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Fuzzy theory based resource allocation problem with possibilistic approach: A case study of sandwich factory

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ABSTRACT

Now a day many service industries do not aim the optimal data as in real situation. Many decision making is done using fuzzy data if we want to select the products like mobile, food, laptop, clothes, cars, house, property etc we see the rating or stars i.e. is actually fuzzy data. Most of the service provider asks for the feedback from the customer, those data which they collect sometime in the form of (bad, good, excellent) some time (bad, average, good, excellent) sometime in the form of stars etc. In this chapter, we have formulated the problem of resource allocation problem using triangular and trapezoidal numbers in the objective function. We have deployed possibilistic approach to convert fuzzy problems into single-valued and then linear and exponential membership function is used as tools. Lingo 15 is used to solve the problem. Using different sap parameter optimal solution is obtained with a degree of satisfaction which helps Decision maker to take a decision.

Keywords: *Resource allocation problem, Triangular and trapezoidal fuzzy numbers, Lingo 15, Possibilistic approach*

1. INTRODUCTION

The human being has the ability to handle complex processes in its daily routine, which often involve approximate reasoning. The ways adopted by human operators to manage such situations has also inaccurate sources, due to the fact that people commonly use linguistic terms in their decision making, using words such as “high”, “low”, “very”, “little”, among others. The classical logic described by Aristotle, also known as standard logic, classifies objects in well-defined categories, in which “everything” has to be or not to be “something”, either now or in the future. Although this binary logic has the ability to solve an extraordinary range of problems, it is necessary to fulfill remaining gaps that are not adequately addressed by these traditional methods. Fuzzy logic concepts bring more flexibility to these binary classifications, in which new “degrees of truth” are available between “yes” and “no”. These degrees can be compared as shades of gray between black and white, which gives a generalisation of the Aristotelian logic.

Important philosophers, such as Bertrand Russell and Albert Einstein, highlighted the inability of standard logic to manage real-world problems. The following thoughts are attributed to Russell showing his position: “Every language is vague”, “All traditional logic habitually assumes that precise symbols are being employed”. “Therefore, this is not applicable to terrestrial life, but only to an imaginary heavenly existence” and “... you cannot imagine how it is vague until you try to do it accurately”. The following statement is attributed to Einstein: “When the laws of mathematics refer to the reality, they are not correct. But, when these laws are correct, they do not refer to the reality”.

2. FORMULATION OF MULTI OBJECTIVE RESOURCE ALLOCATION PROBLEM OF SANDWICH FACTORY WITH TRIANGULAR FUZZY NUMBERS USING POSSIBILISTIC

In this section, we analyse multi-objective resource allocation problem of the sandwich factory with two objectives cost and risk factor, associated with the product. The objective is to minimize the cost and risk factor associated with the product with some realistic constraints. To formulate the multi-objective resource allocation problem of sandwich factory related optimization model, some notations are used defined as follows: [3]

- i indicate the index of product (i=1, 2… n)
- a_{ij} , is the quantity associated with i^{th} product and j^{th} constraint
- b_j is the quantity required for j^{th} constraints
- c_i is a cost associated with i^{th} the product
- r_i is risk associated with i^{th} the product

2.1 Multi-objective resource allocation problem of the sandwich factory with two objectives cost and risk factor
Model -1

$$Min z_1 = \sum_{i=1}^{n} c_i x_i \quad (1)$$

$$Min z_2 = \sum_{i=1}^{n} r_i x_i \quad (2)$$

Subject to the constraints

$$\sum_{i=1}^{n} a_{i1} x_i \le b_1 \quad \text{(Bread constraint)} \quad (3)$$

$$\sum_{i=1}^{n} a_{i2} x_i \ge b_2 \quad \text{(Tomato Constraint)} \quad (4)$$

$$\sum_{i=1}^{n} a_{i3} x_i \ge b_3 \quad \text{(Cheese Constraint)} \quad (5)$$

$$\sum_{i=1}^{n} a_{i4} x_1 \le b_4 \quad \text{(Potato Constraint)} \quad (6)$$

$$\sum_{i=1}^{n} a_{i5} x_i \le b_5 \quad \text{(Manpower Constraint)} \quad (7)$$

$$\sum_{i=1}^{n} a_{i6} x_i \le b_6 \quad \text{(Chicken Constraint)} \quad (8)$$

$$\sum_{i=1}^{n} a_{i7} x_i \le b_7 \quad \text{(Bread constraint)} \quad (9)$$

$$x_2 \ge b_8 \ , \quad \text{(Minimum constraint)} \quad (10)$$

$$x_1 \ge b_7 \ , \quad \text{(Minimum constraint)} \quad (11)$$

$$x_3 \ge b_9 \ , \quad \text{(Minimum constraint)} \quad (12)$$

$$x_4 \ge b_{10} \quad \text{(Minimum constraint)} \quad (13)$$

Where,

$c_i = (c_{i1}, c_{i2}, c_{i3})$ is a cost associated with the product and $r_i = (r_{i1}, r_{i2}, r_{i3})$ is a risk factor associated with i^{th} the product. z_1 and z_2 indicates the cost $c_i = (c_{i1}, c_{i2}, c_{i3})$ and risk $r_i = (r_{i1}, r_{i2}, r_{i3})$ associated with i^{th} the product.

2.2 Possibilistic distribution for uncertain multi-objective resource allocation problem (RAP) for sandwich factory problem:

The possibility theory lies in such way that a significant part of the data deals with human choices and also depends on possibilistic in nature. The possibility distribution is estimated in the derived form of insufficient data and knowledge of DM. Possibilistic programming approach has used to solve fuzzy optimization model of many important applications. Possibilistic approach converts the fuzzy objective and/or constraints into crisp objective and/or constraints regarding their three scenarios as optimistic, most-likely and pessimistic scenarios. It is also utilized to sustain the uncertainty of the problem until the solution is obtained [5]. Therefore, the possibilistic approach is used to convert the uncertain RAP l into a crisp multi-objective optimization model [5]. Numerous studies in the literature are used possibility distribution to solve fuzzy optimization problem [1, 2, 5, and 6].
To find the solution multi-objective resource allocation problem for the sandwich factory, first, they are converted into crisp multi-objective resource allocation problem by possibility distribution.

2.3 Formulation of fuzzy multi-objective RAP optimization model-1

To convert the model-1 into crisp multi-objective optimization model, the triangular Possibility distribution strategy is used. Thus, the quality objective function can be written as follows [4, 5]:

$$\min Z_1 = \min(z_1^o, z_1^m, z_1^p) = \left(\sum_{i=1}^{n} C_i x_i\right)$$
$$\min\left(\left(\sum_{i=1}^{n} C_i^o x_i\right), \left(\sum_{i=1}^{n} C_i^m x_i\right), \left(\sum_{i=1}^{n} C_i^p x_i\right)\right) \tag{14}$$

Where, $c_j = (c_{11}, c_{12}, c_{13})$, which can be considered as follows.

$$\left(\min z_{11}, \min z_{12}, \min z_{13}\right) = \left(\left(\sum_{i=1}^{n} C_i^o x_i\right), \left(\sum_{i=1}^{n} C_i^m x_i\right), \left(\sum_{i=1}^{n} C_i^p x_i\right)\right) \tag{15}$$

to

Eq. (14) and (15) are related with the optimistic, the most likely scenario and the pessimistic scenarios respectively.

