On $g^+$-semi-open fuzzy sets and fuzzy $g^+$-semi-irresolute maps in fuzzy topological spaces

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ABSTRACT

In this paper, the class of $g^+$-closed fuzzy sets are introduced. This class properly placed between the class of semi-closed fuzzy sets and the class of $gs$-closed fuzzy sets in fuzzy topological space. Further, the concept of fuzzy $g^+$-continuous, fuzzy $g^+$-irresolute mapping, fuzzy $g^+$-closed maps, fuzzy $g^+$-open maps and fuzzy $g^+$-homeomorphisms in fuzzy topological spaces are also introduced, studied and some of their properties are obtained.

Keywords— $g^+$-closed fuzzy sets, $fg^+$-continuous, $fg^+$-irresolute, $fg^+$-open, $fg^+$-closed mappings and $fg^+$-homeomorphism.

1. INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [19] in the year 1965. Subsequently, several researchers have worked on topology using fuzzy sets and developed the theory of fuzzy topological spaces.

The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology introduced by C. L. Chang [5].


In the year 2002, $g^+$-closed sets, $g^+$-continuous, $g^+$-irresolute, $g^+$-closed, $g^+$-open maps were introduced and studied by M. K.R.S. Veera Kumar [18] for general topology.

In this paper, we considered a new concept related to semi-closed fuzzy sets and generalized semi-closed fuzzy sets, namely the class of $g^+$-semi-closed (briefly $fg^+$-closed) fuzzy sets. The class of $g^+$-closed fuzzy sets properly contains the class of semi-closed fuzzy sets and $gs$-closed fuzzy sets.

In this paper, we will study and characterize $g^+$-closed fuzzy sets and associated functions, by introducing and characterizing $fg^+$-continuous, $fg^+$-irresolute mappings and $fg^+$-homeomorphisms.

2. PRELIMINARIES

Let $X, Y$ and $Z$ be sets. Throughout the present paper $(X, T), (Y, \sigma)$ and $(Z, \eta)$ and (or simply $X, Y$ and $Z$) mean fuzzy topological spaces on which no separation axioms is assumed unless explicitly stated. Let $A$ be a fuzzy set of $X$. We denote the closure, interior and complement of $A$ by $cl(A)$, $int(A)$ and $C(A)$ respectively.

Before entering into our work we recall the following definitions, which are due to various authors.

2.1 Definition

A fuzzy set $A$ in a fits $(X, T)$ is called:

1. A semi-open fuzzy set, if $A \subseteq int(cl(A))$

2. A pre-open fuzzy set, if $A \subseteq cl(int(A))$

3. A $\alpha$-open fuzzy set, if $A \subseteq cl(int(cl(A)))$

4. A semipro-open fuzzy set, if $A \subseteq int(cl(int(A)))$ can be found in [4] and [7].
The semi-closure (resp. pre-closure, α-closure fuzzy and semipro closure fuzzy) of a fuzzy set $A$ in a fts $(X, T)$ is the intersection of all semi-closed (resp. pre-closed fuzzy sets, α-closed fuzzy sets and sp-closed fuzzy sets) fuzzy sets containing $A$ and is denoted by $scl(A)$ (resp. $pcl(A)$, $acl(A)$ and $spcl(A)$).

The following definitions are useful in the sequel.

### 2.2 Definition

A fuzzy set $A$ of fts $(X, T)$ is called:

1. A generalized closed $(g$-closed fuzzy) fuzzy set, if $int(A) \subseteq U$
2. A generalized-pre closed $(gp$-closed fuzzy) fuzzy set, if $pint(A) \subseteq U$.
3. An α-generalized closed $(ag$-closed fuzzy) fuzzy set, if $aint(A) \subseteq U$.
4. A generalized $\alpha$-closed $(ga$-closed fuzzy) fuzzy set, if $aint(A) \subseteq U$.
5. A generalized semi-pre closed $(gsp$-closed fuzzy) fuzzy set, if $spint(A) \subseteq U$.
6. A generalized semi-closed $(gs$-closed fuzzy) fuzzy set, if $sint(A) \subseteq U$.
7. A semi-generalized closed $(sg$-closed fuzzy) fuzzy set, if $sint(A)U$.
8. A $\gamma$-closed fuzzy set and
9. A $\psi$-closed fuzzy sets can be found in [7].

