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On $g^{\#}$-semi-open fuzzy sets and fuzzy $g^{\#}$-semi-irresolute maps in fuzzy topological spaces

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ABSTRACT

In this paper, the class of $g^{\#}s$-closed fuzzy sets are introduced. This class properly placed between the class of semi-closed fuzzy sets and the class of gs-closed fuzzy sets in fuzzy topological space. Further, the concept of fuzzy $g^{\#}s$-continuous, fuzzy $g^{\#}s$-irresolute mapping, fuzzy $g^{\#}s$-closed maps, fuzzy $g^{\#}s$-open maps and fuzzy $g^{\#}s$-homeomorphisms in fuzzy topological spaces are also introduced, studied and some of their properties are obtained.

Keywords*— *$g^{\#}s$-closed fuzzy sets, $fg^{\#}s$-continuous, $fg^{\#}s$-irresolute, $fg^{\#}s$-open, $fg^{\#}s$-closed mappings and $fg^{\#}s$-homeomorphism.

1.INTRODUCTION

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [19] in the year 1965. Subsequently, several researchers have worked on topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology introduced by C .L. Chang [5].

N. Levine [9] introduced the concepts of generalized closed sets in general topology in the year 1970. G. Balasubramanian and P. Sundarm [2] introduced and studied generalized closed fuzzy sets in fuzzy topology. K. K. Azad introduced semi-closed fuzzy sets in the year 1981.H.Maki, T. Fukutake, M. Kojima and H. Harada [12, 13] introduced generalized semi-closed fuzzy sets (briefly gs-closed fuzzy set) in fuzzy topological space in the year 1998.

In the year 2002, $g^{\#}s$ -closed sets, $g^{\#}s$ -continuous, $g^{\#}s$ -irresolute, $g^{\#}s$ -closed, $g^{\#}s$ -open maps were introduced and studied by M.K.R.S. Veera Kumar [18] for general topology.

In this paper, we considered a new concept related to semi-closed fuzzy sets and generalized semi-closed fuzzy sets, namely the class of $g^{\#}$ -semi-closed (briefly $fg^{\#}s$ -closed) fuzzy sets. The class of $g^{\#}s$ -closed fuzzy sets properly contains the class of semi-closed fuzzy sets and gs-closed fuzzy sets.

In this paper, we will study and characterizing $g^{\#}s$ -closed fuzzy sets and associated functions, by introducing and characterizing $fg^{\#}s$ -continuous, $fg^{\#}s$ -irresolute mappings and $fg^{\#}s$ -homeomorphisms.

2.PRELIMINARIES

Let X, Y and Z be sets. Throughout the present paper (X,T), (Y,σ) and (Z,η) and (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms is assumed unless explicity stated. Let A be a fuzzy set of X. We denote the closure, interior and complement of A by $cl(A)$, $int(A)$ and $C(A)$ respectively.

Before entering into our work we recall the following definitions, which are due to various authors.

2.1 Definition

A fuzzy set A in a fts (X,T) is called:
(1) A semi-open fuzzy set, if $A \subseteq int(cl(A))$
(2) A pre-open fuzzy set, if $A \subseteq cl(int(A))$
(3) A α-open fuzzy set, if $A \subseteq cl(int(cl(A)))$
(4) A semipro-open fuzzy set, if $A \subseteq int(cl(int(A)))$ can be found in [4] and [7].

The semi-closure (resp. pre-closure fuzzy, α-closure fuzzy and semipro closure fuzzy) of a fuzzy set A in a fts (X, T) is the intersection of all semi-closed (resp. pre-closed fuzzy sets, α-closed fuzzy sets and sp-closed fuzzy sets) fuzzy sets containing A and is denoted by $scl(A)\big(resp. pcl(A), \alpha cl(A) and\ spcl(A)\big)$.

The following definitions are useful in the sequel.

2.2 Definition

A fuzzy set A of fts (X, T) is called:

(1) A generalized closed (g-closed fuzzy) fuzzy set, if $int(A) \subseteq U$

(2) A generalized-pre closed (gp-closed fuzzy) fuzzy set, if $pint(A) \subseteq U$.

(3) A α-generalized closed (αg-closed fuzzy) fuzzy set, if $\alpha int(A) \subseteq U$.

(4) A generalized α-closed ($g\alpha$-closed fuzzy) fuzzy set, if $\alpha int(A) \subseteq U$.

(5) A generalized semi-pre closed (gsp-closed fuzzy) fuzzy set, if $spint(A) \subseteq U$.

