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Some allied gsp-continuous, open and closed functions in topology

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ABSTRACT

In 1995, J.Dontchev has defined and studied the notions of gsp-open sets, gsp-closed sets, gsp-continuous functions and gsp-irresolute functions in topological spaces. In the literature, many topologists have been utilized and defined various concepts using these gsp-closed sets in topology. Quite recently, Navalagi et al have utilized these gsp-closed sets and gsp-continuity to define and study the concepts of gsp-separation axioms, gsp-Hausdorff spaces, allied gsp-regularity axioms and allied gsp-normality axioms in topology. In this paper, we define and study the notions of allied - gsp-continuity, gsp-openness, gsp-closedness, totally – gsp- continuous functions and gsp-compactness in topology.

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Keywords— *Semipreopen sets, gsp-closed sets, Preopen sets, gs-closed sets, rps-closed sets, gsp-irresoluteness, pre-gsp-continuous functions and rps-irresolute functions*

1. INTRODUCTION

In 1990, Arya et al [4] have defined the concepts of gs-closed sets and gs-open sets. In 1995,J. Dontchev [7] has defined and studied the concepts of gsp-closed sets, gsp-continuity and gsp-irresoluteness in topological spaces. In 1998 and 1999, T. Noiri et al [17], Arokiarani et al. [3] have defined and studied the concepts of gp-closed sets, gp-continuity, gp-irresoluteness and pre-gp-continuity in topology. In 2010 and 2011 respectively, T. Shyla Isac Mary et al [20&21] have defined and studied the concepts of rps-closed sets, rps –irresolute functions in topology. Quite recently, Navalagi et. al have utilized these gsp-closed sets and gsp-continuity to define and study the concepts of gsp-separation axioms [15], gsp-Hausdorff spaces[16], allied gsp-regularity axioms[13] and allied gsp-normality axioms[14] in topology. In this paper, we define and study the notions of allied-gsp-continuity, gsp-openness, gsp-closedness, totally – gsp- continuous functions and gsp-compactness in topology.

2. PRELIMINARIES

Throughout this paper (X,τ) and (Y,σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. If A be a subset of X, the closure of A and the interior of A is denoted by Cl(A) and Int(A), respectively. A subset A of a space X is called regular open (in brief, r-open) if A =Int Cl(A) and regular closed (in brief, r-closed) if A = Cl Int(A).

We give the following define are useful in the sequel:

Definition 2.1: The subset of A of X is said to be.
(i) semiopen [9] set if $A \subset Cl\ Int(A)$.
(ii) A pre-open [10]set, if $A \subset Int(Cl(A))$
(iii) A semi-preopen[2] set , if $A \subset Cl(Int(Cl(A)))$
The compliment of a semiopen (resp.,propen , semipreopen) set is called semiclosed[5] [(resp.,preclosed [8], semipreclosed [2]) set in space X. The family of all pre-open (resp. semipre-open) sets of a space X is denoted by PO(X) (resp., SPO(X)) and that of pre-closed (resp.semipre-closed) sets of a space X is denoted by PF(X), (resp.SPF(X)).

Definition 2.2: Let be a subset of a space X. Then,
(i) the semiclosure [5]of A and denoted by sCl(A) is the intersection of all semiclosed sets of X containing subset A.
(ii) the preclosure [8]of A and denoted by pCl(A) is the intersection of all preclosed sets of X containing subset A.
(iii) the semipreclosure[2] of A and denoted by spCl(A) is the intersection of all semipreclosed sets of X containing subset A.

Similarly, one can define the sInt(A), pInt(A), spInt(A) operators.

Definition 2.3: A sub set A of a space X is said to be:
(i) a generalized semiclosed (briefly, gs- closed) **[4]** set if $sCl(A)\subseteq U$, whenever $A\subseteq U$ and U is open set in X .
(ii) a generalized semi-preclosed (briefly, gsp- closed) [7] set if $spCl(A) \subseteq U$ whenever $A\subseteq U$ and U is open in X .
(iii) a generalized pre-closed (briefly, gp- closed) [**17**] set if $pCl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X .
(iv) a regular presemiclosed (briefly, rps-closed) set [20] if $spCl(A) \subset U$, whenever $A \subset U$ and U is rg-open in X.