### 2.3 Definition

Let $X, Y$ be two fuzzy topological spaces. A function $f: X \rightarrow Y$ is called:

1. Fuzzy continuous ($f$-continuous).
2. Fuzzy $\omega$-continuous ($f\omega$-continuous).
3. Fuzzy semi-continuous ($f\omega$-continuous function).
4. Fuzzy pre-continuous ($f\omega$-continuous function).
5. $fg$-continuous function.
6. $fgp$-continuous function.
7. $fgs$-continuous function.
8. $fgs$-continuous function.
9. $fg\alpha$-continuous function.
10. $fg\alpha$-continuous function.
11. $fgsp$-continuous functions.
12. $fgsp$-continuous function.
13. $fgsp$-continuous function.
14. $fg\psi$-continuous and
15. $fg\psi$- irresolute and
16. $fg\psi$- irresolute can be found in [7].

### 2.4 Definition

Let $X, Y$ be two fuzzy topological spaces. A function $f: X \rightarrow Y$ is called:

1. Fuzzy $T^{1/2}$-space, fuzzy $T#$ space and fuzzy $#T$ space can be found in [7].

### 2.5 Definition

Let $X, Y$ be two fuzzy topological spaces. A function $f: X \rightarrow Y$ is called:

1. Fuzzy –homeomorphisms,
2. Fuzzy $g\alpha$-homeomorphisms,
3. Fuzzy $g\#$-homeomorphisms,
4. Fuzzy $g\#\alpha$-homeomorphisms and
5. Fuzzy $g\#\alpha$-homeomorphisms can be found in [7].

### 3. $g\#$-SEMI CLOSED FUZZY SETS IN FTs

#### 3.1 Definition

A fuzzy set $A$ of a fuzzy topological spaces $(X, T)$ is called $g\#$ semi-closed fuzzy (briefly $g\#s$-closed fuzzy) set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $ag\#$-open fuzzy set in $(X, T)$.

#### 3.2 Theorem

Every closed (resp:semi- closed fuzzy set and $\alpha$-closed fuzzy set) fuzzy set is $g\#s$-closed fuzzy set in any fts $X$.

**Proof:** Follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

#### 3.3 Example

Let $X = \{a, b, c\}$ and the fuzzy sets $A$ and $B$ be defined as follows: $A = \{(a, 0.3), (b, 0.5), (c, 0.6)\}$, $B = \{(a, 1), (b, 0.8), (c, 0.7)\}$. Consider the fts$(X, T)$, where $T = \{0, 1, A\}$. Note that the fuzzy subset $B$ is $g\#s$-closed fuzzy set but not a closed (not a semi-closed fuzzy set and not a $\alpha$-closed fuzzy set) fuzzy set in $(X, T)$.

#### 3.4 Theorem

Every $g\#s$-closed fuzzy set is $gs$-closed (gsp- closed fuzzy set) fuzzy set in fts $X$. 

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3.5 Example
Let $X = \{a, b, c\}$ and the fuzzy sets $A$ and $B$ be defined as follows: $A = \{(a, 0.3), (b, 0.5), (c, 0.4)\}$, $B = \{(a, 0.5), (b, 0.3), (c, 0.4)\}$. Consider $T = \{0, 1\}$. Then $(X, T)$ is fts. Here the fuzzy set $B$ is $gs$-closed (resp. $gsp$-closed fuzzy set) fuzzy set but not $g^s$-closed fuzzy set in $(X, T)$.