(6) A generalized semi-closed (gs-closed fuzzy) fuzzy set, if $sint(A) \subseteq U$.

(7) A semi-generalized closed (sg-closed fuzzy) fuzzy set, if $sint(A)U$.

(8) A $\hat{g}$-closed fuzzy set and

(9) A ψ-closed fuzzy sets can be found in [7].

2.3 Definition

Let X, Y be two fuzzy topological spaces. A function $f: X \to Y$ is called:

(1) Fuzzy continuous (f-continuous),

(2) Fuzzy α-continuous ($f\alpha$-continuous),

(3) Fuzzy semi-continuous (fs-continuous) function,

(4) Fuzzy pre-continuous (fp-continuous) function,

(5) fg-continuous function,

(6) fgp-continuous function,

(7) fgs-continuous function,

(8) fsg-continuous function,

(9) $fg\alpha$-continuous function,

(10) $f\alpha g$-continuous function,

(11) $fgsp$-continuous functions,

(12) $f\hat{g}$-continuous function,

(13) $f\psi$-continuous function,

(14) $f\psi$-irresolute,

(15) fgc-irresolute and

(16) $f\hat{g}$-irresolute can be found in [7].

2.4 Definition

Let X, Y be two fuzzy topological spaces. A function $f: X \to Y$ is called:

(1) Fuzzy $T^{1/2}$-space, fuzzy $T\#$ space and fuzzy $\#T$ space can be found in [7].

2.5 Definition

Let X, Y be two fuzzy topological spaces. A function $f: X \to Y$ is called:

(1) Fuzzy –homeomorphisms,

(2) Fuzzy $g^{\#}s$-homeomorphisms,

(3) Fuzzy $g\#$-homeomorphisms,

(4) Fuzzy $g\#s$-homeomorphisms and

(5) Fuzzy $g\#\alpha$-homeomorphisms can be found in [7].

3. $g\#$-SEMI CLOSED FUZZY SETS IN FTs

3.1 Definition

A fuzzy set A of a fuzzy topological spaces (X, T) is called $g\#$ semi-closed fuzzy (briefly $g\#s$-closed fuzzy) set if $scl(A) \leq U$ whenever $A \leq U$ and U is αg-open fuzzy set in (X, T).

3.2 Theorem

Every closed ($resp$:semi- closed fuzzy set and α-closed fuzzy set) fuzzy set is $g^{\#}s$-closed fuzzy set in any fts X.

Proof: Follows from the definition.

The converse of the above theorem need not be true as seen from the following example.

3.3 Example

Let $X = \{a, b, c\}$ and the fuzzy sets A and B be defined as follows: $A = \{(a, 0.3), (b, 0.5), (c, 0.6)\}, B = \{(a, 1), (b, 0.8), (c, 0.7)\}$. Consider the fts$(X, T)$, where $T = \{0,1, A\}$. Note that the fuzzy subset B is $g^{\#}s$ -closed fuzzy set but not a closed (not a semi-closed fuzzy set and not a α-closed fuzzy set) fuzzy set in (X, T).

3.4 Theorem

Every $g^{\#}s$ -closed fuzzy set is gs-closed ($gsp\text{-}closed\ fuzzy\ set$)fuzzy set in fts X.

Proof: Follows from the definitions.
The converse of the above theorem need not be true as seen from the following example.

3.5 Example
Let $X = \{a, b, c\}$ and the fuzzy sets A and B be defined as follows: $A = \{(a, 0.3), (b, 0.5), (c, 0.4)\}, B = \{(a, 0.5), (b, 0.3), (c, 0.4)\}$. Consider $T = \{0,1, A\}$. Then (X, T) is fts. Here the fuzzy set B is gs -closed $(resp: gsp - closed\ fuzzy\ set)$ fuzzy set but not $g^{\#}s$ -closed fuzzy set in (X, T).

3.6 Theorem
In a fts X,if a fuzzy set A is both αg-open fu zzy set $and\ g^{\#}s$-closed fuzzy set, then A is semi-closed fuzzy set.

3.7 Theorem
If A is $g^{\#}s$ -closed fuzzy set and $scl(A) \wedge (1 - scl(A)) = 0$.Then there is no non-zero αg-closed fuzzy set F such that $F \leq scl(A) \wedge (1 - A)$.