The complement of a g-closed (resp, gs-closed, gsp-closed, gp-closed, rps-closed) set in X is called g-open (resp. gs-open, gsp-open, gp- open, rps-open) set in X. The family of all gsp-open sets of X is denoted by GSPO(X).

Definition 2.4 [15]: The gsp-closure of A and is denoted by gspCl(A) is the intersection of all gsp-closed sets that contain A.

Definition 2.5: A function $f: X\rightarrow Y$ is called:
(i) semiprecontinuous [11] if the inverse image of each open set of Y is semipreopen in X.
(ii) semipre-irresolute [11] if the inverse image of each semipreopen set of Y is semipreopen in X

Definition 2.6: A function $f: X\rightarrow Y$ is called:
(i) gp-continuous [3] if the inverse image of each closed set of Y is gp-closed in X.
(ii) gp- irresolute [3] if the inverse image of each gp-closed set of Y is gp-closed in X.
(iii) gsp-continuous [7] if the inverse image of each closed set of Y is gsp-closed in X.
(iv) gsp-irresolute [7] if the inverse image of each gsp-closed set of Y is gsp-closed in X.
(v) rps-continuous [21] if the inverse image of each closed set of Y is rps-closed in X.
(vi) rps-irresolute [21] if the inverse image of each rps-closed set of Y is rps-closed in X.
(vii) (rps, gsp)-continuous[16] if the inverse image of rps-open set of Y is gsp-open set in X.

Definition 2.7[12]: A function $f: X\rightarrow Y$ is said to be generalized semipreopen (briefly, gsp-open) if for each open set U of X, f(U) is gsp-open in Y

Definition 2.8[12]: A function $f: X\rightarrow Y$ is said to be generalized semipreclosed (briefly, gsp-closed) if for each closed set F of X, f(F) is gsp-closed in Y.

Definition 2.9[19]: A subset A of a space X is called a sg-clopen if it is both sg-open set and sg-closed set.

Definition 2.10[19]: A function $f: X\rightarrow Y$ is called totally continuous if the inverse imagge of each open subset of Y is clopen subset of X.

Definition 2.11[19]: A function $f: X\rightarrow Y$ is called
(i) a totally semi generalized continuous(briefly, totally sg-continuous) if the inverse image of every open subset of Y is a sg-clopen(ie sg-open and sg-closed) subset of X.
(ii) a strongly semi generalized continuous(briefly, strongly sg-continuous) if the inverse image of every subset of Y is a sg-clopen subset of X.

Definition 2.12[15]: A space X is said to be gsp-T_2 if for any pair of distinct points x,y of X, there exist disjoint gsp-open sets U and V such that $x\in U$ and $y\in V$.

3. ALLIED GSP-CONTINUOUS FUNCTIONS
In view of Definition 2.6, we have following:

Lemma 3.1: Let A be subset of a space X. Then,
(i) If A is semi-preclosed set then it is rps –closed.
(ii) If A is preclosed set then it is rps –closed.
(iii) If A is rps-closed set then it is gsp –closed.
(iv) If A is gs-closed set then it is gsp-closed set.
(v) If A is gp-closed set then it is gsp-closed set.
Then, it is clear from the above lemma -3.1 that the folloing hold good:

Lemma 3.2: Let $f: X \rightarrow Y$ be a function, then
(i) If f is semipre-continuous, then f is rps continuous
(ii) If f is pre-continuous, then f is rps continuous
(iii) If f is rps -continuous, then f is gsp continuous
(iv) If f is gs -continuous, then f is gsp-continuous
(v) If f is gp -continuous, then f is gsp- continuous
Easy proof of the above lemma is trivial.
We, define the following.

Definition 3.3: A function f: X→Y is called:

(i) (s, gsp)-continuous if the inverse image of each semi open set of Y is gsp-open in X.
(ii) (gs, gsp)-continuous if the inverse image of gs open set of Y is gsp-open set in X.
(iii) (gp, gsp)-continuous if the inverse image of gp- open set of Y is gsp-open set in X.