3.6 Theorem
In a fts $X$, if a fuzzy set $A$ is both $ag$-open fuzzy set and $g^s$-closed fuzzy set, then $A$ is semi-closed fuzzy set.

3.7 Theorem
If $A$ is $g^s$-closed fuzzy set and $scl(A) \wedge \left(1 - scl(A)\right) = 0$. Then there is no non-zero $ag$-closed fuzzy set $F$ such that $F \leq scl(A) \wedge \left(1 - A\right)$.

3.8 Theorem
If a fuzzy set $A$ is $g^s$-closed fuzzy set in $X$ such that $A \leq B \leq scl(A)$, then $B$ is also a $g^s$-Closed fuzzy set in $X$.

3.9 Definition
A fuzzy set $A$ of a fts $(X, T)$ is called $g^s$-open fuzzy set (resp. $g^s$-open fuzzy set) if its complement $1 - A$ is $g^s$-closed fuzzy set.

3.10 Theorem
A fuzzy set $A$ of a fts is $g^s$-open iff $F \leq \sin t(A)$, whenever $F$ is $ag$-closed fuzzy set and $F \leq A$.

3.11 Theorem
Every open (resp. semi-open fuzzy set and $\alpha$-open fuzzy set) fuzzy set is a $g^s$-open fuzzy set but not conversely.

Proof: Follows from the definitions
The converse of the above theorem need not be true as seen from the following example.

3.12 Example
In example 3.3, the fuzzy subset $1 - B = \{(a, 0), (b, 0.2), (c, 0.3)\}$ is $g^s$-open fuzzy set but not open (resp. not a semi open fuzzy set and not an $\alpha$-open fuzzy set) fuzzy set in $(X, T)$.

3.13 Theorem
In fts $X$, every $g^s$-open fuzzy set $t$ is $gs$-open fuzzy (resp. $gsp$-open fuzzy set) but not conversely.

The converse of the above theorem need not be true as seen from the following example.

3.14 Example
In example 3.5, the fuzzy subset $1 - B = \{(a, 0.5), (b, 0.7), (c, 0.6)\}$ is $gs$-open fuzzy (resp. $gsp$-open fuzzy set) set but not $g^s$-open fuzzy set in $(X, T)$.

3.15 Theorem
If $\sin t(A) \leq B \leq A$ and if $A$ is $g^s$-open fuzzy set, then $B$ is $g^s$-open fuzzy set in a fts $(X, T)$.

3.16 Theorem
If $A \leq B \leq X$ where $A$ is $g^s$-open fuzzy relative to $B$ and $B$ is $g^s$-open fuzzy relative to $X$, then $A$ is $g^s$-open fuzzy relative to fts $X$.

3.17 Theorem
Finite intersection of $g^s$-open fuzzy set is a $g^s$-open fuzzy set.

3.18 Definition
fts $(X, T)$ is called a fuzzy-$T$s# space if every $g^s$-closed fuzzy set is a closed fuzzy set.

3.19 Definition
fts $(X, T)$ is called a fuzzy-$#T$s space if every $gs$-closed fuzzy set is a $g^s$-closed fuzzy set.

4. FUZZY $g^s$-SEMI-CLOSURE AND FUZZY $g^s$-SEMI-INTERIOR FUZZY SETS IN FTs
In this section we introduce the concepts of fuzzy $g^s$-closure ($fg^s cl$) and fuzzy $g^s$-interior ($fg^s int$) and investigate their properties.

4.1 Definition
For any fuzzy set $A$ in any fts is said to be fuzzy $g^s$-closure and is denoted by $fg^s cl(A)$, defined by $fg^s cl(A)=\wedge \{U: U$ is $g^s$-closed fuzzy set and $A \leq U\}$.
4.2 Definition
For any fuzzy set $A$ in any fts is said to be fuzzy $g^s$ -interior and is denoted by $fg^s$-int$(A)$, defined by $fg^s$-int$(A) = \{ \vert : V \text{ is } g^s - \text{interior fuzzy set and } A \geq V \}$. 