3.8 Theorem
If a fuzzy set A is $g^{\#}s$-closed fuzzy set in X such that $A \leq B \leq scl(A)$, then B is also a $g^{\#}s$ -closed fuzzy set in X.

3.9 Definition
A fuzzy set A of a fts (X, T) is called $g^{\#}s$ -open fuzzy ($g^{\#}s$ -open fuzzy set) set if its complement $1 - A$ is $g^{\#}s$ -closed fuzzy set.

3.10 Theorem
A fuzzy set A of a fts is $g^{\#}s$ -open iff $F \leq \sin t(A)$, whenever F is αg-closed fuzzy set and $F \leq A$.

3.11 Theorem
Every open (resp. semi-open fuzzy set and α-open fuzzy set) fuzzy set is a $g^{\#}s$ -open fuzzy set but not conversely.

Proof: Follows from the definitions
The converse of the above theorem need not be true as seen from the following example.

3.12 Example
In example 3.3, the fuzzy subset $1 - B = \{(a, 0),)(b, 0.2), (c, 0.3)\}$ is $g^{\#}s$ -open fuzzy set but not open (resp. not a semi open fuzzy set and not a α-open fuzzy set) fuzzy set in (X, T).

3.13 Theorem
In fts X, every $g^{\#}s$ -open fuzzy se the t is gs-open fuzzy (resp. gsp-open fuzzy set) set but not conversely.
The converse of the above theorem need not be true as seen from the following example.

3.14 Example
In example 3.5, the fuzzy subset $1 - B = \{(a, 0.5),)(b, 0.7), (c, 0.6)\}$ is gs-open fuzzy (resp. gsp-open fuzzy set) set but not $g^{\#}s$-open fuzzy set in (X, T).

3.15 Theorem
If $\sin t(A) \leq B \leq A$ and if A is $g^{\#}s$ -open fuzzy set, then B is $g^{\#}s$ -open fuzzy set in a fts (X, T).

3.16 Theorem
If $A \leq B \leq X$ where A is $g^{\#}s$ open fuzzy relative to $B\ and\ B$ is $g^{\#}s$ -open fuzzy relative to X, then A is $g^{\#}s$ -open fuzzy relative to fts X.

3.17 Theorem
Finite intersection of $g^{\#}s$ -open fuzzy set is a $g^{\#}s$ -open fuzzy set.

3.18 Definition
fts (X, T) is called a fuzzy-$Ts\#$ space if every $g^{\#}s$ -closed fuzzy set is a closed fuzzy set.

3.19 Definition
fts (X, T) is called a fuzzy-$\#Ts$ space if every gs-closed fuzzy set is a $g^{\#}s$ -closed fuzzy set.

4. FUZZY $g^{\#}$ -SEMI-CLOSURE AND FUZZY $g^{\#}$ -SEMI-INTERIOR FUZZY SETS IN FTs

In this section we introduce the concepts of fuzzy $g^{\#}s$ -closure ($fg^{\#}s\text{-}cl$) and fuzzy $g^{\#}s$ -interior ($fg^{\#}s\text{-}int$) and investigate their properties.

4.1 Definition
For any fuzzy set A in any fts is said to be fuzzy $g^{\#}s$ -closure and is denoted by $fg^{\#}s\text{-}cl(A)$, defined by $fg^{\#}s\text{-}cl(A)= \wedge \{U: U\ is\ g^{\#}s - closed\ fuzzy\ set\ and\ A \leq U\}$.

4.2 Definition

For any fuzzy set A in any fts is said to be fuzzy $g^{\#}s$ -interior and is denoted by $fg^{\#}s\text{-}int(A)$, defined by $fg^{\#}s\text{-}int(A)=\vee\{V: V \text{ is } g^{\#}s - interior\ fuzzy\ set\ and\ A \geq V\}$.

4.3 Theorem

Let A be any fuzzy set in fts (X,T). Then $fg^{\#}s\text{-}cl(A)$=$fg^{\#}s\text{-}cl(1 - A$_=1-$fg^{\#}s\text{-}int(A)$=$fg^{\#}s\text{-}int(1 - A)$=1-$fg^{\#}s\text{-}cl(A)$.

4.4 Theorem

In a fts (X,T), a fuzzy set A is $g^{\#}s$ -closed iff $A = fg^{\#}s\text{-}cl(A)$.

4.5 Theorem

In fts X the following results hold for $g^{\#}s$ -closure.

