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On bipolar-valued fuzzy algebra over bipolar-valued fuzzy field

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ABSTRACT

In this paper, we apply the notion of a bipolar-valued fuzzy set to algebra over a field. We introduce the concept of a bipolar-valued L-fuzzy set of algebra and investigate some properties. We give relations between a bipolar-valued fuzzy algebra and bipolar-valued L-fuzzy set. We also give the characterization of bipolar-valued fuzzy ideal and discuss some related properties.

Keywords— *Bipolar-valued fuzzy algebra, Fuzzy field, Sup property, Bipolar-valued L-fuzzy set, Bipolar-valued fuzzy ring, Ideal*

1. INTRODUCTION

The concept of fuzzy sets was first introduced by Zadeh and subsequently, several authors including zadeh have discussed various aspects of the theory and applications of fuzzy sets. A. Rosenfield proposed the concept of a fuzzy group in order to establish the algebraic structure of fuzzy sets. Fuzzy fields and fuzzy linear spaces over fuzzy fields were introduced and studied by nanda. In this note concept of fuzzy field is introduced and discussed. The purposes of the present note are to introduced and discuss the concept of a bipolar-valued fuzzy algebra over a fuzzy field. The notion of bipolar-valued fuzzy algebra and fuzzy field is introduced, and several properties are investigated. The concept of complete lattice on the intersection of a family of bipolar-valued fuzzy algebra is considered, and some properties are discussed.

2. PRELIMINARIES

2.1 Definition [6]

Let Y be an algebra over a field X and let A be a fuzzy subset of Y. Then A is called a fuzzy algebra over a fuzzy field F of a field X in Y if for all $x, y \in Y$ and $\lambda \in X$.

i) $A(x+y) \geq \min\{A(x), A(y)\}$

ii) $A(\lambda x) \geq \min\{F(\lambda), A(x)\}$

iii) $A(xy) \geq \min\{A(x), A(y)\}$.

If F is an ordinary field then (ii) is replaced by $A(\lambda x) \geq A(x)$, for all $x \in Y, \lambda \in X$.

2.2 Example

Let Xbe a field of real number $(\mathbb{R}, +, .)$ and y be an algebra over a field X. Then fuzzy subset A of Y is defined by:

$$A(x) = \begin{cases} 0.6 & if\ x\ is\ irrational \\ 1 & if\ x = 0 \\ 0.7 & if\ x\ is\ rational \end{cases}$$

$$F(\lambda) = \begin{cases} 0.5 & if\ \lambda\ is\ irrational \\ 0 & if\ \lambda = 0 \\ 0.8 & if\ \lambda\ is\ rational \end{cases}$$

Then A is a fuzzy algebra over a fuzzy field F in Y.

2.3 Definition [6]

Let A be a fuzzy algebra in Y. Then A is called a fuzzy left ideal if for all $x, y \in Y, A(xy) \geq A(y)$, a fuzzy right ideal if $A(xy) \geq A(x)$, and a fuzzy ideal if it is both a fuzzy left and right ideal.

2.4 Example

Let $\mathbb{Z}$ be the set of all integers. Define a fuzzy subset $A: \mathbb{Z} \longrightarrow [0,1]$ by:

$$A(x) = \begin{cases} 0.7 & if\ x\ is\ even \\ 1 & if\ x = 0 \\ 0.5 & if\ x\ is\ odd \end{cases}$$

Then A is a fuzzy ideal

2.5 Definition [6]

A fuzzy set A in a fuzzy algebra Y is said to have the sup property, if for any subset T in Y, there exists $t_0 \in T$ such that $A(t_0) = sup_{t \in T} A(t)$.

2.6 Definition [7]

Let X be a field and F a fuzzy set of field X. If the following conditions hold:

i) $F(x+y) \geq \min\{F(x), F(y)\}$, *for all* $x, y \in X$

ii) $F(-x) \geq F(x)$,

iii) $F(xy) \geq \min\{F(x), F(y)\}$,

iv) $F(x^{-1}) \geq F(x)$.

Then F is a fuzzy field of a field X.

2.7 Example

Let X be a field and let $\mathbb{R}$ be a set of all real numbers of a field X. Then the fuzzy subset F of a field X is define by:

$$F(x) = \begin{cases} 0.5 & if \quad x\ is\ irrational \\ 0.8 & if \quad x = 0 \\ 0.7 & if \quad x\ is\ rational \end{cases}$$

Then F is a fuzzy field of a field X.