4.3 Theorem
Let $A$ be any fuzzy set in fts $(X, T)$. Then $fg^s$-cl$(A) = fg^s$-cl$(1 - A) = fg^s$-int$(1 - A) = 1 - fg^s$-cl$(A)$. 

4.4 Theorem
In a fts $(X, T)$, a fuzzy set $A$ is $g^s$ -closed iff $A = fg^s$-cl$(A)$. 

4.5 Theorem
In fts $X$ the following results hold for $g^s$ -closure.
(1) $fg^s$-cl$(0) = 0$.
(2) $fg^s$-cl$(A)$ is $g^s$ -closed fuzzy set in $X$.
(3) $fg^s$-cl$(A)$ is $g^s$ -closed fuzzy set if $A \leq B$.
(4) $fg^s$-cl$(fg^s$-cl$(A)) = fg^s$-cl$(A)$.
(5) $fg^s$-cl$(A \cup B) \geq fg^s$-cl$(A) \cup fg^s$-cl$(B)$
(6) $fg^s$-cl$(A \cap B) \leq fg^s$-cl$(A) \cap fg^s$-cl$(B)$.

Proof: The easy verification is omitted.

4.6 Theorem
In a fts $X$, a fuzzy set $A$ is $g^s$ -open iff $A = fg^s$-int$(A)$

4.7 Theorem
In a fts $X$, the following results hold for $g^s$ -interior.
(1) $fg^s$-int$(0) = 0$.
(2) $fg^s$-int$(A)$ is $g^s$ -open fuzzy set $X$.
(3) $fg^s$-int$(A)$ is $g^s$ -open fuzzy set if $A \leq B$.
(4) $fg^s$-int$(fg^s$-int$(A)) = fg^s$-int$(A)$.
(5) $fg^s$-int$(A \cup B) \geq fg^s$-int$(A)fg^s$-int$(B)$
(6) $fg^s$-int$(A \cap B) \leq fg^s$-int$(A)fg^s$-int$(B)$.

Proof: The routine proof is omitted.

5. FUZZY $g^s$ -SEMI-CONTINUOUS AND FUZZY $g^s$ -SEMI-IRRESOLUITE MAPPINGS IN FTS

5.1 Definition
A function $f: X \rightarrow Y$ is said to be fuzzy $g^s$ -continuous $(fg^s$-continuous) if the inverse image of every open fuzzy set in $Y$ is $g^s$ -open fuzzy set in $X$.

5.2 Theorem
A function $f: X \rightarrow Y$ is $g^s$ -continuous if the inverse image of every closed fuzzy set in $Y$ is $g^s$ -closed fuzzy set in $X$.

5.3 Theorem
Every fuzzy continuous (resp: $f\alpha$-continuous and $fsemi$ -continuous) function is fuzzy $g^s$ -continuous.

Proof: The following proof is omitted.
The converse of the above theorem need not be true as seen from the following example.

5.4 Example
Let $X = Y = \{ a, b, c \}$ and the fuzzy sets $A, B$ and $C$ defined as follows. $A = \{(a, 0), (b, 0.2), (c, 0.3), B = \{(a, 0.3), (b, 0.5), (c, 0.6), C = \{(a, 1), (b, 0.8), (c, 0.7)\}$. Consider $T = \{0, 1, B\}$, and $\sigma = \{0, 1, A\}$. Then $X, T$ and $Y, \sigma$ are fts. Define $f: X \rightarrow Y$ by $f(a) = a, f(b) = b \text{ and } f(c) = c$. Then $f$ is $fg^s$ -continuous but not $f$ -continuous (resp: not $f\alpha$ -continuous and not $f$ semi-continuous). As the fuzzy set $C$ is closed fuzzy set in $Y$ and $f^{-1}(C)$ is not closed fuzzy set in $X$ but $g^s$ -closed (resp: $\alpha$-closed fuzzy set and semi-closed fuzzy set) fuzzy set in $X$. Hence $f$ is fuzzy $g^s$-continuous.

