**INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY**

**ISSN: 2454-132X**
**Impact factor: 4.295**
***(Volume 4, Issue 4)***
Available online at: www.ijariit.com

# Skin friction analysis of MHD flow past on exponentially accelerated vertical plate with variable temperature and mass diffusion in the presence of chemical reaction

*P. Senthamil*
*pstamil15@gmail.com*
*Arignar Anna Government Arts College For Women*
*Walajah, Vellore, Tamil Nadu*

*Dr. V. Visalakshi*
*visa.mathematics@gmail.com*
*Arignar Anna Government Arts College For Women*
*Walajah, Vellore, Tamil Nadu*

## ABSTRACT

***In this paper the unsteady laminar free-convection flow of a viscous incompressible fluid, past exponentially accelerated infinite vertical plate with variable temperature and mass diffusion in the presence of chemical reaction and the magnetic field is considered. The Laplace transform method is used to obtain the expression for skin-friction, Nusselt number, and Sherwood number. The effect of velocity profiles are studied for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time***

***Keywords***— Accelerated, Isothermal, Vertical Plate, Exponential, Heat & Mass Transfer, Chemical Reaction, Magnetic Field

## 1. INTRODUCTION

MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. Important applications in the study of geological formations, in exploration and thermal recovery of oil, and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. Also, it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering, and electronics. The effects of the transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate was studied by Soundalgekar et al (1979). MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar et al (1981). MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as the accelerated motion of the plate was studied by Raptis and Singh (1983). The dimensionless governing equations were solved using Laplace transform technique.

Free convection effects on flow past an exponentially accelerated vertical plate were studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Gupta (1960) studied the flow of an electrically conducting fluid near a uniformly accelerated vertical plate in the presence of a uniform magnetic field. Raptis et al (1981) discussed the MHD flow past an accelerated vertical plate with variable suction and heat flux. Singh (1984) analyzed the MHD flow past an exponentially accelerated vertical plate with uniform temperature.

The Effect of a chemical reaction depends on whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous if it takes place in solution. Chambre and Young (1958) analyzed a first-order chemical reaction in the neighborhood of a horizontal plate. Das et al. (1994) have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving the isothermal vertical plate in the presence of chemical reaction studied by Das et al. (1999). The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

Skin friction analysis of parabolic started infinite vertical plate with variable temperature and variable mass diffusion in the presence of magnetic field was studied by Indira Priyatharshini *et al* (2017). The solutions for velocity, temperature concentrations fields, Sherwood number and Nusselt number are derived in terms of exponential and complementary error functions.

Effect of parabolic motion of isothermal vertical plate with constant mass flux was discussed by Muthucumaraswamy and Geetha (2014). The effects of skin friction were also discussed.

 

Hence it is proposed to study the effects of skin friction on unsteady MHD flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of homogeneous chemical reaction of first order. The dimension less governing equations are solved using the Laplace- transform technique. The solutions are in terms of the exponential and complementary error function

## 2. MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical plate with uniform mass diffusion in the presence of magnetic field has been considered. The x′-axis is taken along the plate in the vertically upward direction and the y′-axis taken normal to the plate. At time $t' \leq 0$ the plate and fluid are at the same temperature $T\infty$. At time $t' > 0$, the plate is exponentially accelerated with a velocity u=u0exp(a′t′) in its own plane against gravitational field. The temperature from the plate, as well as concentration near the plate, is made to increase linearly with respect to time. A transverse magnetic field of uniform strength B0 is assumed to be applied normal to the plate. It is assumed that the effect of viscous dissipation is negligible in the energy equation and there is a first-order chemical reaction between the diffusing species and the fluid. Then under the usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - MU \qquad (1)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \qquad (2)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \qquad (3)$$

The corresponding initial and boundary conditions in dimensionless form are as follows:

$$\begin{aligned} &U = 0,\ \theta = 0,\ C = 0 \text{ for all } Y, t \leq 0 \\ &t > 0: \quad U = e^{at}, C = t \text{ at } Y = 0 \\ &U \to 0,\ \theta \to 0,\ C \to 0 \text{ as } Y \to \infty \end{aligned} \qquad (4)$$

By introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\ t = \frac{u_0^2 t'}{\nu}\quad Y = \frac{yu_0}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^3}, Gc = \frac{g\nu\beta^*(C'_w - C'_\infty)}{u_0^3},$$

$$\Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D},\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2},\ K = \frac{K_l \nu}{u_0^2}, a = \frac{a'\nu}{u_0^2} \qquad (5)$$