We, prove the following:

Lemma 3.4: The following statements are equivalent for a function f: X→Y :

(i) f is (s,gsp)-continuous ,
(ii) For each point x of X and each semi-neighbourhood V of f(x), there exists a gsp-neighbourhood U of x such that f(U)⊂V.
(iii) For each x in X and each semiopen set V of Y , there exists a gsp-open set U in X such that f(U) ⊂ V.

Proof: Obvious.

We, define the following:

Definition 3.5: A set U ⊂ X is said to be a gs-neighbourhood of a point x ∈X if there exists gs-open set A in X such that x∈A⊂U.

Lemma 3.6: The following statements are equivalent for a function f: X→Y:

(i) f is (gs, gsp)-continuous
(ii) inverse image of each gs-closed set of Y is gsp-closed set in X.
(iii) f(gspCl(A)) ⊂ gsCl(f(A)) for each set A in X.
(iv) gspCl(f^{-1}(B)) ⊂ f^{-1}(gsCl(B)) for each set B in Y.
(v) f^{-1}(gsInt(B))⊂gspInt(f^{-1}(B) for each set B in Y.
(vi) For each point x_a in X and each gs-open set B of Y with f(x_a) ∈B,there exists a gsp-open set A in X such that x∈A and f(A)⊂B.

Proof: Obvious.

Lemma 3.7: Let f: X→Y be a bijective function. The f is (gp,gsp)-continuous iff gpInt(f(A)) ⊂ f(gspInt(A)) for each set A in X.
Proof: Obvious.

Lemma 3.8: Let f: X→ Y and g: Y→ Z be two functions:

(i) If f is gsp-irresolute function and g is (gs, gsp) -continuous functions, then their composition gof is (gs, gsp) -continuous.
(ii) If f is gsp-irresolute function and g is (gp, gsp) -continuous functions, then their composition gof is (gp, gsp) -continuous.
(iii) If f is gsp-irresolute function and g is (rps, gsp) -continuous functions, then their composition gof is (rps, gsp) -continuous.
(iv) If f is (gs, gsp) - continuous function and g is gs -continuous functions, then their composition gof is gsp -continuous.
(v) If f is (gp, gsp) - continuous function and g is gp -continuous functions, then their composition gof is gsp -continuous.
(vi) If f is (rps, gsp) - continuous function and g is rps -continuous functions, then their composition gof is gsp -continuous.
(vii) If f is (s, gsp) - continuous function and g is semi-continuous functions, then their composition gof is gsp -continuous.

Proof: Obvious.

We, prove the following:

Theorem 3.9 : If function f :X→Y is gsp-irresolute , then for every subset B of X , f(gspC(A) ⊂ gspCl(f(A)).
Proof is easy and hence omitted.

We, define the following:

Definition 3.10:

(i) A collection {A_α:α∈∇} of gsp-open sets in a space X is called gsp-open cover of B of X if B ⊂ ∪{A_α :α∈∇ } holds.
(ii) A space X is called gsp-compact if every gsp-open cover of X has a finite subcover.
(iii) A subset B of X is called gsp-compact relative to X if for every collection {A_α : α∈∇} of gsp-open subsets of X such that B ⊂ ∪{A_α : α∈∇ }, there exists a finite subset ∇_0 of ∇ such that B ⊂ ∪{A_α : α∈∇_0 }.
(iv) A subset B of X is said to be gsp-compact if B is gsp-compact as a subspace of X.

Now, we prove the following:

Theorem 3.11: Every gsp-closed subset of gsp-compact space X is gsp-compact relative to X.
Proof: Let A be gsp-closed subset of X, then X-A is gsp-open set in X. Let K = {H_α :α∈∇ } be a cover of A by gsp-open subsets of X. Then, G= K ∪ (X-A) is a gsp-open cover of X, i.e., X = ∪ ({H_α :α∈∇ } ∪ (X-A)). By hypothesis, X is gsp-compact, hence G has a finite subcover of X say , ($H_1 \cup H_2 \cup H_3 \cup \ldots\ldots \cup H_n$) ∪ (X-A). But A and (X-A) are disjoint, hence A ⊂ ($H_1 \cup H_2 \cup H_3 \cup \ldots\ldots \cup H_n$). So, K contains a finite subcover for A, therefore A is gsp-compact relative to X.