5.5 Theorem
Every fuzzy $g^s$ -continuous function is fuzzy $gs$-continuous function and is fuzzy $gsp$-continuous function.

Proof: The following proof is omitted.
The converse of the above theorem need not be true as seen from the following example.

5.6 Example
Let $X = Y = \{ a, b, c \}$ and the fuzzy sets $A, B$ and $C$ defined as follows. $A = \{(a, 0.6), (b, 0.5), (c, 0.3), B = \{(a, 0.6), (b, 0.7), (c, 0.4), C = \{(a, 0.4), (b, 0.3), (c, 0.6)\}$. Consider $T = \{0, 1, A\}$, and $\sigma = \{0, 1, C\}$. Then $X, T$ and $Y, \sigma$ are
A function \( f: X \rightarrow Y \) is \( fg^s \)-continuous and \( X \) is fuzzy-Ts# space. Then \( f \) is \( f \)-continuous.

**5.8 Theorem**

If \( f: X \rightarrow Y \) is \( fg^s \)-continuous and \( X \) is fuzzy-Ts# fts, then \( f \) is \( fg^s \)-continuous.

**5.9 Theorem**

If \( f: X \rightarrow Y \) is \( fg^s \)-continuous and \( g: Y \rightarrow Z \) is \( f \)-continuous, then \( g \circ f: X \rightarrow Z \) is \( fg^s \)-continuous.

**5.10 Definition**

A function \( f: X \rightarrow Y \) is said to be fuzzy \( g^s \) semi-irresolute (\( fg^s \)-irresolute) if the inverse image of every \( g^s \)-closed fuzzy set in \( Y \) is \( g^s \)-closed fuzzy set in \( X \).

**5.11 Theorem**

A function \( f: X \rightarrow Y \) is \( fg^s \)-irresolute function iff the inverse image of every \( g^s \)-open fuzzy set in \( Y \) is \( g^s \)-open fuzzy set in \( X \).

**5.12 Theorem**

Every \( fg^s \)-irresolute function is \( fg^s \)-continuous.

**Proof:** Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

**5.13 Example**

Let \( X = Y = \{a, b, c\} \) and the fuzzy sets \( A, B, C \) and \( D \) defined as follows. \( A = \{(a, 0.2), (b, 0.5), (c, 0.3)\}, B = \{(a, 0.8), (b, 0.5), (c, 0.7)\}, C = \{(a, 0.5), (b, 0.2), (c, 0.3)\}, D = \{(a, 0.5), (b, 0.8), (c, 0.7)\} \) Let \( T = \{0,1\}, \) and \( \sigma = \{0,1,A,B\} \).

Then \( (X,T) \) and \( (Y,\sigma) \) are fts. Define \( f: X \rightarrow Y \) by \( f(a) = b, f(b) = a \) and \( f(c) = c \) Then \( f \) is \( fg^s \)-continuous but not \( fg^s \)-irresolute as the fuzzy set is \( g^s \)-closed fuzzy set in \( Y \) but its inverse image \( f^{-1}(C) = A \) is not \( g^s \)-closed fuzzy set in \( X \).

**5.14 Theorem**

Let \( f: X \rightarrow Y, g: Y \rightarrow Z \) be two fuzzy functions. Then:

1. \( f \circ g: X \rightarrow Z \) is fuzzy \( g^s \)-continuous if \( f \) is fuzzy \( g^s \)-irresolute and \( g \) is fuzzy \( g^s \)-continuous.
2. \( f \circ g: X \rightarrow Z \) is fuzzy \( g^s \)-irresolute if both \( f \) and \( g \) are fuzzy \( g^s \)-irresolute.
3. \( f \circ g: X \rightarrow Z \) is fuzzy \( g^s \)-continuous if \( f \) is fuzzy \( g^s \)-continuous and \( g \) is fuzzy continuous.