(1) $g^{\#}s\text{-}cl(0) = 0$.
(2) $g^{\#}s\text{-}cl(A)$ is $g^{\#}s$ -closed fuzzy set in X.
(3) $g^{\#}s\ –cl(A) \leq g^{\#}s\text{-}cl(B)$ if $A \leq B$.
(4) $g^{\#}s\ –cl(g^{\#}s\text{-}cl(A)) = g^{\#}s\ \text{-}cl(A)$.
(5) $g^{\#}s\ \text{-}cl(A \vee B) \geq g^{\#}s\text{-}cl(A) \vee g^{\#}s\text{-}cl(B)$
(6) $g^{\#}s\ \text{-}cl(A \wedge B) \leq g^{\#}s\text{-}cl(A) \wedge g^{\#}s\text{-}cl(B)$.

Proof: The easy verification is omitted.

4.6 Theorem

In a fts X, a fuzzy set A is $g^{\#}s$ -open iff $A = fg^{\#}s\text{-}int(A)$

4.7 Theorem

In a fts X, the following results hold for $g^{\#}s$ -interior.

(1) $g^{\#}s\ –int(0)$=0.
(2) $g^{\#}s\ –int(A)$ is $g^{\#}s$-open fuzzy set X.
(3) $g^{\#}s\text{-}int(A) \leq g^{\#}s\text{-}int(B)$ if $A \leq B$.
(4) $g^{\#}s\ –int(g^{\#}s\text{-}int(A))$= $g^{\#}s\text{-}int(A)$.
(5) $g^{\#}s\ \text{-}int(A \vee B) \geq g^{\#}s\text{-}int(A) g^{\#}\text{-}s\text{-}int(B)$
(6) $g^{\#}s\ \text{-}int(A \wedge B) \leq g^{\#}s\ \text{-}int(A) \wedge g^{\#}s\text{-}int(B)$.

Proof: The routine proof is omitted.

5. FUZZY $g^{\#}$ -SEMI-CONTINUOUS AND FUZZY $g^{\#}$ -SEMI-IRRESOLUTE MAPPINGS IN FTS

5.1 Definition

A function $f: X \to Y$ is said to be fuzzy $g^{\#}s$ -continuous $fg^{\#}s$-continuous) if the inverse image of every open fuzzy set in Y is $g^{\#}s$ -open fuzzy set in X.

5.2 Theorem

A function $f: X \to Y$ is $g^{\#}s$ -continuous if the inverse image of every closed fuzzy set in Y is $g^{\#}s$ -closed fuzzy set in X.

5.3 Theorem

Every fuzzy continuous (resp: $f\alpha$-continuous $and\ fsemi\ \text{-}continuous$) function is fuzzy $g^{\#}s$ -continuous.

Proof: The following proof is omitted.

The converse of the above theorem need not be true as seen from the following example.

5.4 Example

Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B\ and\ C$ defined as follows. $A = \{(a, 0), (b, 0.2), (c, 0.3)\}, B = \{(a, 0.3), (b, 0.5), (c, 0.6)\}, C = \{(a, 1), (b, 0.8), (c, 0.7)\}$. Consider $T = \{0,1, B\}, and\ \sigma = \{0,1, A\}$. Then $(X,T)\ and\ (Y,\sigma)$ are fts.Define $f: X \to Y$ by $f(a) = a, f(b) = b\ and\ f(c\ ) = c$. Then f is $fg^{\#}s$-continuous but not f-continuous (resp: not $f\alpha$-continuous and not f semi-continuous). As the fuzzy set C is closed fuzzy set in $Y\ and\ f^{-1}(C) = C$ is not closed fuzzy set in X but $g^{\#}s$ -closed (resp: α-closed fuzzy set and semi-closed fuzzy set) fuzzy set in X. Hence f is fuzzy $g^{\#}s$-continuous.

5.5 Theorem

Every fuzzy $g^{\#}s$ -continuous function is fuzzy gs-continuous function and is fuzzy gsp-continuoa us function.

Proof: The following proof is omitted.

The converse of the above theorem need not be true as seen from the following example.

5.6 Example

Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B\ and\ C$ defined as follows. $A = \{(a, 0.6), (b, 0.5), (c, 0.3)\}, B = \{(a, 0.6), (b, 0.7), (c, 0.4)\}, C = \{(a, 0.4), (b, 0.3), (c, 0.6)\}$. Consider $T = \{0,1, A\}, and\ \sigma = \{0,1, C\}$. Then $(X,T)\ and\ (Y,\sigma)$ are fts.Define $f: X \to Y$ by $f(a) = a, f(b) = b \; and \; f(c) = c$.Then f is fgs-continuous (resp: $fgsp$-continuous) but not $fg^{\#}s$-continuous as the fuzzy set B is closed fuzzy set in $Y \; and \; its \; inverse \; image \; f^{-1}(B) = B$ which is not $g^{\#}s$ -closed fuzzy set in X which is fgs-closed($fgsp$-closed set in X) set in X.