2.8 Definition [7]

Let (F, X) be a fuzzy field of the field X, Y an algebra over a field X and A be a fuzzy set of Y. suppose the following conditions holds

i) $A(x+y) \geq \min\{A(x), A(y)\}, x, y \in Y$
ii) $A(\lambda x) \geq \min\{F(\lambda), A(x)\}, \lambda \in X \text{ and } x \in Y$
iii) $A(xy) \geq \min\{A(x), A(y)\},$
iv) $F(1) \geq A(x), x \in Y$

Then we call (A, Y) a fuzzy algebra over fuzzy fa ield (F, X).

2.9 Definition

Let Y be an algebra over a field X, F be a fuzzy field of X. A and B are fuzzy subsets of Y, define $A \cap B, A+B, A.B, \lambda.A, -A$ respectively is the following fuzzy subset of Y, for all $x \in Y, \lambda \in X$.

$(A \cap B)(x) = A(x) \wedge B(x),$
$(A+B)(x) = \cup_{x_1+x_2=x}[A(x_1) \wedge B(x_2)] = \cup_{x_1=x} A(x_1)] \wedge [\cup_{x_2=x} B(x_2)]$
$(A.B)(x) = \cup_{x_1.x_2=x}[A(x_1) \wedge B(x_2)] = [\cup_{x_1=x} A(x_1)] \wedge [\cup_{x_2=x} B(x_2)],$
$(\lambda.A)(x) = \cup_{\lambda x_1=x}[F(\lambda) \wedge A(x_1)] = F(\lambda) \wedge \cup_{\lambda x_1=x} A(x_1),$
$(-A)(x) = A(-x).$

2.10 Definition [4]

Let F be a fuzzy subset of a field X. If for all $\lambda_1, \lambda_2 \in X$.

i) $F(\lambda_1 - \lambda_2) \geq \min\{F(\lambda_1), F(\lambda_2)\}$
ii) $F(\lambda_1 \lambda_2) \geq \min\{F(\lambda_1), F(\lambda_2)\}, \lambda_2 \neq 0$

Then F is called a fuzzy field of a field X.

2.11 Definition [4]

Let F be a fuzzy field of a field X and Y be an algebra over X and A be a fuzzy subset of Y. If for all $y_1, y_2 \in Y$ and $\lambda \in X$.

i) $A(y_1 - y_2) \geq \min\{A(y_1), A(y_2)\}$
ii) $A(\lambda y_1) \geq \min\{F(\lambda), A(y_1)\}$
iii) $A(y_1 y_2) \geq \min\{A(y_1), A(y_2)\}$

Then A is called a fuzzy algebra of Y over a fuzzy field F.

3. STRUCTURES ON BIPOLAR-VALUED FUZZY ALGEBRA

3.1 Definition

Let Y be a algebra over a field X and let $D = (X, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy subset of Y. Then D is called a bipolar-valued fuzzy algebra over a fuzzy field $F = (X, \mu_F^+, \mu_F^-)$ of a field X in Y if for all $x, y \in Y$ and $\lambda \in X$

i) $\mu_D^+(x+y) \geq \min\{\mu_D^+(x), \mu_D^+(y)\}, \mu_D^-(x+y) \leq \max\{\mu_D^-(x), \mu_D^-(y)\}$
ii) $\mu_D^+(\lambda x) \geq \min\{\mu_F^+(\lambda), \mu_D^+(x)\}, \mu_D^-(\lambda x) \leq \max\{\mu_F^-(\lambda), \mu_D^-(x)\}$
iii) $\mu_D^+(xy) \geq \min\{\mu_D^+(x), \mu_D^+(y)\}, \mu_D^-(xy) \leq \max\{\mu_D^-(x), \mu_D^-(y)\}$