Using the α -level sets concepts ($0 \le \alpha \le 1$), where, $\left(C_{ij}\right)_\alpha = \left((C_i)_\alpha^o, (C_i)_\alpha^m, (C_i)_\alpha^p\right)$,

Where, $(C_i)_\alpha^o = C_i^o + \alpha\left(C_i^m - C_i^o\right)$, $(C_i)_\alpha^m = C_i^m$, $(C_i)_\alpha^p = C_i^p - \alpha\left(C_i^p - C_i^m\right)$

Hence, Eq. (12) can be written as:

$$\left(\min z_{11}, \min z_{12}, \min z_{13}\right) = \left(\left(\sum_{i=1}^{n} (C_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^p x_i\right)\right) \tag{16}$$

Similarly, multi-objective optimization model of purchasing cost objective function is as follows

$$\left(\min z_{21}, \min z_{22}, \min z_{23}\right) = \left(\left(\sum_{i=1}^{n} (r_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^o x_i\right)\right) \tag{17}$$

2.4 Crisp multi-objective optimization model:

To reflect the ththe ree different scenarios with $\alpha-$set concept, the uncertain multi-objective RAP problem is converted into crisp multi-objective resource allocation problem is defined as follows:

Model-1.1:

$$\left(\min Z_{11}, \min\ Z_{12}, \min Z_{13}, \min Z_{21}, \min Z_{22}, \min Z_{23}\right) =$$
$$\left(\left(\sum_{i=1}^{n} (C_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^p x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^o x_j\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^m x_i\right)\right) \tag{18}$$

Subject to the constraints: Equation (3) to (13)

2.4.1 Solution approach for solving crisp Multi-objective resource allocation problem of the sandwich factory with two objectives cost and risk factor in triangular fuzzy number:

This section presented a fuzzy technique for uncertain multi-objective RAP for the sandwich factory to determine the best efficient solution with possibility distribution. This approach also gives greater flexibility to solve the multi-objective optimization problem in terms of considered the different choices of shape parameter for each objective function. This approach forces to optimize the maximum product of membership values of each objective to provide better assignment plans.

2.4.2 Steps for finding the solution of multi-objective optimization models of resource allocation problem for sandwich factory:

Step-1: Formulate the model-1 of multi-objective resource allocation problem(s), using appropriate triangular possibility distribution.

Step-2: Define the crisp objective functions of model-1, according to the confidence level α .

Step-3: Convert maximization problem into minimization form of model-1.1.

Step-4: Find out the positive ideal solution (PIS) and negative ideal solution (NIS) [2, 4, 5] for all objective functions of the model-1.1.

Step-5: Determine fuzzy exponential membership value for Z_{ij} .

$$\mu_{Z_{ij}}^E(x) = \begin{cases} 1; & \text{if } Z_{ij} \le Z_{ij}^{\text{PIS}} \\ \dfrac{e^{-S\psi_{ij}(x)} - e^{-S}}{1 - e^{-S}}, & \text{if } Z_{ij}^{\text{PIS}} < Z_{ij} < Z_{ij}^{\text{NIS}} \\ 0; & \text{if } Z_{ij} \ge Z_{ij}^{\text{NIS}} \end{cases}$$

Where, $\psi_{ij}(x) = \dfrac{Z_{ij} - Z_{ij}^{\text{PIS}}}{Z_{ij}^{\text{NIS}} - Z_{ij}^{\text{PIS}}}$ and S($S > 0\,(\text{S} < 0)$) is the shape parameter which given by DM and $0 \le \mu_{Z_i}(x) \le 1$. For $S > 0\,(\text{S} < 0)$, the membership function is strictly concave (convex) in [$Z_{ij}^{\text{PIS}}, Z_{ij}^{\text{NIS}}$]. The value of the membership function permits us to define the model grades of precision in consequent objective function.

Step-6: In this step, multi-objective optimization models of resource allocation problem on sandwich factory model-1.1 converted into the single-objective optimization problem(s) (SOPs) which are as follows.

(Model-1.1.1)

$$\max \lambda \tag{19}$$

Subject to constraints:

$$\lambda \le \mu_{Z_{ij}}^{E}(x) \tag{20}$$

and constraints: (7.3) to (7.10)

Step-7: To find different assignment plans for the model(s) (model-1.1.1) developed in step-6 using LINGO software with different choices of the shape parameter.

If the obtained solution is accepted by DM, consider it as the ideal compromise solution and stop solution process else change the value of shape parameter, confidence level and repeat the step 2 to 7 till satisfactory solution achieved for model-1.1.

2.5 Algorithm:

Input: *Parameters:* $(Z_1, Z_2, ..., Z_m, n)$

Output: ***S****olution of uncertain multi-objective programming problem*

Solve uncertain multi-objective programming problem ($Z_k \downarrow, X \uparrow$ *)*

begin

read: *problem*

while *problem = Uncertain multi-objective programming problem do*

for *k=1 to m do*

enter matrix Z_k

end

-| ***Find triangular distribution for each objective function.***

-| ***Define the crisp multi-objective programming problem according to*** $\alpha -$ ***level.***

the PIS and NIS for each objective.

for *k=1 to m do*

$z_{ij}^{\text{PIS}} = \min (z_i)_{\alpha}^{0}$

Under given constraints

end

for *k=1 to m do*

$z_{ij}^{\text{NIS}} = \max (z_i)_{\alpha}^{0}$

Under given constraints

end

-| ***Define exponential membership function for each objective.***

for *k=1 to m do*

$$\mu_{z_{ij}}^{E}(x) = \begin{cases} 1; & \text{if } z_{ij} \le z_{ij}^{\text{PIS}} \\ \dfrac{e^{-S\psi_{ij}(x)} - e^{-S}}{1 - e^{-S}}, & \text{if } z_{ij}^{\text{NIS}} < z_{ij} < z_{ij}^{\text{PIS}} \\ 0; & \text{if } z_{ij} \ge z_{ij}^{\text{NIS}} \end{cases}$$

end

-| ***find single objective optimization models under given constraints from multi-objective optimization models.***

for*k=1 to m do*

$\max \lambda$

Subject to constraints:

$\lambda \le \mu_{Z_{ij}}^{E}(x)$

Under the given constraints

end

|- *find the solution SOP(s) using LINGO software.*

end

2.6 Case study of food factory with triangular fuzzy numbers

Here in this chapter we have consider the case study of food industry involved in production of sandwich. The production capacity of the factory is 200000 per day which involves more than 1000 worker. This production is divided into 20 units. We have considered the data of one unit in which from the past experience certain fuzzy data were

Table 1: Cost of product with trapezoidal fuzzy numbers

Cost of product 1	[17.5 23.75 30.0]
Cost of product 2	[25.5 31.25 37]
Cost of product 3	[16.5 20.25 24]
Cost of product 4	[37.5 45.25 53]

Table 2: Risk factor associated with trapezoidal fuzzy numbers

Risk factor associated with product 1	[18 20.5 23.0]
Risk factor associated with product 2	[15.5 17.5 19.5]
Risk factor associated with product 3	[5 7.0 9]
Risk factor associated with product 4	[8 10 12]

Table 3: Amount of product required and its availability

Product	Bread	Tomato	Potato	Cheese	Tiki	Chicken	Pasta	Manpower	Machine	Miscall
Cheese s/w	1	30	30	20	0	0	0	3	2	50
Cheese burger	1	30	0	25	50	0	0	3	2	50
Allotiki	1	50	0	25	50	0	0	3	0	50
Chicken pasta	1	50	0	25	0	40	25	3	2	50
avaibility	210	4000	15000	4000	110	4000	3000	960	600	210

2.6.1 Problem Formulation

$$Minz_1 = [17.5, 23.75, 30.0]x_1 + [25.5, 31.25, 37]x_2 + [16.5, 20.25, 24]x_3 + [37.5, 45.25, 53]x_4$$

$$Minz_2 = [18, 20.5, 23.0]x_1 + [15.5, 17.5, 19.5]x_2 + [5, 7, 9]x_3 + [8, 10, 12]x_4$$