5.7 Theorem
A function $f: X \to Y$ is $fg^{\#}s$-continuous and X is fuzzy-$Ts\#$ space. Then f is f-continuous.

5.8 Theorem
If $f: X \to Y$ is fgs-continuous and X is fuzzy-$Ts\#$ fts, then f is $fg^{\#}s$-continuous.

5.9 Theorem
If $f: X \to Y$ is $fg^{\#}s$-continuous and $g: Y \to Z$ is f-continuous, then $g \circ f: X \to Z$ is $fg^{\#}s$-continuous.

5.10 Definition
A function $f: X \to Y$ is said to be fuzzy $g^{\#}$ semi-irresolute ($fg^{\#}s$-irresolute) if the inverse image of every $g^{\#}s$ -closed fuzzy set in Y is $g^{\#}s$ -closed fuzzy set in X.

5.11 Theorem
A function $f: X \to Y$ is $fg^{\#}s$-irresolute function iff the inverse image of every $g^{\#}s$ -open fuzzy set in Y is $g^{\#}s$ -open fuzzy set in X.

5.12 Theorem
Every $fg^{\#}s$-irresolute function is $fg^{\#}s$-continuous.

Proof: Follows from the definitions
The converse of the above theorem need not be true as seen from the following example.

5.13 Example
Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B, C \; and \; D$ defined as follows. $A = \{(a, 0.2), (b, 0.5), (c, 0.3)\}, B = \{(a, 0.8), (b, 0.5), (c, 0.7)\}, C = \{(a, 0.5), (b, 0.2), (c, 0.3), D = \{(a, 0.5), (b, 0.8), (c, 0.7)\}$ Let $T = \{0,1, A\}, and \; \sigma = \{0,1, A, B\}$. Then $(X, T) \; and \; (Y, \sigma)$ are fts.Define $f: X \to Y$ by $f(a) = b, f(b) = a \; and \; f(c) = c$.Th en f is $fg^{\#}s$-continuous but not $fg^{\#}s$-irresolute as the fuzzy set is $g^{\#}s$ -closed fuzzy set in $Y \; but \; its \; inverse \; image \; f^{-1}(C) = A$ is not $g^{\#}s$ -closed fuzzy set in X.

5.14 Theorem
Let $f: X \to Y, g: Y \to Z$ be two fuzzy functions. Then:
(1) $f \circ g: X \to Z$ is fuzzy $g^{\#}s$ -continuous if f is fuzzy $g^{\#}s$ -irresolute and g is fuzzy $g^{\#}s$ -continuous.
(2) $f \circ g: X \to Z$ is fuzzy $g^{\#}s$ -irresolute if both f and g are fuzzy $g^{\#}s$ -irresolute.
(3) $f \circ g: X \to Z$ is fuzzy $g^{\#}s$ -continuous if f is fuzzy $g^{\#}s$ -continuous and g is fuzzy continuous.

Proof:
The following proof is omitted.

5.15 Theorem
If $f: X \to Y, g: Y \to Z$ be two fuzzy functions. If f is $fg^{\#}s$-continuous and g is $fg^{\#}s$-irresolute and Y is fuzzy-$Ts\#$ space, then $g \circ f: X \to Z$ is $fg^{\#}s$-irresolute function.

5.16 Theorem
Let $f: X \to Y$ be a fgc-irresolute and a semi-closed fuzzy map. Then $f(A)$ is a $g^{\#}s$ -closed fuzzy set of Y, for every $g^{\#}s$ -closed fuzzy set A of X.

5.17 Theorem
Let $f: X \to Y$ be a fgc-irresolute and a semi-closed map. Then $f(A)$ is a $g^{\#}s$ -closed fuzzy set of Y, for every $g^{\#}s$ -closed fuzzy set A of X.

6. FUZZY $g^{\#}$ -SEMI OPEN MAPS AND FUZZY $g^{\#}$ -SEMI-CLOSED MAPS IN FTs
Every open and fuzzy closed maps were introduced and studied by C. Wong [20]. This study was further carried out by Sadanand, N. Patil [10]. We introduced the following concepts.