The solutions of equations under the boundary condition have been obtained by Muthucumaraswamy and Valliammal. The solutions are in terms of the exponential and complementary error function

$$\theta = t\left[(1 + 2\eta^2 \Pr) erfc(\eta\sqrt{\Pr}) - 2\eta\sqrt{\frac{\Pr}{\pi}}\exp(-\eta^2 \Pr)\right] \qquad (6)$$

$$\begin{aligned} C = &\frac{t}{2}\left[\exp(2\eta\sqrt{KtSc}) erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\, erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \\ &\frac{\eta\sqrt{Sct}}{2\sqrt{K}}\left[\exp(2\eta\sqrt{KtSc}) erfc(\eta\sqrt{Sc} + \sqrt{Kt}) - \exp(-2\eta\sqrt{KtSc}) erfc(\eta\sqrt{Sc} - \sqrt{Kt})\right] \end{aligned} \qquad (7)$$

 

$$U = e^{at}\begin{bmatrix} \exp(2\eta\sqrt{(M+a)t})erfc(\eta+\sqrt{(M+a)t}) \\ +\exp(-2\eta\sqrt{(M+a)t})erfc(\eta-\sqrt{(M+a)t}) \end{bmatrix}$$

$$+[(e+d)+(ec+bd)t]\begin{bmatrix} \exp(2\eta\sqrt{Mt})erfc(\eta+\sqrt{Mt}) \\ +\exp(-2\eta\sqrt{Mt})erfc(\eta-\sqrt{Mt}) \end{bmatrix}$$

$$-\frac{(ec+bd)\eta\sqrt{t}}{\sqrt{M}}\begin{bmatrix} \exp(-2\eta\sqrt{Mt})erfc(\eta-\sqrt{Mt}) \\ -\exp(2\eta\sqrt{Mt})erfc(\eta+\sqrt{Mt}) \end{bmatrix}$$

$$-de^{bt}\begin{bmatrix} \exp(2\eta\sqrt{(M+b)t})erfc(\eta+\sqrt{(M+b)t}) \\ +\exp(-2\eta\sqrt{(M+b)t})erfc(\eta-\sqrt{(M+b)t}) \end{bmatrix} - 2derfc(\eta\sqrt{\Pr})$$

$$-ee^{ct}\begin{bmatrix} \exp(2\eta\sqrt{(M+c)t})erfc(\eta+\sqrt{(M+c)t}) \\ +\exp(-2\eta\sqrt{(M+c)t})erfc(\eta-\sqrt{(M+c)t}) \end{bmatrix}$$

$$-2bdt\left[(1+2\eta^2\Pr)erfc(\eta\sqrt{\Pr})-2\eta\sqrt{\frac{\Pr}{\pi}}\exp(-\eta^2\Pr)\right]$$

$$+de^{bt}\begin{bmatrix} \exp(2\eta\sqrt{\Pr bt})erfc(\eta\sqrt{\Pr}+\sqrt{bt}) \\ +\exp(-2\eta\sqrt{\Pr bt})erfc(\eta\sqrt{\Pr}-\sqrt{bt}) \end{bmatrix}$$

$$-(1+t)e\begin{bmatrix} \exp(2\eta\sqrt{KSct})erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \\ +\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \end{bmatrix}$$

$$+\frac{ce\eta\sqrt{Sct}}{\sqrt{\pi}}\begin{bmatrix} \exp(-2\eta\sqrt{KSct})erfc(\eta\sqrt{Sc}-\sqrt{Kt}) \\ -\exp(-2\eta\sqrt{KtSc})erfc(\eta\sqrt{Sc}+\sqrt{Kt}) \end{bmatrix}$$

$$+ee^{ct}\begin{bmatrix} \exp(2\eta\sqrt{Sc(K+c)t})erfc(\eta\sqrt{Sc}+\sqrt{(K+c)t}) \\ +\exp(-2\eta\sqrt{Sc(K+c)t})erfc(\eta\sqrt{Sc}-\sqrt{(K+c)t}) \end{bmatrix}$$

Where, $b=\frac{M}{\Pr-1}, c=\frac{M-KSc}{Sc-1}, d=\frac{Gr}{2b^2(1-\Pr)}, e=\frac{Gc}{2c^2(1-Sc)}, \eta=\frac{Y}{2\sqrt{t}}$ (8)