3.2 Example

Let X be a field of real number $(\mathbb{R}, +, .)$ and Y be an algebra over a field X. Then bipolar-valued fuzzy subset $D = (X, \mu_D^+, \mu_D^-)$ of Y is defined by

$$\mu_D^+(x) = \begin{cases} 0.6 & if\ x\ is\ irrational \\ 1 & if\ \ x = 0 \\ 0.7 & if\ \ x\ is\ rational \end{cases}$$

$$\mu_D^-(x) = \begin{cases} 0.03 & if\ \ x\ is\ irrational \\ 1 & if\ \ \ x = 0 \\ 0.04 & if\ \ x\ is\ rational \end{cases}$$

$$\mu_F^+(\lambda) = \begin{cases} 0.5 & if\ \lambda\ is\ irrational \\ 0 & if\ \ \lambda = 0 \\ 0.8 & if\ \lambda\ is\ rational \end{cases}$$

$$\mu_F^-(\lambda) = \begin{cases} 0.04 & if\ \lambda\ is\ irrational \\ 0 & if\ \ \lambda = 0 \\ 0.06 & if\ \ \lambda\ is\ rational \end{cases}$$

Then $D = (X, \mu_D^+, \mu_D^-)$ is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field $F = (X, \mu_F^+, \mu_F^-)$ in Y

3.3 Definition

Let X be a field and $D = (X, \mu_D^+, \mu_D^-)$ a bipolar-valued fuzzy field X. If the following conditions holds

i) $\mu_D^+(x+y) \geq \min\{\mu_D^+(x), \mu_D^+(y)\}, \mu_D^-(x+y) \leq \max\{\mu_D^-(x), \mu_D^-(y)\}$
ii) $\mu_D^+(-x) \geq \mu_D^+(x), \mu_D^-(-x) \leq \mu_D^-(x)$
iii) $\mu_D^+(xy) \geq \min\{\mu_D^+(x), \mu_D^+(y)\}, \mu_D^-(xy) \leq \max\{\mu_D^-(x), \mu_D^-(y)\}$
iv) $\mu_D^+(x^{-1}) \geq \mu_D^+(x), \mu_D^-(x^{-1}) \leq \mu_D^-(x).$

Then $D = (X, \mu_D^+, \mu_D^-)$ a bipolar-valued fuzzy field of a field X denoted by (D, X).

3.4 Example

Let X be a field and let $\mathbb{R}$ be set of all real numbers of a filed X. Then the bipolar-valued fuzzy set $D = (X, \mu_D^+, \mu_D^-)$ of a field X is define by

$$\mu_D^+(x) = \begin{cases} 0.5 & if\ \ x\ \ is\ irrational \\ 0.8 & if\ \ x = 0 \\ 0.7 & if\ \ x\ \ is\ \ rational \end{cases}$$

$$\mu_D^-(x) = \begin{cases} 0.03 & if\ x\ is\ irrational \\ 0.05 & if\ \ x = 0 \\ 0.08 & if\ \ x\ is\ rational \end{cases}$$

Then $D = (X, \mu_D^+, \mu_D^-)$ is a bipolar-valued fuzzy field of a field X.

3.5 Definition

Let $D = (X, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy algebra in Y. Then D is called a bipolar-valued fuzzy left ideal if for all $x, y \in Y$, $\mu_D^+(xy) \geq \mu_D^+(y)$ and $\mu_D^-(xy) \leq \mu_D^-(y)$, a bipolar-valued fuzzy right ideal if $\mu_D^+(xy) \geq \mu_D^+(x), \mu_D^-(xy) \leq \mu_D^-(x)$ and a bipolar-valued fuzzy ideal if it is both a bipolar-valued fuzzy left and right ideal.

3.6 Example

Let $\mathbb{Z}$ be the set of all integers. Define a bipolar-valued fuzzy subset $D = (X, \mu_D^+, \mu_D^-)$ by

$$\mu_D^+(x) = \begin{cases} 0.7 & if\ x\ is\ even \\ 1 & if\ \ x = 0 \\ 0.5 & if\ x\ is\ odd \end{cases}$$

$$\mu_D^-(x) = \begin{cases} 0.06 & if\ \ x\ is\ even \\ 1 & if\ \ x = 0 \\ 0.04 & if\ \ x\ is\ odd \end{cases}$$

Then $D = (X, \mu_D^+, \mu_D^-)$ is a bipolar-valued fuzzy ideal.