Subject to:

$x_1 + x_2 + x_3 + x_4 \leq 210$ (Bread constraint) (21)

$30x_1 + 30x_2 + 50x_3 + 50x_4 \geq 4000$ (tomato constraint) (22)

$20x_1 + 25x_2 + 25x_3 + 25x_4 \geq 4000$ (cheese constraint) (23)

$30x_1 \leq 15000$ (potato constraint) (24)

$3x_1 + 3x_2 + 3x_3 + 3x_4 \leq 960$ (manpower constraint) (25)

$0.04x_4 \leq 4000$ (chicken constraint) (26)

$0.025x_4 \leq 3000$ (Bread constraint) (27)

$x_1 \geq 20, x_2 \geq 20,$
$x_3 \geq 20, x_4 \geq 20$ (Minimum constraint) (28, 29, 30, 31)

2.6.2: Solution for the Triangular fuzzy number using possibility approach

To evaluate multi-objective linear programming problem, the model is coded.Table-1 gives the PIS and NIS for each objective function of model-1.1 at $\alpha = 0.1, 0.5$ and 0.9 which are used in the exponential membership function of each objective. Each combination of the shape parameters is shown in Table-5.

Table 4: PIS and NIS value for each objective at $\alpha = 0.1, 0.5$ and $\alpha = 0.9$

$\alpha -$ value		Z_{11}	Z_{12}	Z_{13}	Z_{21}	Z_{22}	Z_{23}
$\alpha = 0.1$	**PIS**	3173.06	3931.875	4690.69	1425.8	1730	2034.2
	NIS	6962.75	8292.5	9622.25	3520.5	3975	4429.5
$\alpha = 0.5$	**PIS**	3510.31	3931.88	4353.44	1561	1730	1899
	NIS	7553.75	8292.5	9031.25	3722.5	3975	4227.5
$\alpha = 0.9$	**PIS**	3847.56	3931.88	4016.19	1696.2	1730	1763.8
	NIS	8144.75	8292.5	8440.25	3924.5	3975	4025.5

Table 5: Different values of shape parameters

Case	Shape Parameter (K_1, K_2)
1	(-5, -5)
2	(-3, -2)
3	(-2, -1)
4	(-1, -3)
5	(-5, 1)
6	(5, -2)

Table 6: Summary results of a different scenario for each objective at $\alpha = 0.1$, 0.5 and 0.9

$\alpha -$ value	Case	The degree of satisfaction level	Membership function	Objective values	Allocations
$\alpha =$ 0.1	1	0.9962	(0.9962, 0.9969, 0.9973) (0.9962, 0.9963, 0.9963)	(3506.89, 4264.49, 5022.12) (1610.30, 1927.29, 2244.27)	$x_1 = 35.7831,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 21.3735$
	2	0.9805	(0.9805, 0.9835, 0.9857) (0.9805, 0.9806, 0.9806)	(3572.68, 4330.05, 5087.42) (1548.75, 1861.47, 2174.19)	$x_1 = 30.5179,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 25.5856$
	3	0.9637	(0.9637, 0.9689, 0.9729) (0.9637, 0.9638, 0.9638)	(3568.54, 4325.92, 5083.31) (1552.63, 1865.62, 2178.61)	$x_1 = 30.8495,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 25.3204$
	4	0.9698	(0.9698, 0.9739, 0.9771) (0.9698, 0.9699, 0.9699)	(3364.64, 4122.76, 4880.88) (1743.39, 2069.59, 2395.80)	$x_1 = 47.1679,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 12.2657$
	5	0.6659	(1, 1, 1) (0.6659, 0.6667, 0.6672)	(3173.06, 3931.88, 4690.69) (1922.63, 2261.25, 2599.88)	$x_1 = 62.5,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 0$
	6	0.4929	(0.4929, 0.5421, 0.5831) (1, 1, 1)	(3704.1 , 4461, 5217.9) (1425.8, 1730, 2034.2)	$x_1 = 20,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 34$
$\alpha =$ 0.5	1	0.9964	(0.9964, 0.9966, 0.9970) (0.9964, 0.9964, 0.9964)	(3851.73, 4272.63, 4693.52) (1743.55, 1919.37, 2095.19)	$x_1 = 35.1461,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 21.8875$
	2	0.9816	(0.9816, 0.9832, 0.9845) (0.9816, 0.9816, 0.9816)	(3916.09, 4336.85, 4757.59) (1681.16, 1854.65, 2028.14)	$x_1 = 29.9721,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 26.0223$
	3	0.9656	(0.9656, 0.9684, 0.9708) (0.9656, 0.9656, 0.9657)	(3911.99, 4332.74, 4753.50) (1685.14, 1858.77, 2032.41)	$x_1 = 30.3018,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 25.7586$
	4	0.9701	(0.9701, 0.9723, 0.9742) (0.9703, 0.9703, 0.9704)	(3712.74, 4134.47, 4556.19) (1884.64, 2065.82, 2247.01)	$x_1 = 46.7507,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 12.7435$
	5	0.6663	(1, 1, 1) (0.6663, 0.6666, 0.6669)	(3510.31, 3931.88, 4353.44) (2073.13, 2261.25, 2449.38)	$x_1 = 62.5,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 0$
	6	0.5159	(0.5159, 0.5421, 0.5658) (1, 1, 1)	(4040.5, 4461, 4881.5) (1561, 1730, 1899)	$x_1 = 20,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 34$
$\alpha =$ 0.9	1	0.9966	(0.9966, 0.9967, 0.9967) (0.9966, 0.9966, 0.9966)	(4195.71, 4279.88, 4364.06) (1876.73, 1911.84, 1946.95)	$x_1 = 35.5473,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 22.3621$
	2	0.9826	(0.9826, 0.9829, 0.9832) (0.9826, 0.9826, 0.9826)	(4258.86, 4343.00, 4427.15) (1813.82, 1848.47, 1883.13)	$x_1 = 29.4778,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 26.4178$
	3	0.9673	(0.9673, 0.9679, 0.9684) (0.9673, 0.9673, 0.9673)	(4254.87, 4339.02, 4423.17) (1817.95, 1852.63, 1887.32)	$x_1 = 29.8048,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 26.1562$
	4	0.8282	(0.9715, 0.9719, 0.9724) (0.9715, 0.9715, 0.9715)	(4053.06, 4137.29, 4221.52) (2018.89, 2055.02, 2091.16)	$x_1 = 46.0012,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 13.1991$
	5	0.6667	(0.9999, 1.0000, 1.0000) (0.6667, 0.6666, 0.6667)	(3847.56, 3931.88, 4016.19) (2223.63, 2261.25, 2298.88)	$x_1 = 62.5,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 0$
	6	0.5370	(0.5370, 0.5421, 0.5469) (1, 1, 1)	(4376.9, 4461, 4545.1) (1696.2, 1730, 1763.8)	$x_1 = 20,\ x_2 = 20,$ $x_3 = 90,\ x_4 = 34$

According to triangular possibility distribution, the assignment plans for multi-objective programming problem-1 are reported in Table-3 with different values of the shape parameters which are specified by the DM. The different values of confidence level $\alpha = 0.1, \alpha = 0.5$ and $\alpha = 0.9$ is used to reflect the different situation of DM's confidence on fuzzy decision.

