isl_0.
We assume that the impulsive instants on $(t_0, t_0+d]$ are $t_i, i = 1,2, \dots, m, m \le l_0$.
Note that $|x(t_0)| = |\phi(0)| < \rho$.

By the continuity of $x(t)$ on $[t_0, t_1)$, for small enough $\xi > 0$.
We have $|x(t)| < \rho$ for $t \in [t_0-\tau, t_0+\xi)$.we assert that $|x(t)| < \rho$ for$[t_0-\tau, t_1)$.

Otherwise, there exists $t_1^* \in (t_0, t_1)$ such that $|x(t)| < \rho$ for $t \in [t_0-\tau, t_1^*)$ and $|x(t_1^*)| = \rho$.
For $t \in [t_0, t_1^*]$ and $\theta \in [-\tau, 0]$. when $t+\theta \le t_0$.
We have

$$|x(t+\theta)| \le \|\phi\|_\tau$$

When $t+\theta > t_0$,
By

$$\dot{x}(t) = f(t, x_0), t > t_0, t \ne t_{k,}$$

And using (A_4), we get

$$|x(t+\theta)| = \left|X(t_0) + \int_{t_0}^{t+\theta} f(s, X_s)\right| \le \|\phi\|_\tau + \int_{t_0}^{t} K_1\|X_s\|_\tau ds$$

It follows that,

$$\|X_t\|_\tau \le \|\phi\|_\tau + \int_{t_0}^{t} K_1\|X_s\|_\tau ds \qquad for\ t \in [t_0, t_1^*].$$

Applying the Gronwall inequality gives

$$\|X_t\|_\tau \le \|\phi\|_\tau e^{K_1(t-t_0)}, \qquad t \in [t_0, t_1^*],$$

This implies

$$|X(t_1^*)| \le \left({}^{\rho}/_{\varrho}\right)e^{K_1 d} < \rho$$

This is a contradiction.
Thus $|X(t)| < \rho$ for $t \in [t_0-\tau, t_1)$.

We have,

$$\|X_t\|_\tau \le \|\phi\|_\tau e^{K_1(t-t_0)} \quad t \in [t_0, t_1).$$

By

$$X(t) = g_k\left(X(t^-)\right), X((t-d_k)^-), \quad t = t_k, k \in \mathbb{N}$$

And using (A_5),
We get

$$|X(t_1)| \le \left(1+\bar{h}\right)\|\phi\|_\tau e^{K_1(t_1-t_0)}.$$

Hence

$$|X(t)| \le \left(1+\bar{h}\right)\|\phi\|_\tau e^{K_1(t_1-t_0)}, \quad t \in [t_0, t_1].$$

Repeating the above argument gives that for $t \in [t_0, t_m]$,

$$|X(t)| \le \left(1+\bar{h}\right)^m\|\phi\|_\tau e^{K_1(t_1-t_0)} \le \left(1+\bar{h}\right)^{l_0}\|\phi\|_\tau e^{K_1(t_1-t_0)}.$$

Since there are no impulses on $(t_m, t_0+d]$, We obtain,

$$|X(t)| \leq (1+\bar{h})^{l_0}\|\phi\|_\tau e^{K_1(t-t_0)} \leq (1+\bar{h})^{l_0} e^{K_1 d}\|\phi\|_\tau, \qquad for\ t \in [t_0-\tau, t_0+d].$$

Hence the proof

2. CONCLUSION

In this paper, we defined the exponential stability and exponential stability of impulsive time-delay systems. We show that the exponential stability of impulsive time-delay systems. We magnitude of the delayed impulses is sufficiently small and the impulse input delays are bounded. For the sake of simplicity in the exponential stability of the system

3. REFERENCES

[1] Bainov, D.D., and Simeonov, P.S. (1989). Systems with impulse effect: stability, theory, and applications. Chichester: Ellis Horwood.

[2] Ballinger, G., & Liu, X. (1999).Existence and uniqueness results for impulsive delay differential equations. Dynamics of continuous. Discrete and Impulsive Systems, 5(1-4), 579-591.

[3] Chen, W.-H., Wang, J.-G., Tang, Y.-J., & Lu, X. (2008). Robust H_∞ control of uncertain linear impulsive stochastic systems. International Journal of Robust Nonlinear Control, 18(3), 1348-1371.

[4] Chen, W.-H., and Zheng, W.X. (2009a). Ro'bust stability and H_∞-control of uncertain impulsive systems with time-delay. Automatica, 45(1), 109-117.

[5] Anokhin , A., Berezansky, L., and Braverman, E. (1995). Exponential stability of linear delay impulsive. Journal of Mathematical Analysis and Applications, 193(3), 923-941.