3.7 Proposition

Let Y be an algebra over a field X and let $D = (X, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy subset of Y is a bipolar-valued fuzzy algebra in Y over a bipolar-valued fuzzy field F if and only if for all $x, y \in Y$ and $\lambda_1, \lambda_2 \in X$

i) $\mu_D^+(\lambda_1 x + \lambda_2 y) \geq \min\begin{bmatrix}\min(\mu_F^+(\lambda_1), \mu_D^+(x)), \\ \min(\mu_F^+(\lambda_2), \mu_D^+(y))\end{bmatrix} \mu_D^-(\lambda_1 x + \lambda_2 y) \leq \max[\max(\mu_F^-(\lambda_1), \mu_D^-(x)), \max(\mu_F^-(\lambda_2), \mu_D^+(y))]$
ii) $\mu_D^+(xy) \geq \min(\mu_D^+(x), \mu_D^+(y)), \mu_D^-(xy) \leq \max(\mu_D^-(x), \mu_D^-(y))$

Proof: Assume that $D = (X, \mu_D^+, \mu_D^-)$ is a bipolar-valued fuzzy algebra in Y over a bipolar-valued fuzzy field $F = (X, \mu_F^+, \mu_F^-)$.

We have to prove that:

(i) $\mu_D^+(\lambda_1 x+\lambda_2 y)\geq \min(\mu_D^+(\lambda_1 x),\mu_D^+(\lambda_2 y))$,
$\mu_D^-(\lambda_1 x+\lambda_2 y)\leq \max(\mu_D^-(\lambda_1 x),\mu_D^-(\lambda_2 y))$
(Since D is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field by first condition)

$\mu_D^+(\lambda_1 x+\lambda_2 y)\geq$
$\min[\min(\mu_F^+(\lambda_1),\mu_D^+(x)),\min(\mu_F^+(\lambda_2),\mu_D^+(y))]$,
$\mu_D^-(\lambda_1 x+\lambda_2 y)\leq$
$\max[\max(\mu_F^-(\lambda_1),\mu_D^-(x)),\max(\mu_F^-(\lambda_2),\mu_D^-(y))]$

(Since $D=(x,\mu_D^+,\mu_D^-)$ is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field by second condition)

Since $D=(x,\mu_D^+,\mu_D^-)$ is a bipolar-valued fuzzy algebra (ii) holds.

Therefore, $\mu_D^+(xy)\geq \min(\mu_D^+(x),\mu_D^+(y)),\mu_D^-(xy)\leq \max(\mu_D^-(x),\mu_D^-(y))$.

Conversely, assume that given conditions are holds, we have to prove that $D=(X,\mu_D^+,\mu_D^-)$ is a bipolar-valued fuzzy algebra. Let $\lambda_1=1,\lambda_2=1$ in (ii) we have:

i) $\mu_D^+(x+y)\geq \min\begin{bmatrix}\min(\mu_F^+(1),\mu_D^+(x)),\\ \min(\mu_F^+(1),\mu_D^+(y))\end{bmatrix}\geq$

$\min\begin{bmatrix}\min(\mu_D^+(x),\mu_D^+(x)),\\ \min(\mu_D^+(y),\mu_D^+(y))\end{bmatrix}\geq \min(\mu_D^+(x),\mu_D^+(y))$.

Similarly, $\mu_D^-(x+y)\leq \max(\mu_D^-(x),\mu_D^-(y))$.

ii) $\mu_D^+(\lambda x)=\mu_D^+(\lambda x+0x)\geq \mathrm{Min}\begin{bmatrix}\min(\mu_F^+(\lambda),\mu_D^+(x)),\\ \min(\mu_F^+(0),\mu_D^+(x))\end{bmatrix}\geq$

$\min[\min(\mu_F^+(\lambda),\mu_D^+(x)),\min(\mu_F^+(x),\mu_D^+(x))]$
$\geq \min[\min(\mu_F^+(\lambda),\mu_D^+(x)),\mu_D^+(x)]\geq \min(\mu_F^+(\lambda),\mu_D^+(x))$.

Therefore $\mu_D^+(\lambda x)\geq \min(\mu_F^+(\lambda),\mu_D^+(x)),\mu_D^-(\lambda x)\leq \max(\mu_F^-(\lambda),\mu_D^-(x))$.

iii) $\mu_D^+(xy)\geq \min(\mu_D^+(x),\mu_D^+(y)),\mu_D^-(xy)\leq \max(\mu_D^-(x),\mu_D^-(y))$.

Hence $D=(X,\mu_D^+,\mu_D^-)$ is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field $F=(x,\mu_F^+,\mu_F^-)$ in Y.