By the expression (8) the skin-friction at the plate is given by

$$\tau=-\left(\frac{dU}{dy}\right)_{y=0}=-\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0}$$

$$=\frac{e^{at}}{2\sqrt{t}}\left[\sqrt{(M+a)t}+\frac{2}{\sqrt{\pi}}e^{-(M+a)t}\right]$$

$$+\frac{2}{\sqrt{t}}[(e+d)+(ec+bd)t]\left[\sqrt{Mt}erf(\sqrt{Mt})+\frac{e^{-Mt}}{\sqrt{\pi}}\right]-\frac{ec+bd}{\sqrt{M}}erf(\sqrt{Mt})$$

$$-\frac{2}{\sqrt{t}}de^{bt}\left[\sqrt{(M+b)t}erf(\sqrt{(M+b)t})+\frac{e^{-(M+b)t}}{\sqrt{\pi}}\right]-\frac{2de}{\sqrt{t\pi}}$$

$$-\frac{2e}{\sqrt{t}}e^{ct}\left[\sqrt{(M+c)t}erf(\sqrt{(M+c)t})+\frac{2e^{-(M+c)t}}{\sqrt{\pi}}\right]-2bd\sqrt{\frac{t}{\pi}}(1+\sqrt{\Pr})$$

$$+\frac{2}{\sqrt{t}}de^{bt}\left[\sqrt{\Pr bt}erf(\sqrt{bt})+\frac{e^{-bt}}{\sqrt{\pi}}\right]+\frac{(1+ct)e}{\sqrt{t}}\left[\sqrt{KSct}erf(\sqrt{Kt})+\frac{e^{-Kt}}{\sqrt{\pi}}\right]$$

$$-\frac{ce\sqrt{Sc}}{\sqrt{K}}erf(\sqrt{Kt})+\frac{2e}{\sqrt{t}}e^{ct}\left[\sqrt{Sc(K+c)t}erf(\sqrt{(K+c)t})+\frac{e^{-(K+c)t}}{\sqrt{\pi}}\right]$$

(9)

By the expression (6), the rate of heat transfer in terms of Nusselt number in non-dimensional form is given by:

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{d\theta}{d\eta}\right)_{\eta=0} = \sqrt{\frac{t}{\pi}}(1+\sqrt{\Pr}) \tag{10}$$

By the expression (7), the rate of mass transfer in terms of Sherwood number in non-dimensional form is given by:

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{dC}{d\eta}\right)_{\eta=0} = \sqrt{t}\left[\sqrt{KtSc}\, erf(\sqrt{Kt}) + \frac{e^{-Kt}}{\sqrt{\pi}}\right] + \frac{1}{2}\sqrt{\frac{Sc}{K}} erf(\sqrt{Kt}) \tag{11}$$

**Table 1: Numerical values of nusselt number**

| Pr | t | Nu |
|---|---|---|
| 10 | 0.2 | 1.050198 |
| 10 | 0.3 | 1.286224 |
| 10 | 0.4 | 1.485204 |
| 10 | 0.6 | 1.818996 |
| 60 | 0.2 | 2.206723 |
| 60 | 0.3 | 2.702673 |
| 60 | 0.4 | 3.120778 |
| 60 | 0.6 | 3.822157 |

**Table 2: Numerical Values of Sherwood Number for K=6**

| Sc | K | t | Sh |
|---|---|---|---|
| 2.01 | 3 | 0.2 | 0.792765 |
| 2.01 | 3 | 0.3 | 1.065646 |
| 2.01 | 3 | 0.4 | 1.330144 |
| 2.01 | 3 | 0.6 | 1.846092 |
| 2.01 | 4 | 0.2 | 0.845159 |
| 2.01 | 4 | 0.3 | 1.151939 |
| 2.01 | 4 | 0.4 | 1.451053 |
| 2.01 | 4 | 0.6 | 2.036870 |