6.1 Definition
A function $f: X \to Y$ is said to be fuzzy $g^{\#}$ -semi open (briefly $fg^{\#}s$ -open) if the image of every open fuzzy set in X is $g^{\#}s$ -open fuzzy set in Y.

6.2 Definition
A function $f: X \to Y$ is said to be fuzzy $g^{\#}$ -semi closed (briefly $fg^{\#}s$-closed) if the image of every closed fuzzy set in X is $g^{\#}s$ -closed fuzzy set in Y.

6.3 Theorem
Every fuzzy-open map is fuzzy $g^{\#}s$ -open map.

Proof: The proof follows from the definition 6.1
The converse of the above theorem need not be true as seen from the following example.

6.4 Example
Let$X = Y = \{a, b, c\}$. Fuzzy sets A, B and C be defined as follows.$A = \{(a, 0), (b, 0.2), (c, 0.3)\}, B = \{(a, 0.3), (b, 0.5), (c, 0.6)\}$, $C = \{(a, 1), (b, 0.8), (c, 0.7)\}$ Consider $T = \{0,1, A\}, and\ \sigma = \{0,1, B\}$.Then (X, T) and (Y, σ) are fts.Define $f: X \to Y$ by $f(a) = a, f(b) = b$ and $f(c) = c$.Then f is $fg^{\#}s$-open map but not an f-open map as the fuzzy set A is open fuzzy set in X, and its $image$ $f(A) = A$ is not open fuzzy set in Y which is $g^{\#}s$ -open fuzzy set in Y.

6.5 Theorem
If $f: X \to Y$ is $fg^{\#}s$-open map and Y is a fuzzy-$Ts\#$, then f is a fuzzy open map.

6.6 Theorem
Every fuzzy $g^{\#}s$ -open map is fuzzy gs-open map.

Proof: The proof follows from the definition 6.1
The converse of the above theorem need not be true as seen from the following example.

6.7 Example
Let $X = Y = \{a, b, c\}$ .Fuzzy sets A, B and C be defined as follows. $A = \{(a, 0.3), (b, 0.5), (c, 0.4)\}, B = \{(a, 0.7), (b, 0.5), (c, 0.6)\}, C = \{(a, 0.5), (b, 0.3), (c, 0.4)\}$. Consider $T = \{0,1, A\}, and\ \sigma = \{0,1, A, B\}$. Then (X, T) and (Y, σ) are fts.Define $f: X \to Y$ by $f(a) = b, f(b) = a$ and $f(c) = c$ .Then the function f is fuzzy gs-open map but not an fuzzy $g^{\#}s$ -open map as the image of open fuzzy set A in X is $f(A) = C$ open fuzzy set in Y but not $g^{\#}s$ -open fuzzy set in Y.

6.8 Theorem
If $f: X \to Y$ is fuzzy gs-open map and Y is a fuzzy-$\#Ts$ space, then $fa\ a\ a\ a$ is fuzzy $g^{\#}s$ -open fuzzy map.

6.9 Theorem
Every fuzzy-closed map is fuzzy $g^{\#}s$ -closed map.

Proof: The proof follows from the definition 6.2
The converse of the above theorem need not be true as seen from the following example.

6.10 Example
In example 6.4, the function f is fuzzy $g^{\#}s$ -closed map but not closed fuzzy map as the fuzzy set C is closed fuzzy set in X and its image $f(C) = C$ is $g^{\#}s$ -closed fuzzy set in Y but not closed fuzzy set in Y.

6.11 Theorem
If $f: X \to Y$ is fuzzy $g^{\#}s$ -closed map and Y is a fuzzy-$Ts\#$, then f is fuzzy closed fuzzy map.

6.12 Theorem
A function $f: X \to Y$ is $fg^{\#}s$-closed iff for each fuzzy set S of Y and for each open fuzzy set U such that $f^{-1}(S) \leq U$, there is a $g^{\#}s$ -open fuzzy set V of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

6.13 Theorem
If a map $f: X \to Y$ is fuzzy gc-ieersolute and $fg^{\#}s$-closed and A is $g^{\#}s$ -closed fuzzy set in X and Y is fuzzy-$T^{1/2}$ then $f(A)$ is $g^{\#}s$ -closed fuzzy set in Y.

