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Analysis of isotropic plates for finite element large amplitude free flexural vibration

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ABSTRACT

Structural components are generally subjected to dynamic loadings in their working life. Very often these components may have to perform in a severe dynamic environment wherein the maximum damage results from the resonant vibration. Susceptibility to fracture of materials due to vibration is determined by stress and frequency. The maximum amplitude of the vibration must be in the limited for the safety of the structure. Hence vibration analysis has become very important in designing a structure to know in advance its response and to take necessary steps to control the structural vibrations and its amplitudes.

The non-linear or large amplitude flexural vibration of plates has received considerable attention in recent years because of the great importance and interest attached to the structures of low flexural rigidity. These easily deformable structures vibrate at large amplitudes. The solution obtained based on the lineage models provide no more than a first approximation to the actual solutions. The increasing demand for more realistic models to predict the responses of elastic bodies combined with the availability of super computational facilities has enabled researchers to abandon the linear theories in favor of non-linear methods of solutions. In the present investigation, large amplitude vibration of several rectangular and skew plates has been studied using an isoparametric quadratic plate bending element for the finite element method. The formulations have incorporated the shear deformation of the plates. Plates with various boundary conditions have been considered in the study. The effect of variations in the Poisson's ratio, thickness parameter & plate aspect ratio on the non-linear frequency ratio has also been included in the research.

Keywords: *Isotropic Plates, Finite Element Large Amplitude, Flexural Vibration.*

1. INTRODUCTION

1.1 General

Structural components are generally subjected to dynamic loadings in their working life. Very often these components may have to perform in a severe dynamic environment wherein the maximum damage results from the resonant vibration. In the vehicles used in transport such as ships and aircraft, vibration results in discomfort to the crew and passengers. Human reaction to vibration in the range of 4Hz to 16Hz is more sensitive because natural frequencies of the body can be excited and may cause serious physiological effects. Susceptibility to fracture of materials due to vibration is determined by stress and frequency. The maximum amplitude of the vibration must be in the limited for the safety of the structure. Hence vibration analysis has become very important in designing a structure to know in advance its response and to take necessary steps to control the structural vibrations and its amplitudes.

1.2 Present Investigation

In the current investigation, the main objective is to find out the non-linear frequency ratios of free un-damped vibration of plates. Finite element method has been adopted for the current analysis. An isoparametric quadratic plate bending element has been used. It also considers the shear deformation of the plate. Hence the formulation is applicable to both thin as well as thick plates. Consistent mass matrix has been used. As the higher order terms in the strain-displacement relations are not known, in order to obtain the solution for non-linear free vibration problem an iterative procedure is adopted using linear strain-displacement relations for the first iteration. For the successive iterations, the higher order terms of the strain-displacement relations have been evaluated from the scaled eigenvectors corresponding to a given amplitude at a prescribed point of the previous iterations. The iteration process is continued until required convergence is reached. In the present investigation, non-linear free vibration analysis is done for several quadrangular plates. Various boundary conditions have been considered. The effect of variations in the same material and/or geometric properties of the plate has also been studied.

2. LITERATURE REVIEW

The large amplitude vibration of plates and shells has received considerable attention in recent years because of great importance and interest attached to the structures of low flexural rigidity. As a result of which large amplitude vibrations of plates of various geometries have received much attention [18, 19, 20].