3.8 Theorem

Let Y and Z be a anlgebra over a bipolar-valued fuzzy field F in a field X and f an algebraic homomorphism of Y into Z. Let D be a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field Fin Z, then the inverse image $f^{-1}(D)=\{(X,f^{-1}(\mu_D^+)(x),f^{-1}(\mu_D^-)(x))/x\in Y\}$ of D is a bipolar-valued fuzzy algebra over F in Y.

Proof: For all $x,y\in Y$ and $\lambda_1,\lambda_2\in X$,
$f^{-1}(\mu_D^+)(\lambda_1 x+\lambda_2 y)=\mu_D^+(f(\lambda_1 x+\lambda_2 y))=\mu_D^+(\lambda_1 f(x)+\lambda_2 f(y))$ (Since f is an algebraic homomorphism)

$$\geq \min\begin{bmatrix}\min\left(\mu_F^+(\lambda_1),\mu_D^+(f(x))\right),\\ \min\left(\mu_F^+(\lambda_2),\mu_D^+(f(y))\right)\end{bmatrix}$$

(Since D is a bipolar-valued fuzzy algebra over F)

$$\geq \min\begin{bmatrix}\min(\mu_F^+(\lambda_1),f^{-1}(\mu_D^+)(x)),\\ \min(\mu_F^+(\lambda_2),f^{-1}(\mu_D^+)(y))\end{bmatrix}$$

Similarly,$f^{-1}(\mu_D^-)(\lambda_1 x+\lambda_2 y)$

$$\leq \max\begin{bmatrix}\max(\mu_F^-(\lambda_1),f^{-1}(\mu_D^-)(x)),\\ \max(\mu_F^-(\lambda_2),f^{-1}(\mu_D^-)(y))\end{bmatrix}.$$

$$f^{-1}(\mu_D^+)(xy)=\mu_D^+(f(xy))=\mu_D^+(f(x)f(y))$$
$$\geq \min[\mu_D^+(f(x)),\mu_D^+(f(y))]\geq \min[f^{-1}(\mu_D^+)(x),f^{-1}(\mu_D^+)(y)]$$
$$f^{-1}(\mu_D^-)(xy)=\mu_D^-(f(xy))=\mu_D^-(f(x)f(y))$$
$$\leq \max[\mu_D^-(f(x)),\mu_D^-(f(y))]\leq \max[f^{-1}(\mu_D^-)(x),f^{-1}(\mu_D^-)(y)].$$

Hence the inverse image $f^{-1}(D)$ of D is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field F in Y.

3.9 Definition [2]

A complete lattice is a partially ordered set in which all subsets have both a supremum ⋁ infimum ⋀.

3.10 Definition [2]

Let $(L,\leq,\wedge,\vee)$ denotes a complete distributive lattice with maximal element 1 and minimal element 0. Let X be a non-empty set. A L-fuzzy set μ of X is a function $\mu: X\longrightarrow L$.

3.11 Definition [2]

Let $(L,\leq)$ be the lattice with an involutive order reversing operation $N: L\longrightarrow L$. Let X be a non-empty set. An bipolar-valued L-fuzzy set $D=(X,\mu_D^+,\mu_D^-)$ in X is defined as an object of the form $D=\{(X,\mu_D^+(x),\mu_D^-(x))/x\in X\}$, where $\mu_D^+: X\longrightarrow L$ and $\mu_D^-: X\longrightarrow L$ define the degree of membership and the degree of non-membership for every $x\in X$ satisfying $\mu_D^+(x)\leq N(\mu_D^-(x))$.

3.12 Definition

Let $A=\{(X,\mu_A^+(x),\mu_A^-(x))/x\in X\}$ and $B=\{(X,\mu_B^+(x),\mu_B^-(x))/x\in X\}$ be two bipolar-valued L-fuzzy sets of X. Then we define

i) $A\subseteq B$ if and only if for all $x\in X,\mu_A^+(x)\geq \mu_B^+(x)$ and $\mu_A^-(x)\leq \mu_B^-(x)$
ii) $A=B$ if and only if for all $x\in X,\mu_A^+(x)=\mu_B^+(x)$ and $\mu_A^-(x)\leq \mu_B^-(x)$
iii) $A\cup B=\{(x,(\mu_A^+\cup\mu_B^+)(x),(\mu_A^-\cap\mu_B^-)(x))/x\in X\}$ where $\mu_A^+\cup\mu_B^+=\mu_A^+\vee\mu_B^+,\mu_A^-\cap\mu_B^-=\mu_A^-\wedge\mu_B^-$
iv) $A\cap B=\{(x,(\mu_A^+\cap\mu_B^+)(x),(\mu_A^-\cup\mu_B^-)(x))/x\in X\}$.