**Proof:**

The following proof is omitted.

**5.15 Theorem**

If \( f: X \rightarrow Y, g: Y \rightarrow Z \) be two fuzzy functions. If \( f \) is \( fg^s \)-continuous and \( g \) is \( fg^s \)-irresolute and \( Y \) is fuzzy-Ts# space, then \( g \circ f: X \rightarrow Z \) is \( fg^s \)-irresolute function.

**5.16 Theorem**

Let \( f: X \rightarrow Y \) be a \( fgc \)-irresolute and a semi-closed fuzzy map. Then \( f(A) \) is a \( g^s \)-closed fuzzy set of \( Y \) for every \( g^s \)-closed fuzzy set \( A \) of \( X \).

**5.17 Theorem**

Let \( f: X \rightarrow Y \) be a \( fgc \)-irresolute and a semi-closed map. Then \( f(A) \) is a \( g^s \)-closed fuzzy set of \( Y \) for every \( g^s \)-closed fuzzy set \( A \) of \( X \).

6. FUZZY \( g^s \)-SEMI OPEN MAPS AND FUZZY \( g^s \)-SEMI-CLOSED MAPS IN FTs

Every open and fuzzy closed maps were introduced and studied by C. Wong [20]. This study was further carried out by Sadanand, N. Patil [10]. We introduced the following concepts.

6.1 Definition

A function \( f: X \rightarrow Y \) is said to be fuzzy \( g^s \)-semi open (briefly \( fg^s \)-open) if the image of every open fuzzy set in \( X \) is \( g^s \)-open fuzzy set in \( Y \).

6.2 Definition

A function \( f: X \rightarrow Y \) is said to be fuzzy \( g^s \)-semi closed (briefly \( fg^s \)-closed) if the image of every closed fuzzy set in \( X \) is \( g^s \)-closed fuzzy set in \( Y \).
Every fuzzy-open map is fuzzy $g^s$-open map.

**Proof:** The proof follows from the definition 6.1
The converse of the above theorem need not be true as seen from the following example.

Let $X = Y = [a, b, c]$. Fuzzy sets $A$, $B$ and $C$ be defined as follows. $A = \{(a, 0), (b, 0.2), (c, 0.3)\}$, $B = \{(a, 0.3), (b, 0.5), (c, 0.6)\}$, $C = \{(a, 0.1), (b, 0.8), (c, 0.7)\}$ Consider $T = \{0, 1, A\}$, and $\sigma = \{0, 1, B\}$. Then $(X, T)$ and $(Y, \sigma)$ are fts.

Define $f: X \to Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$. Then $f$ is $fg^s$-open map but not an $f$-open map as the fuzzy set $A$ is open fuzzy set in $X$, and its image $f(A) = A$ is not open fuzzy set in $Y$ which is $g^s$-open fuzzy set in $Y$.

If $f: X \to Y$ is $fg^s$-open map and $Y$ is a fuzzy-$T$'s#, then $f$ is a fuzzy open map.

Every fuzzy $g^s$-open map is fuzzy $gs$-open map.

**Proof:** The proof follows from the definition 6.1
The converse of the above theorem need not be true as seen from the following example.

Let $X = Y = [a, b, c]$. Fuzzy sets $A$, $B$ and $C$ be defined as follows. $A = \{(a, 0.3), (b, 0.5), (c, 0.4)\}$, $B = \{(a, 0.7), (b, 0.5), (c, 0.6)\}$, $C = \{(a, 0.5), (b, 0.3), (c, 0.4)\}$. Consider $T = \{0, 1, A\}$, and $\sigma = \{0, 1, A, B\}$. Then $(X, T)$ and $(Y, \sigma)$ are fts.

