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Higher dimensional cosmological models with variable gravitational and cosmological constants

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ABSTRACT

A higher-dimensional cosmological model with the gravitational and cosmological constants generalized as coupling scalars in Einstein theory is considered in the framework kaluza-klein theory of gravitation. A general method of solving the field Equations is given. Exact solutions for matter distribution in cosmological model satisfying $G = G_o\left(\frac{R}{R_o}\right)^m$ is presented. The corresponding physical interpretations of the cosmological solutions are also discussed.

***Keywords*—** Cosmological model, Cosmological constants, Kaluza-Klein theory of Gravitation

1. INTRODUCTION

Today, a challenging problem is the unification of gravity with other fundamental forces in nature. In this time, the space-time dimensions to be more than four is the most recent efforts. In the context of Kaluza-Klein and superstring theories, higher dimensions have acquired much significant. Kaluza-klein achievements are shown that five-dimensional general relativity contains both Einstein's four-dimensional theory of gravity and Maxwell's theory of electromagnetism.

In recent years, multidimensional cosmological models have been studied by several methods. Kaluza-klein inhomogeneous cosmological models with and without cosmological constants have studied by Chaterzee and Benerjee (1993) and Banerjee et. al (1995) respectively. Kaluza-Klein cosmological models in generalizes scalar-tensor theory and Lyra geometry have been studied by Chakraborty and Ghosh (2000) and Rahaman and Bera (2001), respectively. However, there are a few works in a literature where variable G and $\wedge$ have been consider in ha igher dimension.

A theory of gravitation using G and $\wedge$ as non constant coupling scalar has been used by Beesham (1986) and Abdel Rahaman (1990). The motivation was to include a G coupling constant of graviaty as a pioneer by Dirac in (1937).

The evolution of the universe is described by Einstein's field Equations together with perfect fluid and an Equation of state in relativistic and the observational cosmology. Einstein field Equation contains two parameters, the cosmological constant $\wedge$ and the gravitational constant G. The gravitational constant G plays the role of a coupling constant between geometry of space and matter content in Einstein field Equations. The cosmological constant $\wedge$ interpreted as the energy density of the vacuum in general relativistic quantum field theory. If we assume the equality of gravitational and inertial mass and gravitational time dilation in Einstein theory we must required that the Equation of motion of particle and photon does not contain G and $\wedge$. In any case the strongest constraints are presently observed G_o value and observational limit $\wedge_o$. Sistero (1991) found an exact solution for zero pressure models satisfying $G = G_o\left(\frac{R}{R_o}\right)^m$. We have obtained exact solution for matter distribution in cosmological models in (2009) satisfying $G = G_o\left(\frac{R}{R_o}\right)^m$. Khadekar and Kamdi (2010) have obtained exact solutions for Zeldovich matter distribution in the framework of Kaluza-Klein theory of gravitation.
In this paper, we present an exact solution for matter distribution in higher dimensional cosmological model satisfying

$$G = G_o\left(\frac{R}{R_o}\right)^m$$

2. MODEL AND FIELD EQUATIONS

We consider the five-dimensional Robertson-Walker metric:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right] + A^2(t)dx_n{}^2 \quad (1)$$

Where k is the curvature index which can take up the values $(-1, 0, +1)$ corresponding to the open, flat and closed universe respectively and $R(t)$ is the scale factor. The universe is assumed to be filled with distribution of matter represented by energy-momentum tensor of perfect fluid

$$T_{ab} = -pg_{ab} + (p+\rho)u_a u_b \tag{2}$$

Where p and ρ are the pressure and energy density of the cosmic matter respectively and u_a is $(n+4)$velocithe ty vector such that $u_a u^a = 1$.

The Einstein field Equations are given by:

$$R_{ab} - \frac{1}{2}Rg_{ab} = 8\wedge GT_{ab} + \wedge g_{ab} \tag{3}$$

Where T_{ab} the matter energy-momentum tensor, g_{ab} the metric tensor, G and $\wedge$ are coupling scalars.