**Table 3: Skin friction profiles for Water (Pr=7.0)**

| t | Gr | Gc | Sc | M | K | a | Ʈ |
|---|---|---|---|---|---|---|---|
| 0.2 | 2 | 5 | 0.16 | 3 | 5 | 3 | 85.168153 |
| 0.3 | 2 | 5 | 0.16 | 3 | 5 | 3 | 66.590962 |
| 0.4 | 2 | 5 | 0.16 | 3 | 5 | 3 | 50.250770 |
| 0.2 | 2 | 5 | 0.16 | 4 | 5 | 3 | 265.086609 |
| 0.3 | 2 | 5 | 0.16 | 4 | 5 | 3 | 188.783581 |
| 0.4 | 2 | 5 | 0.16 | 4 | 5 | 3 | 121.578087 |
| 0.2 | 2 | 5 | 0.16 | 3 | 4 | 3 | 98.706726 |
| 0.3 | 2 | 5 | 0.16 | 3 | 4 | 3 | 75.993396 |
| 0.4 | 2 | 5 | 0.16 | 3 | 4 | 3 | 55.927604 |
| 0.2 | 2 | 5 | 0.16 | 3 | 5 | 4 | 85.309300 |
| 0.3 | 2 | 5 | 0.16 | 3 | 5 | 4 | 67.003829 |
| 0.4 | 2 | 5 | 0.16 | 3 | 5 | 4 | 51.244368 |
| 0.2 | 5 | 5 | 0.16 | 3 | 5 | 3 | 161.156587 |
| 0.2 | 5 | 10 | 0.16 | 3 | 5 | 3 | 331.429353 |

**Table 4: Skin friction profiles (Pr=60)**

| t | Gr | Gc | Sc | M | K | a | Ʈ |
|---|---|---|---|---|---|---|---|
| 0.2 | 2 | 5 | 0.16 | 3 | 5 | 3 | 39.648234 |
| 0.3 | 2 | 5 | 0.16 | 3 | 5 | 3 | 29.552732 |
| 0.2 | 2 | 5 | 0.16 | 4 | 5 | 3 | 87.196636 |
| 0.3 | 2 | 5 | 0.16 | 4 | 5 | 3 | 43.956404 |
| 0.2 | 2 | 5 | 0.16 | 3 | 5 | 4 | 39.789381 |
| 0.3 | 2 | 5 | 0.16 | 3 | 5 | 4 | 29.965599 |
| 0.2 | 2 | 5 | 0.6 | 3 | 5 | 4 | 0.266656 |
| 0.3 | 2 | 5 | 0.6 | 3 | 5 | 4 | 1.050111 |

## 3. RESULTS AND DISCUSSION

For a physical understanding of the problem, numerical computations are carried out for different physical parameters *Gr*, *Gc*, *Sc*, M, K, and *t* upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.16. Also, the values of Prandtl number Pr are chosen for liquid organic fluids (Pr = 60). The numerical values of the velocity are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number, magnetic field parameter, chemical reaction parameter and time.

Table I shows the Nusselt number (rare of heat transfer) for different values of Prandtl number and time t. From the table, it is clear that the rate of heat transfer increases with increasing values of Prandtl number and time t.

Table II depicts the Sherwood number (rate of mass transfer) for different values of Schmidt number (Sh), Chemical reaction parameter (K) and time (t). It shows that Sherwood number enhances with increasing values of Schmidt number and time. It is also observed that Sherwood number increases with increasing value of chemical reaction parameter.

The skin friction is tabulated in Table 3 and Table 4. The effect of skin friction for the different values of thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, Chemical reaction parameter, magnetic field parameter and time *t* was analyzed. Table III displays the effect of skin friction in the presence of water (Pr=7.0). It is clear that skin-friction enhances with an increase of Schmidt number. It is also observed that skin-friction increases with the increase of Prandtl number. As time t advances the value of skin-friction decreases. Moreover, the value of the skin friction increases with increasing thermal Grashof number or mass Grashof number. It is also clear that skin friction increases with increasing values of the magnetic field parameter and chemical reaction parameter.

Table 4 shows the effect of skin friction in the presence of liquid organic fluids (Pr=60). It is clear that skin friction increases with increasing values of Gr, Gc, M, K, and exponential parameter a. The trend is reversed with respect to time t. It is clear from Table 3 and Table 4 that the values of skin friction are more in water than in liquid organic fluids.

## 4. CONCLUSION

The effects of skin friction on unsteady MHD flow past an exponentially accelerated infinite vertical plate with variable temperature and mass diffusion, in the presence of homogeneous chemical reaction of first order have been discussed. The dimensionless governing equations are solved using the Laplace- transform technique. The solutions are in terms of the exponential and complementary error function.