Define $f: X \to Y$ by $f(a) = b, f(b) = a$ and $f(c) = c$. Then the function $f$ is fuzzy $gs$-open map but not an fuzzy $g^s$-open map as the image of open fuzzy set $A$ in $X$ is $f(A) = C$ open fuzzy set in $Y$ but not $g^s$-open fuzzy set in $Y$.

If $f: X \to Y$ is fuzzy $gs$-open map and $Y$ is a fuzzy-$#T$s space, then $faa$ is fuzzy $g^s$-open fuzzy map.

Every fuzzy-closed map is fuzzy $g^s$-closed map.

**Proof:** The proof follows from the definition 6.2
The converse of the above theorem need not be true as seen from the following example.

In example 6.4, the function $f$ is fuzzy $g^s$-closed map but not closed fuzzy map as the fuzzy set $C$ is closed fuzzy set in $X$ and its image $f(C) = C$ is $g^s$-closed fuzzy set in $Y$ but not closed fuzzy set in $Y$.

If $f: X \to Y$ is fuzzy $g^s$-closed map and $Y$ is a fuzzy-$Ts$#, then $f$ is fuzzy closed fuzzy map.

A function $f: X \to Y$ is $fg^s$-closed iff for each fuzzy set $S$ of $Y$ and for each open fuzzy set $U$ such that $f^{-1}(S) \subseteq U$, there is a $g^s$-open fuzzy set $V$ of $Y$ such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

If a map $f: X \to Y$ is fuzzy $gc$-ieersolute and $fg^s$-closed and $A$ is $g^s$-closed fuzzy set in $X$ and $Y$ is fuzzy-$T^{1/2}$ then $f(A)$ is $g^s$-closed fuzzy set in $Y$.

Let $f: X \to Y$ is fuzzy continuous and fuzzy $g^s$-closed. If $A$ is $g^s$-closed fuzzy set in $X$ and $Y$ is fuzzy-$T^{1/2}$ then $f(A)$ is $g^s$-closed fuzzy set in $Y$.

If $f: X \to Y$ is $f$-closed a map and $g: Y \to Z$ is $fg^s$-closed maps, then $g \circ f: X \to Z$ is $fg^s$-closed map.

If $f: X \to Y$ and $g: Y \to Z$ are $fg^s$-closed maps and $Y$ is fuzzy $Ts$# space, then $g \circ f: X \to Z$ is $fg^s$-closed map.

Proof: The proof follows from the definition.

Let $f: X \to Y, g: Y \to Z$ be two maps such that $g \circ f: X \to Z$ is $fg^s$-closed map.
6.18 Theorem
The composition \( g \circ f \) of \( f: X \to Y \) and \( g: Y \to Z \) is \( fg^g\)-open map, if \( f \) is \( fg^g\)-irresolute and \( g \) is \( fg^g\)-open map.

6.19 Theorem
If \( f: X \to Y \) and \( g: Y \to Z \) be two mappings and \( g \circ f: X \to Z \) be composition of those two mappings. Then if \( f \) the the is fuzzy-open and \( g \) is \( fg^g\)-open maps, then \( g \circ f \) is \( fg^g\)-open.

6.20 Theorem
If \( A \) is a fuzzy set closed fuzzy set in \( X \) and \( f: X \to Y \) is bijective, \( f \)-continuous and \( fg^g\)-closed, then \( f(A) \) is a fuzzy set -closed fuzzy set in \( Y \).

6.21 Theorem
If a function \( f: X \to Y \) is \( f \)-continuous and \( fg^g\)-closed in \( X \) and \( A \) is a \( fg^g\)-closed fuzzy set in \( X \), then \( f[A] \) is \( f \)-continuous and \( fg^g\)-closed map.

6.22 Definition [3]
Let \( X \) and \( Y \) be two fts. A bijective map \( f: X \to Y \) is closed fuzzy-homeomorphism (briefly \( f \)-homeomorphism) if \( f \) and \( f^{-1} \) are fuzzy-continuous.