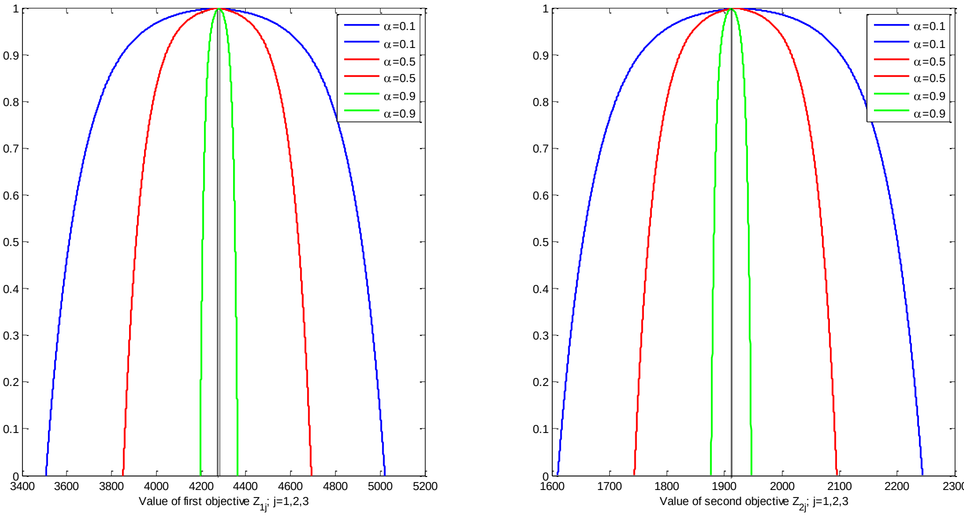

Fig. 1: Possibilities distribution for both objectives with (--5, -5) shape parameter

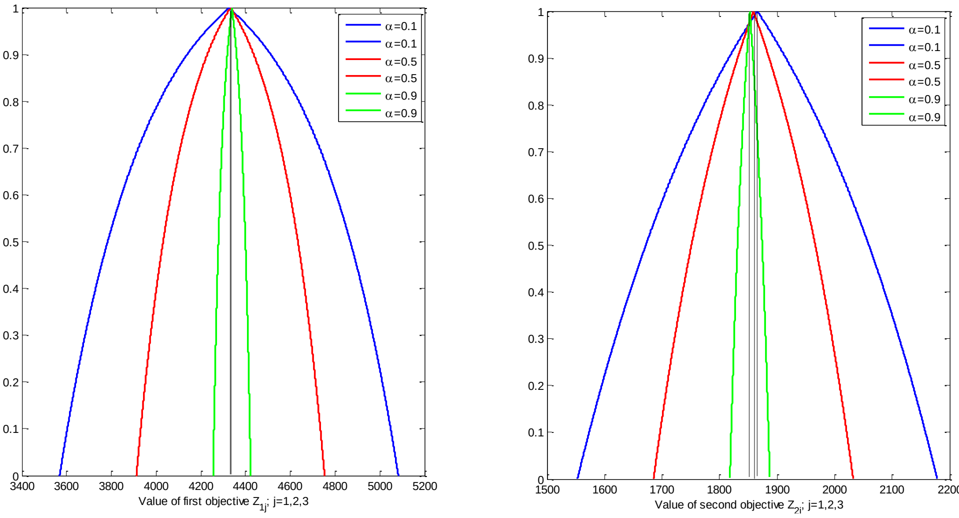

Fig.2: Possibilities distribution for both objective with (--2, -1) shape parameter

Figure-1 indicates that the efficient solutions of objectives as (3506.89, 4264.49, 5022.12) and (1610.30, 1927.29, 2244.27) at $\alpha = 0.1$, (3851.73, 4272.63, 4693.52) and (1743.55, 1919.37, 2095.19) at $\alpha = 0.5$, (4195.71, 4279.88, 4364.06) and (1876.73, 1911.84, 1946.95) at $\alpha = 0.9$ for (-5, -5) shape parameter respectively. Similarly, Figure-2 provides the efficient solutions of the objectives (3568.54, 4325.92, 5083.31) and (1552.63, 1865.62, 2178.61) at $\alpha = 0.1$, (3911.99, 4332.74, 4753.50) and (1685.14, 1858.77, 2032.41) at $\alpha = 0.5$, (4254.87, 4339.02, 4423.17) and (1817.95, 1852.63, 1887.32) at $\alpha = 0.9$ for (-2, -1) shape parameters respectively.

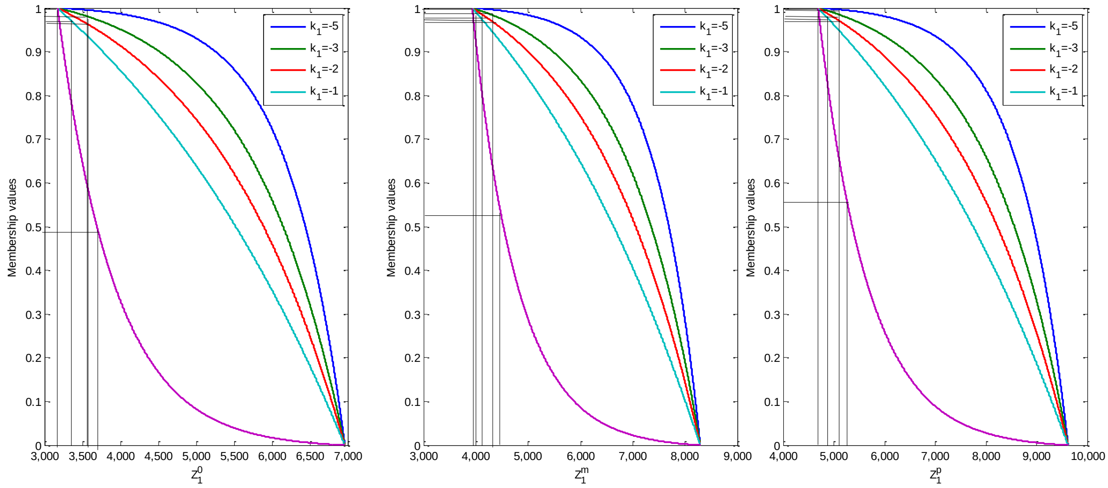

Fig. 3: The degree of satisfaction of goal of the 1st objective at $\alpha = 0.1$

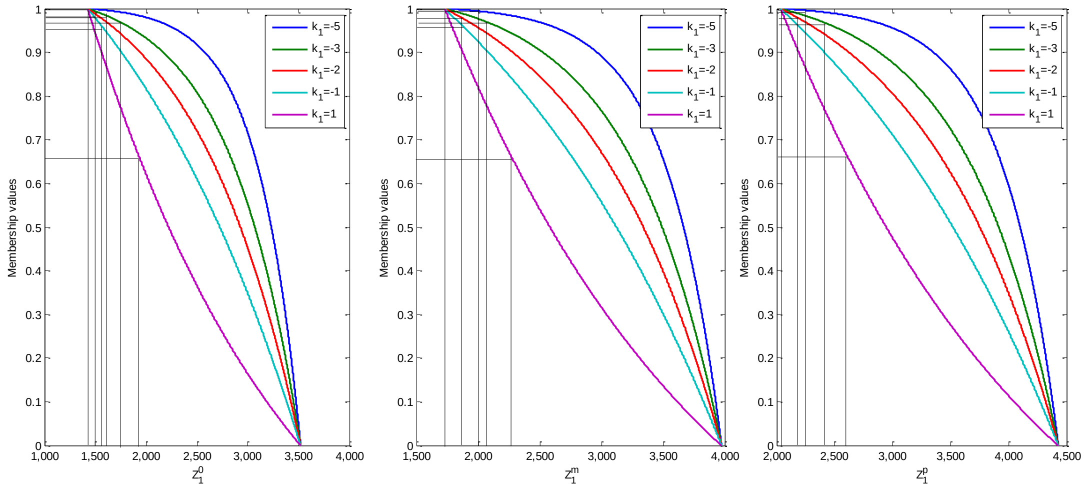

Fig. 4: The degree of satisfaction of goal of the 2nd objective at $\alpha = 0.1$

Figure-3 and 4 indicate the degree of satisfaction levels of the objectives for three different scenario optimistic, most-likely and pessmistic at (-5, -5), (-3, -2), (-2, -1), (-1, -3), (-5, 1) and (5, -2) shape parameters for $\alpha = 0.1$.