6.14 Theorem
Let $f: X \to Y$ is fuzzy continuous and fuzzy $g^{\#}s$ -closed. If A is $g^{\#}s$ -closed fuzzy set in X and Y is fuzzy-$T^{1/2}$ then $f(A)$ is $g^{\#}s$ -closed fuzzy set in Y.

6.15 Theorem
If $f: X \to Y$ is f-closed a map and $g: Y \to Z$ is $fg^{\#}s$-closed maps, then $g \circ f: X \to Z$ is $fg^{\#}s$-closed map.

6.16 Theorem
If $f: X \to Y$ and $g: Y \to Z$ are $fg^{\#}s$-closed maps and Y is fuzzy $Ts\#$ space, then $g \circ f: X \to Z$ is $fg^{\#}s$-closed map.

Proof: The proof follows from the definition.

6.17 Theorem
Let $f: X \to Y, g: Y \to Z$ be two maps such that $g \circ f: X \to Z$ is $fg^{\#}s$-closed map.

(1) If f is f-continuous and surjective, then g is $fg^{\#}s$-closed map.
(2) If g is $fg^{\#}s$-irresolute and injective, then f is $fg^{\#}s$-closed map.

6.18 Theorem
The composition $g \circ f\ of\ f: X \to Y\ and\ g: Y \to Z$ is $fg^{\#}s$-open map, if f is $fg^{\#}s$-irresolute and g is $fg^{\#}s$-open map.

6.19 Theorem
If $f: X \to Y\ and\ g: Y \to Z$ be two mappings and $g \circ f: X \to Z$ be composition of those two mappings. Then if f the the is fuzzy-open and g is $fg^{\#}s$-open maps, then $g \circ f\ is\ fg^{\#}s - open$.

6.20 Theorem
If A $g^{\#}s$ -closed fuzzy set in X and $f: X \to Y$ is bijective, f-continuous and $fg^{\#}s$-closed, then $f(A)$ is $g^{\#}s$ -closed fuzzy set in Y.

6.21 Theorem
If a function $f: X \to Y$ is f-continuous and $fg^{\#}s$-closed in X and A is a $g^{\#}s$ -closed fuzzy set in X, then $f_A: A \to Y$ is f-continuous and $fg^{\#}s$-closed map.

6.22 Definition [3]
Let X and Y be two fts. A bijective map $f: X \to Y$ is closed fuzzy-homeomorphism (briefly f-homeomorphism) if f and f^{-1} are fuzzy-continuous.

We introduce the following.

6.23 Definition
A function $f: X \to Y$ is called fuzzy $g^{\#}$ semi-homeomorphism (briefly $fg^{\#}s$-homeomorphism) if f and f^{-1} are $fg^{\#}s$-continuous.

6.24 Theorem
Every f-homeomorphism is $fg^{\#}s$-iomeomorphism

Proof: The proof is follows the definition.
The converse of the above theorem need not be true as seen from the following example.

6.25 Example
Let $X = Y = \{a, b, c\}$ and the fuzzy sets $A, B\ and\ C$ be defined as follows. $A = \{(a, 1), (b, 0), (c, 0)\}, B = \{(a, 1), (b, 1), (c, 0)\}, C = \{(a, 1), (b, 0), (c, 1)\}$. Consider $T = \{0,1, A, C\}\ and\ \sigma = \{0,1, B\}$. Then $(X, T) and\ (Y, \sigma)$ are fts. Define $f: X \to Y$ by $f(a) = a, f(b) = c\ and\ f(c) = b$.Then f is $fg^{\#}s$-hemeomorphism but not f-hemeomorphism as A is open fuzzy set in X and its image $f(A) = A$ is not open in Y,$f^{-1}: Y \to X$ is not f-continuous.

6.26 Theorem
Let $f: X \to Y$ be a bijective function. Then the following are equivalent:
(1) f is $fg^{\#}s$-hemeomorphism
(2) f is $fg^{\#}s$-continuous and $fg^{\#}s$-open maps.
(3) f is $fg^{\#}s$-continuous and $fg^{\#}s$-closed maps.

6.27 Theorem
If $f: X \to Y\ fg^{\#}s$-homeomorphism and $g: Y \to Z$ is $fg^{\#}s$-homeomorphism and Y is fuzzy-$Ts\#$ space, then $g \circ f: X \to Z$ is $fg^{\#}s$-homeomorphism.

6.28 Definition
Let $X\ and\ Y$ be two fts. A bijective map $f: X \to Y$ is called fuzzy $g^{\#}$ semi-homeomorphism (briefly $f\ \ g^{\#}$semi-homeomorphism) if $f$and f^{-1} are fuzzy $g^{\#}s$ -irresolute.