We introduce the following.

6.23 Definition
A function \( f: X \to Y \) is called fuzzy \( g^g \) semi-homeomorphism (briefly \( fg^g\)-homeomorphism) if \( f \) and \( f^{-1} \) are \( fg^g\)-continuous.

6.24 Theorem
Every \( f \)-homeomorphism is \( fg^g\)-homeomorphism

Proof: The proof is follows the definition.
The converse of the above theorem need not be true as seen from the following example.

6.25 Example
Let \( X = Y = \{a, b, c\} \) and the fuzzy sets \( A, B \) and \( C \) be defined as follows. \( A = \{(a, 1), (b, 0), (c, 0)\} \), \( B = \{(a, 1), (b, 1), (c, 0)\} \), \( C = \{(a, 1), (b, 0), (c, 1)\} \). Consider \( T = \{0, 1, A, C\} \) and \( \sigma = \{0, 1, B\} \). Then \( (X, T) \) and \( (Y, \sigma) \) are fts. Define \( f: X \to Y \) by \( f(a) = a, f(b) = c \) and \( f(c) = b \). Then \( f \) is \( fg^g\)-homeomorphism but not \( f \)-homeomorphism as \( A \) is open fuzzy set in \( X \) and its image \( f(A) = A \) is not open in \( Y, f^{-1} : Y \to X \) is not \( f \)-continuous.

6.26 Theorem
Let \( f: X \to Y \) be a bijective function. Then the following are equivalent:
(1) \( f \) is \( fg^g\)-homeomorphism
(2) \( f \) is \( fg^g\)-continuous and \( fg^g\)-open maps.
(3) \( f \) is \( fg^g\)-continuous and \( fg^g\)-closed maps.

6.27 Theorem
If \( f: X \to Y, g: Y \to Z \) are \( fg^g\)-homeomorphism and \( g: Y \to Z \) is \( fg^g\)-homeomorphism and \( Y \) is fuzzy-T's# space, then \( g \circ f: X \to Z \) is \( fg^g\)-homeomorphism.

6.28 Definition
Let \( X \) and \( Y \) be two fts. A bijective map \( f: X \to Y \) is called fuzzy \( g^g \) semi-homeomorphism (briefly \( fg^g\)-semi-homeomorphism) if \( f \) and \( f^{-1} \) are fuzzy \( g^g \)-irresolute.

6.29 Theorem
Let \( X, Y, Z \) be fuzzy topological spaces and \( f: X \to Y, g: Y \to Z \) is \( fg^g \)-semi-homeomorphisms then their composition \( g \circ f: X \to Z \) is \( fg^g \)-semi-homeomorphism.

Proof: The following proof is omitted.

6.30 Theorem
Every \( fg^g \)-semi-homeomorphism is \( fg^g\)-homeomorphism.

Proof: The proof follows from the definition.

7. CONCLUSION
In this paper, the attempt has been made to study that on \( g^g\)-semi open fuzzy sets and fuzzy \( g^g\)-semi-irresolute maps in fuzzy topological spaces. We have discussed some basic definitions. We have discussed some theorems and results based on basic properties of \( g^g\)-semi-closed fuzzy sets in fts. We have discussed some theorems and results based on fuzzy \( g^g\)-semi-closure and
fuzzy $g^\#-$semi interior fuzzy sets in fts. We have discussed some theorems and results based on fuzzy $g^\#-$semi-continuous and fuzzy $g^\#-$semi-irresolute mappings in fts. We have discussed some theorems and results based on fuzzy $g^\#$-semi-open maps and fuzzy $g^\#$-semi-closed maps in fts.

8. REFERENCES


[10] Sadanand. N. Patil, on $\mu$-closed fuzzy sets, Fuzzy $\mu$-continuous maps and fuzzy $\mu$ irresolute maps in fuzzy topological space, Submitted.