3. FORMULATION OF MULTI OBJECTIVE RESOURCE ALLOCATION PROBLEM OF SANDWICH FACTORY WITH TRAPEZOIDAL FUZZY NUMBERS USING POSSIBILISTIC DISTRIBUTION

In this section, we analyse multi-objective resource allocation problem of the sandwich factory with two objectives cost and risk factor, associated with the product. The objective is to minimize the cost and risk factor associated with the product with some realistic constraints. To formulate the multi-objective resource allocation problem of sandwich factory related optimization model, some notations are used defined as follows:[5]

- i indicate the index of product (i=1, 2… n)
- a_{ij} , is the quantity associated with i^{th} product and j^{th} constraint
- b_j is the quantity required for j^{th} constraints
- c_i is a cost associated with i^{th} the product
- r_i is risk associated with i^{th} the product

3.1 Multi-objective resource allocation problem of the sandwich factory with two objectives cost and risk factor

$$Minz_1 = \sum_{i=1}^{n} c_i x_i \quad (32)$$

$$Minz_2 = \sum_{i=1}^{n} r_i x_i \quad (34)$$

Subject to the constraints:

$$\sum_{i=1}^{n} a_{i1} x_i \leq \text{ b}_1 \qquad (\text{Bread constraint}) \quad (35)$$

$$\sum_{i=1}^{n} a_{i2} x_i \geq b_2 \qquad (\text{tomato constraint}) \quad (36)$$

$$\sum_{i=1}^{n} a_{i3} x_i \geq b_3 \qquad (\text{cheese constraint}) \quad (37)$$

$$\sum_{i=1}^{n} a_{i4} \text{x}_1 \leq b_4 \qquad (\text{potato constraint}) \quad (38)$$

$$\sum_{i=1}^{n} a_{i5} x_i \leq b_5 \qquad (\text{manpower constraint}) \quad (39)$$

$$\sum_{i=1}^{n} a_{i6} x_i \leq b_6 \qquad (\text{chicken constraint}) \quad (40)$$

$$\sum_{i=1}^{n} a_{i7} x_i \leq b_7 \qquad (\text{Bread constraint}) \quad (41)$$

$x_1 \geq b_7$, (Minimum constraint) (42)

$x_2 \geq b_8$, (Minimum constraint) (43)

$x_3 \geq b_9$, (Minimum constraint) (44)

$x_4 \geq b_{10}$ (Minimum constraint) (45)

Where,

$c_i = (c_{i1}, c_{i2}, c_{i3}, c_{i4})$ is a cost associated with the product and $r_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4})$ is a risk factor associated with i^{th} the product. z_1 and z_2 indicates the cost $c_i = (c_{i1}, c_{i2}, c_{i3}, c_{i4})$ and risk $r_i = (r_{i1}, r_{i2}, r_{i3}, r_{i4})$ associated with i^{th} the product

3.2 Formulation of fuzzy multi-objective RAP optimization model-2

To convert the model-1 into crisp multi-objective optimization model, the triangular Possibility distribution strategy is used. Thus, the quality objective function can be written as follows [4, 10]:

$$\min Z_1 = \min(z_1^o, z_1^m, z_1^{\bar{m}}, z_1^p) = \left(\sum_{i=1}^{n} C_i x_i\right)$$
$$\min\left(\left(\sum_{i=1}^{n} C_i^o x_i\right), \left(\sum_{i=1}^{n} C_i^m x_i\right), \left(\sum_{i=1}^{n} C_i^{\bar{m}} x_i\right), \left(\sum_{i=1}^{n} C_i^p x_i\right)\right) \quad (46)$$

Where $\tilde{c}_i = \left(c_i^o, c_i^m, c_i^{\bar{m}}, c_{ij}^p\right)$ which can be considered as follows.

$$\left(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{14}\right) = \left(\left(\sum_{i=1}^{n} C_i^o x_i\right), \left(\sum_{i=1}^{n} C_i^m x_i\right), \left(\sum_{i=1}^{n} C_i^{\bar{m}} x_i\right), \left(\sum_{i=1}^{n} C_i^p x_i\right)\right) \quad (47)$$

Eq. (46) and (47) are associated with the optimistic scenario, the lower value of the interval of mostthe likely scenario, upper value of the interval of most likely scenario and the pessimistic scenario respectively.

Using the α -level sets concepts ($0 \leq \alpha \leq 1$), each q_{ij} can be stated as $\left(c_{ij}\right)_\alpha$ = $\left((c_i)_\alpha^o, (c_i)_\alpha^m, (c_i)_\alpha^{\bar{m}}, (c_i)_\alpha^p\right)$, where,

$$(c_i)_\alpha^o = c_i^o + \alpha\left(c_i^m - c_i^o\right),\ (c_i)_\alpha^m = c_i^m,\ (c_i)_\alpha^{\bar{m}} = c_i^{\bar{m}},\ (c_i)_\alpha^p = c_i^p - \alpha\left(c_i^p - c_i^{\bar{m}}\right).$$

Hence, Eq. (47) can be written as:

$$\left(\min z_{11}, \min z_{12}, \min z_{13}, \min z_{14}\right) = \left(\left(\sum_{i=1}^{n} C_i^o x_i\right), \left(\sum_{i=1}^{n} C_i^m x_i\right), \left(\sum_{i=1}^{n} C_i^{\bar{m}} x_i\right), \left(\sum_{i=1}^{n} C_i^p x_i\right)\right) \quad (48)$$

Similarly, multi-objective optimization model of purchasing cost objective function is as follows

$$\left(\min z_{21}, \min z_{22}, \min z_{23}, \min z_{24}\right) = \left(\left(\sum_{i=1}^{n} (r_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^{\bar{m}} x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^o x_i\right)\right) \quad (49)$$

3.3 Crisp multi-objective optimization model:

To reflect the three different scenarios with $\alpha -$the set concept, the uncertain multi-objective RAP is converted into crisp multi-objective resource allocation problem is defined as follows:

Model-2.1:

$$\left(\min Z_{11}, \min Z_{12}, \min Z_{13}, \min z_{14}, \min Z_{21}, \min Z_{22}, \min Z_{23}, \min z_{24}\right) =$$
$$\begin{pmatrix} \left(\sum_{i=1}^{n} (C_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^{\bar{m}} x_i\right), \left(\sum_{i=1}^{n} (C_i)_\alpha^p x_i\right), \\ \left(\sum_{i=1}^{n} (r_i)_\alpha^o x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^m x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^{\bar{m}} x_i\right), \left(\sum_{i=1}^{n} (r_i)_\alpha^m x_i\right) \end{pmatrix} \quad (50)$$

Subject to constraints: (35) to (45)

3.4 Solution approach for solving crisp multi-objective optimization models of RAP with possibility distribution:

This section presented a fuzzy technique for uncertain multi-objective RAP to determine the best efficient solution with possibility distribution. This approach also gives greater flexibility to solve the multi-objective optimization problem in terms of considered the different choices of shape parameter for each objective function. This approach forces to optimize the maximum product of membership values of each objective to provide better assignment plans.

3.5 Steps for finding the solution of uncertain multi-objective optimization models of RAP:

The solution procedure for the uncertain optimization model of multi-objective RAP is summarized as under.

Step-1: Formulate the model-2 of multi-objective RAP, using appropriate triangular possibility distribution and/or trapezoidal possibility distribution.