3.13 Theorem

Let Y be an algebra over a field X and L be a complete lattice. Let any mapping $D: Y\longrightarrow L$ is a bipolar-valued L-fuzzy set of Y, then the intersection of a family of bipolar-valued fuzzy algebra is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field F in Y.

Proof: Let $\{D_i\}_{i\in\Lambda}$ is an family of bipolar-valued fuzzy algebra in Y over a bipolar-valued fuzzy field F in X, let L be a complete lattice and let $\cup_{i\in\Lambda}D_i=\left\{\left(x,\wedge\mu_{D_i}^+(x),\vee\mu_{D_i}^-(x)\right)/x\in X\right\}$ then for all $x,y\in Y$ and $\lambda_1,\lambda_2\in X$

$$\mu_{D_i}^+(\lambda_1 x+\lambda_2 y)=inf_{i\in\Lambda}\mu_{D_i}^+(\lambda_1 x+\lambda_2 y)\geq$$
$$inf_{i\in\Lambda}\left[\min\begin{Bmatrix}\min\left(\mu_F^+(\lambda_1),\mu_{D_i}^+(x)\right),\\ \min\left(\mu_F^+(\lambda_2),\mu_{D_i}^+(y)\right)\end{Bmatrix}\right]$$
$$\geq \min\begin{bmatrix}\min\left(\mu_F^+(\lambda_1),inf_{i\in\Lambda}\mu_{D_i}^+(x)\right),\\ \min\left(\mu_F^+(\lambda_2),inf_{i\in\Lambda}\mu_{D_i}^+(y)\right)\end{bmatrix}$$
$$\geq \min[\min(\mu_F^+(\lambda_1),\mu_D^+(x)),\min(\mu_F^+(\lambda_2),\mu_D^+(y))]$$

Similarly, $\mu_{D_i}^-(\lambda_1 x+\lambda_2 y)\leq \max\begin{bmatrix}\max(\mu_F^-(\lambda_1),\mu_D^+(x)),\\ \max(\mu_F^-(\lambda_2),\mu_D^-(y))\end{bmatrix}$.

$$\mu_D^+(xy)=inf_{i\in\Lambda}\mu_{D_i}^+(xy)inf_{i\in\Lambda}\min\left(\mu_{D_i}^+(x),\mu_{D_i}^+(y)\right)$$
$$\geq \min\left(inf_{i\in\Lambda}\mu_{D_i}^+(x),inf_{i\in\Lambda}\mu_{D_i}^+(y)\right)$$
$$\geq \min(\mu_D^+(x),\mu_D^+(y))$$

Similarly, $\mu_D^-(xy) \leq \max(\mu_D^-(x), \mu_D^-(y))$.
Hence $D = (X, \mu_D^+, \mu_D^-)$ is a bipolar-valued fuzzy algebra over a bipolar-valued fuzzy field $F = (X, \mu_F^+, \mu_F^-)$.

3.14 Definition [5]

Let R be a ring and μ be a fuzzy subset in R. Then μ is called a fuzzy ring of R if for every $x, y \in R$ the following conditions are satisfied

i. $\mu(x+y) \geq mim(\mu(x), \mu(y))$
ii. $\mu(-x) \geq \mu(x)$,
iii. $\mu(xy) \geq \min(\mu(x), \mu(y))$.

3.15 Definition [5]

Let R be a ring and μ be a fuzzy subset in R. Then, μ is called fuzzy ideal over a fuzzy ring R for every $x, y \in R$ the following conditions are satisfied

i. $\mu(x+y) \geq \min(\mu(x), \mu(y))$,
ii. $\mu(-x) \geq \mu(x)$,
iii. $\mu(xy) \geq \max(\mu(x), \mu(y))$.