Theorem 3.12: Let f: X→Y be a function:

(i) If X is gsp-compact and f is gsp-continuous bijective, thn Y is compact.
(ii) If f is gsp-irresolute and G is gsp-compact relative to X, then f(G) is gsp-compact relative to Y.
(iii) If X is compact and f is continuous surjective,then Y is gsp-compact.

Proof is similar to Th.4.3 due to [1]

We, define the following:

Definition 3.13: A sub set A of a space is called
(i) a gsp-clopen if it is both gsp-open set and gsp-closed set.
(ii) a gs-clopen if it is both gs-open set and gs-closed set.
Clearly, every sg-clopen set is gsp-clopen set and every gs-clopen set is gsp-clopen set.

Definition 3.14: A function f: X $\rightarrow$ Y is said to be totally gsp-continuous if inverse image of each open subset of Y is a gsp-clopen subset of X.
Clearly, every totally continuous function is totally gsp-continuous and every totally sg-continuous function is totally gsp-continuous function.

Definition 3.15: A function f: X$\rightarrow$ Y is said to be G- gsp-continuous if inverse image of each subset of Y is a gsp-clopen subset of X.
Clearly, every G-gsp-continuous function is totally gsp-continuous function.

Theorem 3.16: Every totally gsp-continuous functions into T_1-space is G- gsp-continuous.
Proof: In a T_1-space, singltons are closed. Hence f $^{-1}$(A) is gsp-clopen in X for every subset A of Y.

Definition 3.17: A space X is said to be gsp-connected if X is cannot be experessed as the union of two non empty disjoint gsp-open sets.

Theorem 3.18: If f is totally gsp-continuous function from a gsp-connected space X onto any space Y, then Y is an indiscrete space
Proof: Suppose that Y is not indiscrete. Let A be a proper non-empty open subset of Y. Then f $^{-1}$(A) is a proper non empty gsp-open subset of X, which is a contradiction to the fact that X is gsp-connected.

Theorem 3.19: A space X is gsp-T_2 if and only if for any pair of distinct points x,y of X there exist gsp-open sets U and V such that $x\in U$ and $y\in V$ and gspCl(U)$\cap$ gspCl(V)=$\emptyset$.
Proof: Necessity: Suppose that X is gsp-T_2. Let x and y be distinct points of x. There exist gsp-open sets U and V such that $x\in U$ and $y\in V$ and $U\cap V=\emptyset$. Hence gspCl(U)$\cap$ gspCl(V)=$\emptyset$ and and by lemma 3.27. gspCl(U) is gsp-open. Therefore, we obtained gspCl(U)$\cap$ gspCl(V)=$\emptyset$.
Sufficiency: Obvious.

Theorem 3.20: If f: X $\longrightarrow$ Y is a totally gsp-continuous injective and Y is T_0 then X is gsp-T_2
Proof: Let x and y be any pair of distinct points of X, Then f(x) $\neq$ f(y). Since Y is T_0 there exists an open set U containg say, f(x) but not f(y).Then $x\in f^{-1}(U)$ and $y\notin f^{-1}(V)$. Since f is totally gsp-continuous, f $^{-1}$(U) is a gsp-clopen subset of X. Also $x\in f^{-1}(U)$ and $y\in X - f^{-1}(U)$.Then by Theorem 3.10 , X is gsp-T_2.