Step-2: Define the crisp objective functions of model-1 and/or model-2, according to the confidence level α .

Step-3: Convert maximization problem into minimization form of model-2.1.

Step-4: Find out the positive ideal solution (PIS) and negative ideal solution (NIS) [2, 4, 5] for all objective functions of the model-2.1.

Step-5: Determine fuzzy exponential membership value for Z_{ij} .

$$\mu_{Z_{ij}}^{E}(x)=\begin{cases}1; & \text{if } Z_{ij}\le Z_{ij}^{\text{PIS}}\\ \dfrac{e^{-S\psi_{ij}(x)}-e^{-S}}{1-e^{-S}}, & \text{if } Z_{ij}^{\text{PIS}}<Z_{ij}<Z_{ij}^{\text{NIS}}\\ 0; & \text{if } Z_{ij}\ge Z_{ij}^{\text{NIS}}\end{cases}$$

Where, $\psi_{ij}(x)=\dfrac{Z_{ij}-Z_{ij}^{\text{PIS}}}{Z_{ij}^{\text{NIS}}-Z_{ij}^{\text{PIS}}}$ and S($S>0(\text{S}<0)$) is the shape parameter which given by DM and $0\le\mu_{Z_i}(x)\le 1$. For $S>0(\text{S}<0)$, the membership function is strictly concave (convex) in [$Z_{ij}^{\text{PIS}},Z_{ij}^{\text{NIS}}$]. The value of the membership function permits us to define the model grades of precision in consequent objective function.

Step-6: In this step, multi-objective optimization models RAP model 2.1 converted into the single-objective optimization problem(s) (SOPs) which are as follows.

and/or **(model-2.1.1)** $\max\lambda$

Subject to constraints:

$$\lambda\le\mu_{Z_{ij}}^{E}(x) \tag{51}$$

and constraints: (7) - (13)

Step-7: To find different assignment plans for the model(s) model-2.1.1) developed in step-6 using LINGO software with different choices of the shape parameter.

If the obtained solution is accepted by DM, consider it as the ideal compromise solution and stop solution process else change the value of shape parameter, confidence level and repeat the step 2 to 7 till satisfactory solution achieved for model-2.1.

3.6 Algorithm:

Input: *Parameters:* $(Z_1,Z_2,...,Z_m,n)$

Output: *Solution of uncertain multi-objective RAP*

Solve uncertain multi-objective RAP($Z_k\downarrow, X\uparrow$ *)*

begin

read: *problem*

while *problem = Uncertain multi-objective RAP do*

for *k=1 to m do*

enter the matrix Z_k

end

-| ***Find trapezoidal possibilities distribution for each objective function.***

-| ***Define the crisp multi-objective RAP according to*** $\alpha-$ ***level.***

-| ***Convert the maximization problem into minimization form.***

-| ***determine the PIS and NIS for each objective.***

for *k=1 to m do*

$$z_{ij}^{\text{PIS}}=\min(z_i)_{\alpha}^{0}$$

Under given constraints

end

for *k=1 to m do*

$$z_{ij}^{\text{NIS}}=\max(z_i)_{\alpha}^{0}$$

Under given constraints

end

-| ***Define exponential membership function for each objective.***

for *k=1 to m do*

$$\mu_{z_{ij}}^{E}(x)=\begin{cases}1; & \text{if } z_{ij}\le z_{ij}^{\text{PIS}}\\ \dfrac{e^{-S\psi_{ij}(x)}-e^{-S}}{1-e^{-S}}, & \text{if } z_{ij}^{\text{NIS}}<z_{ij}<z_{ij}^{\text{PIS}}\\ 0; & \text{if } z_{ij}\ge z_{ij}^{\text{NIS}}\end{cases}$$

end

-| ***find single objective optimization models under given constraints from multi-objective optimization models.***

for *k=1 to m do*

max λ

Subject to constraints:

$$\lambda \le \mu_{Z_{ij}}^{E}(x)$$

constrains:(3) to (13)

end

|- *find the solution SOP(s) using LINGO software.*

3.7 Case study of food factory with trapezoidal fuzzy numbers

Here in this chapter, we have considered the case study of food industry involved in the production of the sandwich. The production capacity of the factory is 200000 per day which involves more than 1000 worker. This production is divided into 20 units. We have considered the data of one unit in which from the past experience certain fuzzy data were

Table 7: Cost of product with trapezoidal fuzzy numbers

Cost of product 1	[17.5 21.25 26.25 30.0]
Cost of product 2	[25.5 28.95 33.55 37]
Cost of product 3	[16.5 18.75 21.75 24]
Cost of product 4	[37.5 42.15 48.35 53]

Table 8: Risk factor associated with trapezoidal fuzzy numbers

Risk factor associated with product 1	[18 19.5 21.5 23.0]
Risk factor associated with product 2	[15.5 16.7 18.3 19.5]
Risk factor associated with product 3	[5 6.2 7.8 9]
Risk factor associated with product 4	[8 9.2 10.8 12]

Table 9: Amount of product required and its availability

Product	Bread	Tomato	Potato	Cheese	Tiki	Chicken	Pasta	Manpower	Machine	Miscall
Cheese s/w	1	30	30	20	0	0	0	3	2	50
Cheese burger	1	30	0	25	50	0	0	3	2	50
Allotiki	1	50	0	25	50	0	0	3	0	50
Chicken pasta	1	50	0	25	0	40	25	3	2	50
avaibility	210	4000	15000	4000	110	4000	3000	960	600	210

3.7.1 Problem formulation

$$Minz_1 = [17.5, 21.25, 26.25, 30.0]x_1 + [25.5, 28.95, 33.55, 37]x_2 + [16.5, 18.75, 21.75, 24]x_3 + [37.5, 42.15, 48.35, 53]x_4 \quad (51)$$

$$Minz_2 = [18, 19.5, 21.5, 23.0]x_1 + [15.5, 16.7, 18.3, 19.5]x_2 + [5, 6.2, 7.8, 9]x_3 + [8, 9.2, 10.8, 12]x_4 \quad (52)$$

Subject to the constraint (21) to (31)

For a Trapezoidal fuzzy number

To evaluate multi-objective programming problem, the model is coded. Table-6 gives the PIS and NIS for each objective function of model-2 at $\alpha = 0.1, 0.5$ and 0.9.

Table 10: PIS and NIS value for each objective at $\alpha = 0.1, 0.5$ and $\alpha = 0.9$

$\alpha -$ value		Z_{11}	Z_{12}	Z_{13}	Z_{14}	Z_{21}	Z_{22}	Z_{23}	Z_{24}
$\alpha = 0.1$	**PIS**	3139.34	3594.63	4269.13	4724.41	1412.28	1594.8	1865.2	2047.72
	NIS	6903.65	7701.5	8883.5	9681.35	3500.3	3773	4177	4449.7
$\alpha = 0.5$	**PIS**	3341.69	3594.63	4269.13	4522.06	1493.4	1594.8	1865.2	1966.6
	NIS	7258.25	7701.5	8883.5	9326.75	3621.5	3773	4177	4328.5
$\alpha = 0.9$	**PIS**	3544.04	3594.63	4269.13	4319.71	1574.52	1594.8	1865.2	1885.48
	NIS	7612.85	7701.5	8883.5	8972.15	3742.7	3773	4177	4207.3

Each combination of the shape parameters is shown in Table-11.