3.16 Definition

Let R be a ring and $D = (X, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy subset of R. Then, D is called a bipolar-valued fuzzy ring of R if for every $x, y \in R$ the following conditions are satisfied

i. $\mu_D^+(x+y) \geq \min(\mu_D^+(x), \mu_D^+(y))$, $\mu_D^-(x+y) \leq \max(\mu_D^-(x), \mu_D^-(y))$,
ii. $\mu_D^+(-x) \geq \mu_D^+(x)$, $\mu_D^-(-x) \leq \mu_D^-(x)$,
iii. $\mu_D^+(xy) \geq \min(\mu_D^+(x), \mu_D^+(y))$, $\mu_D^-(xy) \leq \max(\mu_D^-(x), \mu_D^-(y))$.

3.17 Definition

A bipolar-valued fuzzy set $D = (X, \mu_D^+, \mu_D^-)$ in X is called a bipolar-valued fuzzy ideal of X, if it satisfies

1) $\mu_D^+(0) \geq \mu_D^+(x)$ *and* $\mu_D^-(0) \leq \mu_D^-(x)$,
2) $\mu_D^+(x) \geq \min(\mu_D^+(x*y), \mu_D^+(y))$,
3) $\mu_D^-(x) \leq \max(\mu_D^-(x*y), \mu_D^-(y))$, *for all* $x, y \in X$.

3.18 Example

Let $X = \{0,1,2,3,4\}$ be a d-algebra and let $D = (X, \mu_D^+, \mu_D^-)$ is the bipolar-valued fuzzy set of X defined by

$$\mu_D^+(x) = \begin{cases} 1 & if \ \ x = 0,2 \\ 0.4 & if \ \ x = 1,2,4 \end{cases}$$

$$\mu_D^-(x) = \begin{cases} -1 & if \ \ x = 0,2 \\ -0.3 & if \ \ x = 1,3,4 \end{cases}$$

Then $D = (X, \mu_D^+, \mu_D^-)$ is a bipolar-valued fuzzy ideal of X.

3.19 Theorem

Let X be a d-algebra and let L be a complete lattice. Let any mapping $D: X \longrightarrow L$ is a bipolar-valued L-fuzzy set on X, then the intersection of family of bipolar-valued fuzzy ideal is also bipolar-valued fuzzy ideal

Proof: Let $\{D_i\}_{i \in I}$ be a family of bipolar-valued fuzzy ideal of X, Let L be a complete lattice then the bipolar-valued L-fuzzy set D in X is characterized by D from X to L and let $D = \bigcap_{i \in I} D_i$ then for all $x, y \in X$

$$\mu_D^+(x*x) = inf_{i \in I} \mu_{D_i}^+(x*x)$$
$$\mu_D^+(0) \geq inf_{i \in I} \min\{\mu_{D_i}^+(x), \mu_{D_i}^+(x)\}$$
$$\geq \min\{inf_{i \in I} \mu_{D_i}^+(x), \mu_{D_i}^+(x)\}$$
$$\geq \min\{\mu_D^+(x), \mu_D^+(x)\}$$

$\mu_D^+(0) \geq \mu_D^+(x)$ and $\mu_D^-(0) \leq \mu_D^-(x)$.

$$\mu_D^+(x) = inf_{i \in I} \mu_{D_i}^+(x) \geq inf_{i \in I}\{\min(\mu_{D_i}^+(x*y), \mu_{D_i}^+(y)\}.$$
$$\geq \min\begin{bmatrix} inf_{i \in I} \mu_{D_i}^+ \\ (x*y), inf_{i \in I} \mu_{D_i}^+(y) \end{bmatrix}$$
$$\geq \min(\mu_D^+(x*y), \mu_D^+(y))$$

$$\mu_D^-(x) = sup_{i \in I} \mu_{D_i}^-(x)$$
$$\leq sup_{i \in I} \max\{\mu_{D_i}^-(x*y), \mu_{D_i}^-(y)\}$$
$$\leq \max[sup_{i \in I} \mu_{D_i}^-(x*y), sup_{i \in I} \mu_{D_i}^-(y)]$$
$$\leq \max(\mu_D^-(x*y), \mu_D^-(y))$$

Hence $D = (x, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy ideal in X.

3.20 Proposition

Let $D = (X, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy ideal of X. If the inequality $x * y \leq z$ holds in X, then $\mu_D^+(x) \geq \min(\mu_D^+(y), \mu_D^+(z))$, $\mu_D^-(x) \leq \max(\mu_D^-(y), \mu_D^-(z))$.