The conservation Equation for variable G and $\wedge$ is given by

$$\dot{\rho} + (3+n)\frac{\dot{R}}{R}(\rho + p) = -\left(\frac{\dot{G}}{G}\rho + \frac{\dot{\wedge}}{8\pi G}\right) \tag{4}$$

Using co-moving co-ordinates

$$u^a = (1, 0, 0, 0 \ldots, 0) \tag{5}$$

In Equation (2) and with the line element (1), Einstein's field Equation (3) becomes

$$8\pi G\rho = 3\left[(n+1)\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right] - \wedge(t) \tag{6}$$

$$8\pi Gp = -(n+2)\frac{\ddot{R}}{R} - (n^2+n+1)\frac{\dot{R}^2}{R^2} - \frac{k}{R^2} + \wedge(t) \tag{7}$$

$$8\pi Gp = -3\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right) + \wedge(t) \tag{8}$$

Where the overdot denotes the differentiation with respect to t. The usual conservation law (i.e. $T^{ab}_{;b} = 0$) yields

$$\dot{\rho} + (3+n)(\rho + p)\frac{\dot{R}}{R} = 0 \tag{9}$$

Using Equation (9) in Equation (4), we have

$$8\pi\dot{G}\rho + \dot{\wedge} = 0 \tag{10}$$

Equations (6), (7) and (10) are the fundamental Equations. Equations (6) and (7) may be written as

$$3(n+2)\ddot{R} = -8\,\pi GR(3p+\rho) - 3n^2\frac{\dot{R}^2}{R} + 2\wedge R \tag{11}$$

$$3(n+1)\dot{R}^2 = 8\,\pi GR^2\left[\rho + \frac{\wedge}{8\,\pi G}\right] - 3k \tag{12}$$

Equation (9) can also be expressed as

$$\frac{d}{dt}(\rho R^{n+3}) + p\frac{d}{dt}(R^{n+3}) = 0 \tag{13}$$

Equations (6), (10) and (13) will be used in the following as fundamental since they are independent. Once the problem is determined the integration constant is characterized by the observable parameters

$$H_o = \frac{\dot{R}_o}{R_o} \tag{14}$$

$$\sigma_o = \frac{4\pi}{3}\frac{G_o\rho_o}{{H_o}^2} \tag{15}$$

$$q_o = -\frac{\ddot{R}_o}{R_o}{H_o}^2 \tag{16}$$

$$\epsilon_o = \frac{p_o}{\rho_o} \tag{17}$$

Which must satisfy Einstein's Equations at present cosmic time t_o:

$$\wedge_o = 3{H_o}^2\left[\sigma_o(3\epsilon_o + 1) - \frac{n+2}{2}q_o + \frac{n^2}{2}\right] \tag{18}$$

$$\frac{k}{{R_o}^2} = {H_o}^2\left[3(1+\epsilon_o)\sigma_o - \frac{n+2}{2}q_o + \frac{(n^2-2n-2)}{2}\right] \tag{19}$$

And the conservation Equation (10) can be written as:

$$\dot{\wedge}G_o + 6\dot{G}_o{H_o}^2\sigma_o = 0 \tag{20}$$

Solutions:

In this work we adopt a method similar to that introduced by Sistero (1971), for G and $\wedge$ constants. We assume the global 'Equation of state',

$$p = \frac{1}{3}\rho\emptyset \tag{21}$$

Where, $\emptyset$ is a function of the scale factor R.

From Equations (13) and (21), we obtain

$$\frac{1}{\Psi}\frac{d\Psi}{dR} + \frac{(n+3)}{3}\frac{\emptyset}{R} = 0 \tag{22}$$

Where,
$$\Psi = \rho R^{n+3} \tag{23}$$

In Equation (22), either $\emptyset$ or Ψ may be taken to be an arbitrary function. If $\emptyset$ is a given explicit function of R, then Equation (21) is determined, and Ψ follows from Equation (22),

$$\Psi = \Psi_o exp\left[-\int \frac{(n+3)}{3}\frac{\emptyset}{R} dR\right] \tag{24}$$

Conversely, if Ψ is given, $\emptyset$ immediately follows from Equation (22)

$$\emptyset = -\frac{3}{(n+3)}\frac{R}{\Psi}\frac{d\Psi}{dR} \tag{25}$$

The Friedman Equation (6) with Equation (23) becomes:

$$3(n+1)\dot{R}^2 = 8\,\pi G\,\Psi R^{-(n+1)} + \wedge R^2 - 3k \tag{26}$$

Equations (10) and (23) with $\frac{d}{dt} = \dot{R}\left(\frac{d}{dR}\right)$ give,

$$8\pi\frac{dG}{dR} + \Psi^{-1}R^{n+3}\frac{d\wedge}{dR} = 0 \tag{27}$$

Finally if $G = G(R)$ is given, we integrate Equation (27) to yield $\wedge = \wedge(R)$. Equation (26) determines $R = R(t)$ and the problem is solved; $\wedge = \wedge(R)$ may be given instead and $G(R)$ derives from Equation (27), giving in turn $R(t)$ from integration of Equation (26the the).