Table 11: Different values of shape parameters

Case	Shape Parameter (K_1, K_2)
1	(-5, -5)
2	(-2, -1)
3	(-3, -2)
4	(5, -3)
5	(-1, 5)

Table 12: Summary results of a different scenario for each objective at $\alpha = 0.1,\ 0.5$ and 0.9

$\alpha -$ value	Case	The degree of satisfaction level	Membership function	Objective values	Allocations
$\alpha =$ 0.1	1	0.9962	(0.9962, 0.9966, 0.9971, 0.9973) (0.9962, 0.9962, 0.9962, 0.9962)	(3472.36, 3926.93, 4600.36, 5054.93) (1596.99, 1787.22, 2069.03, 2259.25)	$x_1 = 35.8501,\ x_2 = 20,\ x_3 = 90,\ x_4 = 21.3199$
	2	0.9635	(0.9635, 0.9669, 0.9709, 0.9732) (0.9635, 0.9635, 0.9636, 0.9636)	(3534.15, 3988.59, 4661.82, 5116.26) (1539.39, 1727.21, 2005.47, 2193.29)	$x_1 = 30.9073,\ x_2 = 20,\ x_3 = 90,\ x_4 = 25.2742$
	3	0.9630	(0.9630, 0.9665, 0.9706, 0.9729) (0.9804, 0.9804, 0.9804, 0.9805)	(3538.29, 3992.72, 4665.94, 5120.37) (1535.53, 1723.19, 2001.20, 2188.86)	$x_1 = 30.5756,\ x_2 = 20,\ x_3 = 90,\ x_4 = 25.5395$
	4	0.3007	(1, 1, 1, 1) (0.3007, 0.3012, 0.3019, 0.3024)	(3139.34, 3594.63, 4269.13, 4724.41) (1907.58, 2110.75, 2411.75, 2614.93)	$x_1 = 62.5,\ x_2 = 20,\ x_3 = 90,\ x_4 = 0$
	5	0.4909	(0.4904, 0.5213, 0.5612, 0.5848) (1, 1, 1, 1)	(3670.46, 4124.6, 4797.4, 5251.54) (1412.28, 1594.8, 1865.2, 2047.72)	$x_1 = 20,\ x_2 = 20,\ x_3 = 90,\ x_4 = 34$
$\alpha =$ 0.5	1	0.9963	(0.9963, 0.9966, 0.9970, 0.9972) (0.9963, 0.9963, 0.9964, 0.9964)	(3679.34, 3931.87, 4605.29, 4857.82) (1676.85, 1782.43, 2063.96, 2169.53)	$x_1 = 35.4553,\ x_2 = 20,\ x_3 = 90,\ x_4 = 21.6358$
	2	0.9646	(0.9646, 0.9665, 0.9706, 0.9719) (0.9647, 0.9647, 0.9647, 0.9648)	(3740.55, 3993.02, 4666.27, 4918.74) (1618.89, 1723.15, 2001.17, 2105.43)	$x_1 = 30.5688,\ x_2 = 20,\ x_3 = 90,\ x_4 = 25.55$
	3	0.9811	(0.9811, 0.9821, 0.9844, 0.9851) (0.9811, 0.9811, 0.9811, 0.9811)	(3744.48, 3996.93, 4670.14, 4922.59) (1614.93, 1719.09, 1996.86, 2101.02)	$x_1 = 30.2380,\ x_2 = 20,\ x_3 = 90,\ x_4 = 25.8096$
	4	0.3009	(1, 1, 1, 1) (0.3009, 0.3012, 0.3019, 0.3022)	(3341.69, 3594.63, 4269.13, 4522.06) (1997.88, 2110.75, 2411.75, 2524.63)	$x_1 = 62.5,\ x_2 = 20,\ x_3 = 90,\ x_4 = 0$
	5	0.5046	(0.5046, 0.5213, 0.5612, 0.5746) (1, 1, 1, 1)	(3872.3, 4124.6, 4797.4, 5049.7) (1493.4, 1594.8, 1865.2, 1966.6)	$x_1 = 20,\ x_2 = 20,\ x_3 = 90,\ x_4 = 34$
$\alpha =$ 0.9	1	0.9965	(0.9965, 0.9965, 0.9969, 0.9969) (0.9965, 0.9965, 0.9965, 0.9965)	(3886.06, 3936.56, 4609.97, 4660.47) (1756.77, 1777.86, 2059.12, 2080.21)	$x_1 = 35.0792,\ x_2 = 20,\ x_3 = 90,\ x_4 = 21.9367$
	2	0.9489	(0.9658, 0.9661, 0.9703, 0.9706) (0.9489, 0.9489, 0.9491, 0.9491)	(3946.29, 3996.78, 4669.99, 4720.48) (1756.77, 1777.86, 2059.12, 2080.21)	$x_1 = 30.2498,\ x_2 = 20,\ x_3 = 90,\ x_4 = 25.8001$
	3	0.9817	(0.9817, 0.9819, 0.9842, 0.9844) (0.9817, 0.9819, 0.9817, 0.9817)	(3950.40, 4000.89, 4674.09, 4724.58) (1694.42, 1715.23, 1992.78, 2013.59)	$x_1 = 29.9204,\ x_2 = 20,\ x_3 = 90,\ x_4 = 26.0637$
	4	0.3012	(1, 1, 1, 1) (0.3012, 0.3012, 0.3019, 0.3019)	(3544.04, 3594.63, 4269.13, 4319.71) (2088.18, 2110.75, 2411.75, 2434.33)	$x_1 = 62.5,\ x_2 = 20,\ x_3 = 90,\ x_4 = 0$
	5	0.5181	(0.5181, 0.5213, 0.5612, 0.5639) (1, 1, 1, 1)	(4074.14, 4124.6, 4797.4, 4847.86) (1574.52, 1594.8, 1865.2, 1885.48)	$x_1 = 20,\ x_2 = 20,\ x_3 = 90,\ x_4 = 34$

According to trapezoidal possibility distribution, the optimal allocation plans for uncertain multi-objective programming problem for trapezoidal numbers are reported in Table-8 with different the shape parameters as given in Table-11. Here $\alpha = 0.1, \alpha = 0.5$ and $\alpha = 0.9$ are used to reflect the different situation of DM's confidence on the fuzzy decision.

From the Table-4 and 12, it is clear that the change in confidence level influence spreads of the objective function. That is, a smallest α -value yields solution having large spreads in the objective function values form the average which shows that the obtained solution corresponding to high level of pessimism and uncertainty while the largest $\alpha -$ value yields solution having smaller spreads in the objective function which shows that obtained solution corresponding to high level of optimism in the fuzzy decision of the DM.

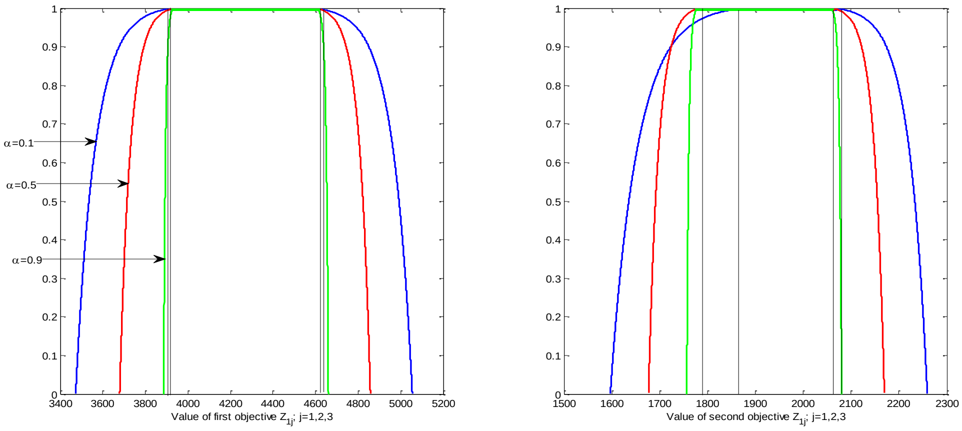

Fig. 5: Possibilities distribution for both objectives with (-5, -5) shape parameter