Proof: Let $x, y, z \in X$ be such that $x * y \leq z$, then $(x*y)*z = 0$ and so

$$\mu_D^+(x) \geq \min(\mu_D^+(x*y), \mu_D^+(y))$$
$$\geq \min[\min\{\mu_D^+((x*y)*z), \mu_D^+(z)\}, \mu_D^+(y)]$$
$$= \min[\min\{\mu_D^+(0), \mu_D^+(z)\}, \mu_D^+(y)]$$
$$= \min(\mu_D^+(y), \mu_D^+(z)),$$

$$\mu_D^-(x) \leq \max(\mu_D^-(x*y), \mu_D^-(y))$$
$$\leq \max[\max\{\mu_D^-((x*y)*z), \mu_D^-(z)\}, \mu_D^-(y)]$$
$$\leq \max[\max\{\mu_D^-(0), \mu_D^-(z)\}, \mu_D^-(y)]$$
$$= \max(\mu_D^-(y), \mu_D^-(z)).$$

3.21 Proposition

Let $D = (X, \mu_D^+, \mu_D^-)$ be a bipolar-valued fuzzy ideal of X. If the inequality $x \leq y$ holds in X, then $\mu_D^+(x) \geq \mu_D^+(y)$ and $\mu_D^-(x) \leq \mu_D^-(y)$.

Proof: Let $x, y \in X$, be such that $x \leq y$. Then

$$\mu_D^+(x) \geq \min(\mu_D^+(x*y), \mu_D^+(y))$$
$$= \min(\mu_D^+(0), \mu_D^+(y)) = \mu_D^+(y)$$

Therefore $\mu_D^+(x) \geq \mu_D^+(y)$.

$$\mu_D^-(x) \leq \max(\mu_D^-(x*y), \mu_D^-(y)) = \max(\mu_D^-(0), \mu_D^-(y))$$
$$= \mu_D^-(y).$$

Therefore $\mu_D^-(x) \leq \mu_D^-(y)$.

3.22 Proposition

Let X be a field and $F = (X, \mu_D^+, \mu_D^-)$ a bipolar-valued fuzzy set of X. If (F, X) is a bipolar-valued fuzzy field of a field X, then

i. $\mu_F^+(0) \geq \mu_F^+(x), \mu_F^-(0) \leq \mu_F^-(x), x \in X$,
ii. $\mu_F^+(1) \geq \mu_F^+(x), \mu_F^-(1) \leq \mu_F^-(x), (x \neq 0) \in X$,
iii. $\mu_F^+(0) \geq \mu_F^+(1), \mu_F^-(0) \leq \mu_F^-(1)$.

Proof: Let (F, X) is a bipolar-valued fuzzy field of X, then

i. For all $x \in X$, $\mu_D^+(0) \geq \mu_F^+(x + (-x)) \geq \min(\mu_F^+(x), \mu_F^+(-x))$ $\geq \min(\mu_F^+(x), \mu_F^+(x)) = \mu_F^+(x)$ (Since F is a bipolar-valued fuzzy field)

Similarly $\mu_F^-(0) = \mu_F^-(x + (-x)) \leq \max(\mu_F^-(x), \mu_F^-(-x)) \leq \max(\mu_F^-(x), \mu_F^-(x)) = \mu_F^-(x)$

Therefore $\mu_F^+(0) \geq \mu_F^+(x), \mu_F^-(0) \leq \mu_F^-(x)$.

ii. For all $x \neq 0 \in X$, $\mu_F^+(1) \geq \mu_F^+(xx^{-1}) \geq \min(\mu_F^+(x), \mu_F^+(x^{-1}))$
$$\geq \min(\mu_F^+(x), \mu_F^+(x)) = \mu_F^+(x)$$

Similarly, $\mu_F^-(1) \leq \mu_F^-(xx^{-1}) \leq \max(\mu_F^-(x), \mu_F^-(x^{-1}))$ $\leq \max(\mu_F^-(x), \mu_F^-(x)) = \mu_F^-(x)$

Therefore $\mu_F^+(1) \geq \mu_F^+(x)$, $\mu_F^-(1) \leq \mu_F^-(x)$.

iii. Let $x = 1$ in $\mu_F^+(0) \geq \mu_F^+(x)$ we have $\mu_F^+(0) \geq \mu_F^+(1)$ and $\mu_F^-(0) \leq \mu_F^-(1)$.

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