As an example of matter distribution, we consider a case of $\emptyset = 2$, a constant (numerical) in Equation (24) thereby giving the relation

$$\Psi = \rho_o\left(\frac{R_o}{R}\right)^{\frac{2}{3}(n+3)} R_o{}^{\frac{1}{3}(n+3)} \tag{28}$$

Substituting the condition

$$G = G_o\left(\frac{R}{R_o}\right)^m \tag{29}$$

Into Equation (27) with Ψ from Equation (28) we have

$$\wedge = \wedge_o + B_m\left\{1-\left(\frac{R}{R_o}\right)^{m-\frac{5}{3}(n+3)}\right\}R_o{}^{-\frac{2}{3}(n+3)} \tag{30}$$

where,

$$B_m = \frac{6m}{m-\frac{5}{3}(n+3)}\sigma_0 H_o{}^2 \tag{31}$$

For $m \neq \frac{5}{3}(n+3)$, B_m is a parameter related to the integration constant of Equation (27).
From Equation (18), we have

$$\wedge_o = 3H_o{}^2\left[3\sigma_o - \frac{n+2}{2}q_o + \frac{n^2}{2}\right] \tag{32}$$

Using Equation (28), (29) and (30), Friedman's Equation (26) gives

$$\dot{R}^2 = \alpha_n R^{m-\frac{5}{3}(n+3)+2} + \beta_n R^2 - \frac{1}{n+1}k \tag{33}$$

Where,

$$\alpha_n = \frac{-10(n+3)}{3(n+1)\left\{m-\frac{5}{3}(n+3)\right\}}H_o{}^2\sigma_o R_o{}^{(n+3)-m} \tag{34}$$

$$\beta_n = \frac{H_o{}^2}{(n+1)}\left[\left\{3+\frac{2m}{m-\frac{5}{3}(n+3)}R_o{}^{-\frac{2}{3}(n+3)}\right\}\sigma_o - \frac{n+2}{2}q_o + \frac{n^2}{2}\right] \tag{35}$$

Equation (19) can be written as:

$$\frac{k}{R_o{}^2} = H_o{}^2\left[5\sigma_o - \frac{(n+2)}{2}q_o + \frac{(n^2-2n-2)}{2}\right] \tag{36}$$

And Equation (20) is also satisfied.

It is clear that the models are completely characterized by the set of parameters $(H_o, G_o, \sigma_o, q_0, m)$ with $m \neq \frac{5}{3}(n+3)$, $B_m < 0$ in Equation (31) and $\alpha_n > 0$ in Equation (34) when $m < \frac{5}{3}(n+3)$ and vice versa; $\beta_n \geq (<) 0$ according to m, n, σ_o and q_o combine in Equation (35);

$\wedge_o \geq (<)0$ As $\sigma_o \geq (<)\left\{\frac{1}{3}\left(\frac{n+2}{2}q_o - \frac{n^2}{2}\right)\right\}$ as given by Equation (32) and the curvature parameter k equals $+1,\ 0,\ -1$ according to $\left[5\sigma_o - \frac{(n+2)}{2}q_o + \frac{(n^2-2n-2)}{2}\right] \geq (<)0$ in Equation (36). These relations determine the integration conditions of the Friedman Equation (33) and the properties of its solutions.

3. CONCLUSION

In summary, Einstein's field Equations are generalized with usual conservation laws $T^{ab}_{;b} = 0$ by considering the gravity with G and $\wedge$ coupling scalars. Its applications is developed to cosmology. The field Equations for perfect fluid cosmology are formally identical to Einstein's Equations for G and $\wedge$ constants including Equation (13). The additional conservation Equation (10) gives the coupling of the scalars fields with matter. We find the exact solutions of the matter distribution of higher-dimensional world in the framework of Kaluza-Klein theory of gravitation with the global Equation of state of the form $p = \frac{1}{3}\rho\emptyset$ by introducing a general method of solving the cosmological field Equations.

The solutions illustrate many interesting cases showing initial singularity $R(0) = 0$ with an ever expanding or finite cosmic era returning to a feature singularity$R(t) = 0$.

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