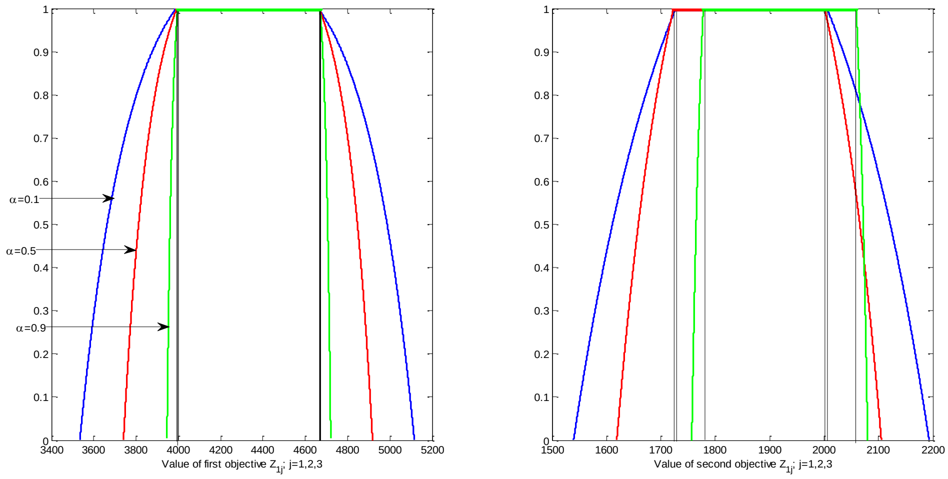

Fig. 6: Possibilities distribution for both objectives with (-2, -1) shape parameter

Figure-7 shows the efficient solutions of both objectives as (3472.36, 3926.93, 4600.36, 5054.93) and (1596.99, 1787.22, 2069.03, 2259.25) at $\alpha = 0.1$, (3679.34, 3931.87, 4605.29, 4857.82) and (1676.85, 1782.43, 2063.96, 2169.53) at $\alpha = 0.5$, (3886.06, 3936.56, 4609.97, 4660.47) and (1756.77, 1777.86, 2059.12, 2080.21) at $\alpha = 0.9$ for (-5, -5) shape parameter respectively. Similarly, Figure-8 indicates that the efficient solutions of both objectives (3534.15, 3988.59, 4661.82, 5116.26) and (1539.39, 1727.21, 2005.47, 2193.29) at $\alpha = 0.1$, (3740.55, 3993.02, 4666.27, 4918.74) and (1618.89, 1723.15, 2001.17, 2105.43) at $\alpha = 0.5$, (3946.29, 3996.78, 4669.99, 4720.48) and (1756.77, 1777.86, 2059.12, 2080.21) at $\alpha = 0.9$ for (-2, -1) shape parameter respectively.

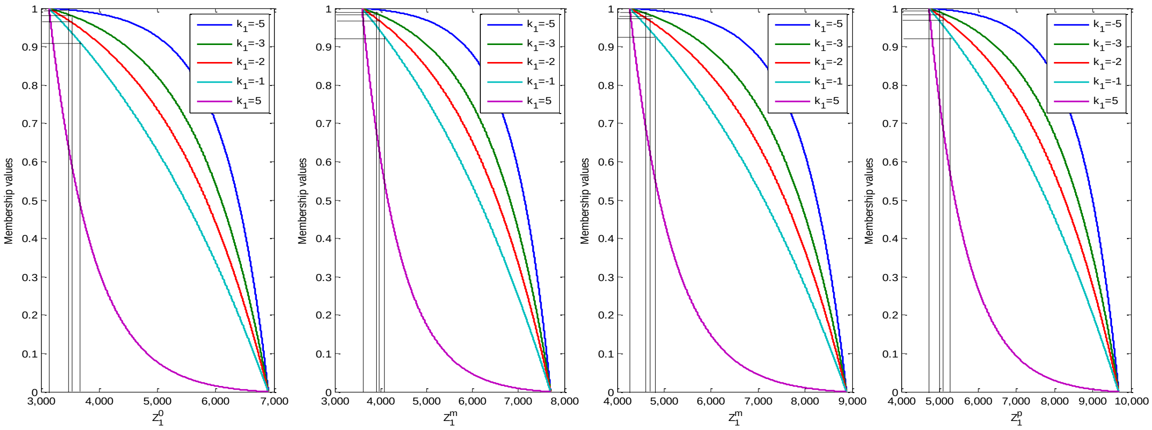

Fig. 7: The degree of satisfaction of goal of the first objective at $\alpha = 0.1$

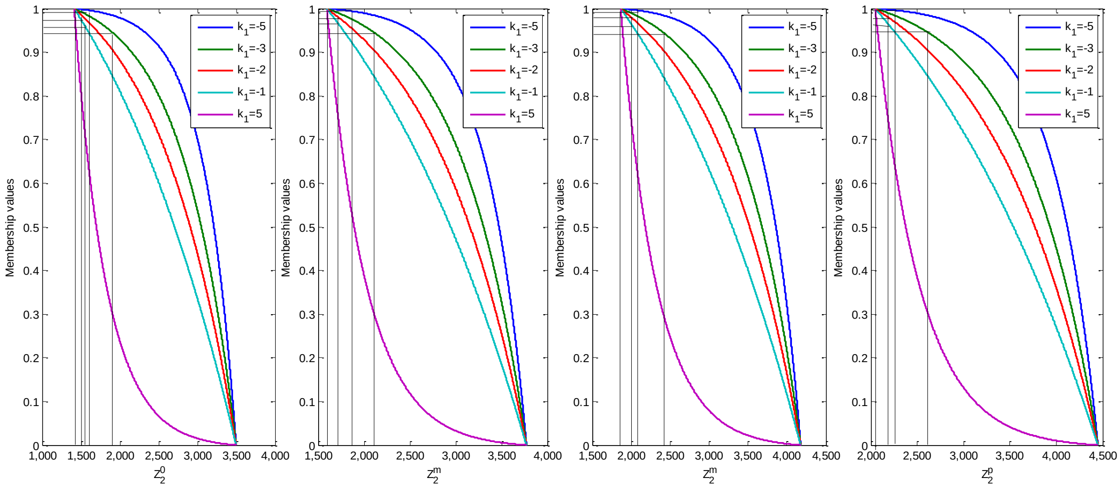

Fig. 8: The degree of satisfaction of goal of the second objective at $\alpha = 0.1$

Figure-9 and 10 indicate the degree of satisfaction levels of the both objectives for four different scenario optimistic, interval of most-likely value and pessmistic at (-5, -5), (-2, -1), (-3, -2), (5, -3) and (-1, 5) shape parameters for $\alpha = 0.1$.
From Figure-1, 2, 7 and 8, it is concluded that as the values of α increases, the influence of uncertainty decrease in DM fuzzy judgment and obtained solution having more influences of optimism than pessimism. The obtained solution and assignment plans from the Table-4 and 8 show the advantages of the exponential membership function with different shape parameter for solution of uncertain multi-objective programming problem. If DM is not satisfied with obtained an assignment plan, more plans can be generated by varying the values of α and values of shape parameters.

From Figures 3, 4, 9 and 10, it is clear that obtained solution having more influence of optimism then pessimism, representing possibility distribution corresponding to each objective function respectively.
Table-4 and 8, as well as Figures 3, 4, 9 and 10 indicate that the proposed solution approach gives flexibility and a large collection of information in the sense of changing the shape parameters. It also provides the different scenario analysis to DM for allocation strategy in uncertain multi-objective programming problem. If DM is not fulfilled by the obtained assignment plans, more allocation plans can be produced by changing the confidence level.

4. REFERENCE

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