The rate of heat and mass transfer and also skin friction were analyzed for different physical parameters like Prandtl number, Schmidt number, thermal Grashof number and mass Grashof number, exponential parameter, magnetic field parameter, chemical reaction parameter and time t. The conclusions of the study are as follows:

- The effect of skin friction increases with increases values of Gr, Gc, Sc, Pr, M, K, a and the trend is reversed with respect to time.
- The Sherwood number increases with increasing values of Schmidt number, chemical reaction parameter and time
- The Nusselt number increases with increasing values of Prandtl number Pr and time t.

## 5. REFERENCES

[1] Soundalgekar VM, Gupta SK, and Birajdar NS. 1979. Effects on mass transfer and free convection currents on MHD Stokes problem for a vertical plate, Nuclear Engineering Design, Vol 53, pp.339-346
[2] Soundalgekar VM., Patil MR, and Jahagirdar MD, 1981, MHD Stokes problem for a vertical plate with variable temperature, Nuclear Engineering Design, Vol 64, pp.39-42
[3] Raptis A and Singh AK, 1983, MHD free convection flow past an accelerated vertical plate, International Communications in Heat and Mass Transfer. Vol.10, pp. 313-321.
[4] Singh AK and Naveen Kumar 1984, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and space, vol.98, pp.245-258
[5] Hossain MA, and Shayo LK, 1986, The skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and space, vol.125, pp.315-324
[6] Gupta AS, 1960, On the flow of an electrically conducting fluid near an accelerated plate in the presence of a magnetic field, Journal of physical society of Japan, Vol.15, pp.894-897
[7] Raptis AA., Tzivanidis GJ and Perdikis CP. 1981, Hydromagnetic free convection flow past an accelerated vertical infinite plate with variable suction and heat flux, Letters in Heat and Mass Transfer, Vol.8, pp. 137-143.
[8] Singh AK, 1984, Hydromagnetic free convection flow near a time-varying accelerated vertical plate, Astrophysics and space, vol.104, pp.259-265.
[9] Chambre PL, and young JD, 1958, On the diffusion of a chemically reactive species in a laminar boundary flow, The Physics of Fluids, Vol.1, pp.48-54.
[10] Das UN, Deka RK and Soundalgekar VM, 1994, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung in ingenieurwesen, Vol.60, pp. 284-287.
[11] Das UN, Deka RK and Soundalgekar VM, 1999, Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction, The Bulleting of GUMA, Vol.5, pp. 13-20.
[12] Muthucumaraswamy R and Geetha E (2014), Effects of parabolic motion on an isothermal vertical plate with constant mass flux, *Ain Shams Engineering Journal* 5, pp. 1317-1323
[13] Indira Priyatharshini S and Neel Armstrong A (2017), Skin friction analysis of parabolic started infinite vertical plate with variable temperature and variable mass diffusion in the presence of magnetic field, *International Journal of Pure and Applied Mathematics* 4, pp. 81-92

 

**APPENDIX**

$A$ : Constant
$D$ : Mass diffusion coefficient
$Gc$ : Mass Grashof number
$Gr$ : Thermal Grashof number
$M$ : Magnetic field parameter
$K$ : Chemical reaction parameter
$g$ : Acceleration due to gravity
$q$ : Heat flux per unit area at the plate
$k$ : Thermal conductivity of the fluid
$Pr$ : Prandtl number
$Sc$ : Schmidt number
$T$ : Temperature of the fluid near the plate
$T_w$ : Temperature of the plate
$T_\infty$ : Temperature of the fluid far away from the plate
$t'$ : Time
$t$ : Dimensionless time
$u$ : Velocity of the fluid in the x-direction
$u_0$ : Velocity of the Plate
$U$ : Dimensionless velocity component in the $x$-direction
$y$ : Coordinate axis normal to the plate
$Y$ : Dimensionless coordinate axis normal to the plate
$\beta$ : Volumetric coefficient of thermal expansion
$\beta^*$ : Volumetric coefficient of expansion with a concentration
$\mu$ : Coefficient of viscosity
$\upsilon$ : Kinematic viscosity
$\rho$ : Density of the fluid
$\theta$ : Dimensionless temperature
$\eta$ : Similarity parameter
$erfc$ : Complementary error function
$C'$ : Species concentration in the fluid
$C$ : Dimensionless concentration
$C'_w$ : Wall concentration
$C'_\infty$ : Concentration in the fluid far away from the plate