6.29 Theorem
Let X, Y, Z be fuzzy topological spaces and $f: X \to Y, g: Y \to Z$ is $f\ \ g^{\#}$ semic-homeomorphisms then their composition $g \circ f: X \to Z$ is $fg^{\#}$ semic-homeomorphism.

Proof: The following proof is omitted.

6.30 Theorem
Every $fg^{\#}$ semic-homeomorphism is $fg^{\#}s$-homeomorphism.

Proof: The proof follows from the definition.

7. CONCLUSION
In this paper, the attempt has been made to study that on $g^{\#}$-semi open fuzzy sets and fuzzy $g^{\#}$-semi-irresolute maps in fuzzy topological spaces. We have discussed some basic definitions. We have discussed some theorems and results based on basic properties of $g^{\#}$-semi-closed fuzzy sets in fts. We have discussed some theorems and results based on fuzzy $g^{\#}$-semi-closure and

fuzzy $g^{\#}$-semi interior fuzzy sets in fts. We have discussed some theorems and results based on fuzzy $g^{\#}$-semi-continuous and fuzzy $g^{\#}$-semi-irresolute mappings in fts. We have discussed some theorems and results based on fuzzy $g^{\#}$-semi-open maps and fuzzy $g^{\#}$-semi-closed maps in fts.

8. REFERENCES

[1] K. K. Azad (1981), On the Fuzzy continuity, Fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl., 82, 14-32.
[2] G. Balasubramanian and P. Sundaram (1997), on some generalizations of fuzzy continuous functions, Fuzzy sets, and systems, 86, 93-100.
[3] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 182-190.
[4] N. Levine (1970), Generalized closed sets in topology, Rend. Circ. Mat. Palermo, 19(2), 89-96.
[5] H. Maki, T. Fukutake (1998), M. Kojima and H. Harada, Generalized closed sets in fts I report for a meeting on topological spaces, Theory and its Applications, 23-26.
[6] Sadanand N. Patil (June-2008), on $g^{\#}$-semi continuous mappings in fuzzy topological spaces, Intl. Jl.Math.Comp.Appls. Special, vol. l.
[7] Sadanand. N. Patil (2009), Thesis, on some recent advances in topology, awarded the doctorate degree in Karnataka University Dharwad.
[8] Sadanand. N. Patil, on pre-semi-closed fuzzy sets in fuzzy topological spaces, submitted.
[9] Sadanand. N. Patil, On $g^{\#}$-closed fuzzy sets and fuzzy $g^{\#}$-continuous maps in fuzzy topological spaces, submitted.
[10] Sadanand. N. Patil, on μ-closed fuzzy sets, Fuzzy μ-continuous maps and fuzzy μ irresolute maps in fuzzy topological space, Submitted.
[11] Sadanand. N. Patil, on fuzzy $g^{\#}$-homeomorphism maps fuzzy topological cues, Submitted.
[12] Sadanand. N. Patil, on $g^{\#}$-semi-closed fuzzy sets and fuzzy $g^{\#}$-semi-irresolute maps in fuzzy topological spaces, submitted.
[13] Sadanand. N. Patil, on α-$g^{\#}$-closed fuzzy sets in fuzzy topological spaces, submitted.
[14] Sadanand. N. Patil, on μ-semi-closed fuzzy sets and fuzzy μ-semi-continuous maps in fuzzy topological spaces, submitted.
[15] Sadanand. N. Patil, on μ-pre-closed fuzzy sets in fuzzy topological spaces, submitted.
[16] Sadanand. N. Patil, on semi-μ-closed fuzzy sets and fuzzy semi-μ-irresolute maps in fuzzy topological spaces, submitted.
[17] Sadanand. N. Patil (2008), on $g^{\#}\psi$-closed fuzzy sets and fuzzy $g^{\#}\psi$- homeomorphism maps in fuzzy, topological spaces, Proc. of int.con. On anal and appl. November 3-5, Aligarh Muslim University, Aligarh, submitted.
[18] M.K.R.S. Veera Kumar (2006), Generalized-μ-closed sets, Antarctica J.Math., 3(1), 67-83.
[19] C.K. Wong (1974), Fuzzy points and local properties of fuzzy topology, Jl. Math. Anal. Appl., 46, 316-328.
[20] L. A. Zadeh (1965), Fuzzy sets, Inform and Control, 8,338-353