We, define the following:
Definition 3.21: A function f : X $\rightarrow$ Y is said to be (rps, gsp)-open if for each rps-open set U of X, f(U) is gsp-open in Y.
Definition 3.22: A function f : X $\rightarrow$ Y is said to be (gsp, rps)-open if for each gsp-open set U of X, f(U) is rps-open in Y.
Definition 3.23: A function f : X $\rightarrow$ Y is said to be (gs, gsp)-open if for each gs-open set U of X, f(U) is gsp-open in Y.
Definition 3.24: A function f : X$\rightarrow$ Y is said to be (gp, gsp)-open if for each gp-open set U of X, f(U) is gsp-open in Y.

We, state the following:
Theorem 3.25: A surjective function f : X $\rightarrow$ Y is (rps, gsp)-open if and only if for each subset B of Y and each rps- closed F of X containing f $^{-1}$(B), there exists gsp-closed set H of Y such that $B\subset U$ and f $^{-1}$(H) $\subset$F.

Theorem 3.26: A surjective function f : X $\rightarrow$ Y is (gs, gsp)-open if and only if for each subset B of Y and each gs-closed F of X containing f $^{-1}$(B), there exists gsp-closed set H of Y such that $B\subset U$ and f $^{-1}$(H) $\subset$F.

Theorem 3.27: A surjective function f : X $\rightarrow$ Y is (gp, gsp)-open if and only if for each subset B of Y and each gp-closed F of X containing f $^{-1}$(B), there exists gsp-closed set H of Y such that $B\subset U$ and f $^{-1}$(H) $\subset$F.

We, define the following:
Definition 3.28: A function f: X$\rightarrow$ Y is said to be (rps,gsp)-closed if for each rps-closed set U of X, f(U) is gsp-closed in Y.
Definition 3.29: A function f: X $\rightarrow$ Y is said to be (gp, gsp)-closed if for each gp-closed set U of X, f(U) is gsp-closed in Y.
Definition 3.30: A function f: X $\rightarrow$ Y is said to be (gs, gsp)-closed if for each gs-closed set U of X, f(U) is gsp-closed in Y.

We, prove the following:
We, recall the following from [12].
Theorem 3.31: A surjective function f : X $\rightarrow$ Y is gsp-closed if and only if for each subset B of Y and each open set U of X containing f $^{-1}$(B), there exists gsp-open set V of Y such that $B\subset V$ and $f^{-1}(V)\subset U$.
Proof: Suppose that f is gsp-closed. Let B be any subset of Y and U be open set of X containing f $^{-1}$(B). Put V= Y-f(X-U). Then the complement of X-VX-V=Y-V = f(X-U). Since X-U is closed in X and f is gsp-closed, f(X-U) = X-V is gsp-closed. Therefore V is gsp-open in Y. It is easy ti see that $B\subset V$ and $f^{-1}(V)\subset U$.

Conversely, Let F be any closed set in X. Put B = Y-f(F), then we have $f^{-1}(B) \subset X-F$ and X-F is open set of in X. Then by the assumption, there exists gsp-open set V of Y such that $B=Y-f(F) \subset V$ and $f^{-1}(V) \subset X-F$. Now $f^{-1}(V) \subset X-F$ implies $V \subset Y-f(F) = B$. Also $B \subset V$ and so B=V Therefore, we obtained f(F)= Y-V and hence f(F) is gsp-closed in Y. This shows that f is gsp-closed.

We, state the following:

Theorem 3.32: A surjective function $f : X \to Y$ is (rps, gsp)-closed if and only if for each subset B of Y and each rps- open F of X containing $f^{-1}(B)$, there exists gsp-open set H of Y such that $B \subset U$ and $f^{-1}(H) \subset F$.

Theorem 3.33: A surjective function $f : X \to Y$ is (gs, gsp)-closed if and only if for each subset B of Y and each gs-open F of X containing $f^{-1}(B)$, there exists gsp-open set H of Y such that $B \subset U$ and $f^{-1}(H) \subset F$.

Theorem 3.34: A surjective function $f : X \to Y$ is (gp, gsp)-closed if and only if for each subset B of Y and each gp-open F of X containing $f^{-1}(B)$, there exists gsp-open set H of Y such that $B \subset U$ and $f^{-1}(H) \subset F$.
Proofs are all similar to Theorem 3.31 above.

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