In the area of non-linear vibration of plates of various shapes, rectangular plates have been studied more frequently than others. Most of the work is based on the governing equations in terms of stress function and lateral displacement. The governing non-linear equations of motion can be suitably modified to include the effects of several complicating factors such as transverse shear, in-plane forces and the like. Large amplitude flexural vibration of thin rectangular plates is studied by Rao et al.[using a direct finite element formulation. The formulation is based on an appropriate linearization of strain displacement relation and an iterative method of solution has been used. They have observed the period ratios for the square as well as rectangular plates under different boundary conditions. [13] The geometrically non-linear free vibrations of thin isotropic and laminated rectangular composite plates with fully clamped edges have been successfully investigated by Bikri et al. using a theoretical model based on Hamilton's principle and spectral analysis. The work has been further extended [3] [2] to the plates with other boundary conditions in order to determine their fundamental non-linear mode shape and associated amplitude-dependent resonant frequencies and flexural stress distribution. Sarma et al. have formulated the non-linear vibration of a rectangular plate by using Kirchhoff's hypothesis and von Karman type strain-displacement relations. In-plane deformations are included and the corresponding inertia terms are neglected in this formulation. The restoring force function in the equation of motion is found to be a cubic polynomial, which is of Duffing type or a combination of quadratic and cubic terms. The exact solution of the equation of motion is presented in an expression which is evaluated numerically for obtaining the frequency corresponding to the specified maximum amplitude. [17] Rao et al. have studied the large amplitude vibration of rectangular plates with and without stiffeners. They have taken the shear deformation of the plate and the stiffeners into due consideration in their formulation. Also, the formulation considers the in-plane inertia of the plate. [14] Large amplitude flexural vibrations clamped as well as simply supported isotropic elastic rhombic plates were investigated by Ray et al.[15], the effects of shear deformation and rotary inertia being included. The governing equations for plates of transversely isotropic material were solved by the well-known Galerkin procedure. A number of cases were solved for different edge conditions of the rhombic plates, indicating the significant influences of skew angle, transverse shear deformation, and rotary inertia. Large amplitude free flexural vibration analysis of composite stiffened plates have been carried out by Goswami & Kant [7] using a nine-noded Lagrangian element. The element is based on the first order shear deformation theory. The large deformation effect of the stiffened plated structures has been taken care of by the dynamic version of von Karman's field equations. The non-linear equations obtained have been solved by the direct iteration technique using the linear mode shapes as the starting vectors. 0 C Harras et al. [have made an extensive study on symmetrically laminated rectangular CFRP plates. They have formulated the geometrically non-linear behavior and analyzed for fully clamped support condition. In their study, they have reported the frequency amplitude dependence. The non-linear solutions have been performed by using the basic plate functions. 8] Non-linear free vibration analysis is carried out by Saha et al. [on square plates with different boundary conditions. They have focused only on geometric non-linearity and proposed a new methodology that can be employed for plate structure problems having any combination of boundary conditions to determine the non-linear frequencies and mode shapes. The large amplitude vibration problem is analyzed in two parts. The static problem corresponding to a uniform transverse loading is solved first and the dynamic problem is subsequently taken up with the known deflection field. Both these problems are formulated [16] through energy method, the underlying principle being the extermination of the total energy of the system in its equilibrium state. The solution methodology employs an iterative numerical scheme using the technique of successive relaxation. A nine-nodded isoparametric plate-bending element has been used by Pandit et al. [12] for the analysis of free un-damped vibration of isotropic and fiber-reinforced laminated composite plates. The effect of shear deformation is incorporated in the formulation by considering the first-order shear deformation theory for the analysis. An effective mass lumping scheme with rotary inertia has been recommended. Two types of mass lumping schemes have been formed. In one lumping scheme, rotary inertia has also been introduced. Large-amplitude free vibration analysis of simply supported thin isotropic skew plates has been presented by Das et al. [5]. In this paper, the large deformation has been imparted statically by subjecting the plate under uniform transverse pressure. The mathematical formulation is based on the variation principle in which the displacement fields are assumed as a combination of orthogonal polynomial or transcendental functions, each satisfying the corresponding boundary conditions of the plate. The large-amplitude dynamic problem has been addressed by solving the corresponding static problem first, and subsequently, with the resultant displacement field, the problem is formulated. The vibration frequencies are obtained from the solution of a standard eigenvalue problem. Entire computational work has been carried out in a normalized square domain obtained through an appropriate domain mapping technique.

3. THEORY AND FINITE ELEMENT FORMULATION

3.1 Finite Element Formulation

In the finite element analysis, the continuum is divided into a finite number of elements having finite dimensions and reducing the continuum having infinite degrees of freedom to a finite number of unknowns. The formulation presented here is based on assumed displacement pattern within the element and can be applied to linear, quadratic, cubic or any other higher order element by incorporating appropriate shape functions. In the following the element mass and stiffness matrices of the plate are derived. The element mass and stiffness matrices are then assembled to form the overall mass and stiffness matrices. Necessary boundary conditions are then incorporated. Reduced integration technique has been used to obtain the element mass and stiffness matrices. An isoparametric quadratic plate bending element (Fig. 3.1) is chosen in the present analysis.

3.2 Assumptions

The formulation is based on the following assumptions:

(i) The material of the plate obeys Hooke's law.
(ii) The bending deformations follow Mindlin's hypothesis, therefore, the normal perpendicular to the middle plane of the plate before bending remains straight, but not necessarily normal to the middle plane of the plate after bending.
(iii) The deflection in the z-direction is a function of x and y only.
(iv) The transverse normal stresses are neglected

3.3 Solution Procedure

Step 1: The fundamental linear frequency and corresponding linear mode shape are calculated by solving equation (3.6.2) with all the terms in [KNL] being set to zero. Step 2: The mode shape is normalized by appropriately scaling the eigenvector ensuring that the maximum displacement is equal to the desired amplitude Wmax/h.

Step 2: The terms in the stiffness matrix [KNL] are computed using the normalized mode shape.

Step 3: The equations are then solved to obtain new eigenvalues and corresponding eigenvectors.

Step 4: Steps 2 to 4 are now repeated with wmax {φ}I until a convergence criterion is satisfied.

4. COMPUTER PROGRAM

4.1 SALIENT FEATURES

The salient features of the computer program are given below:

a. Automatic mesh generation

b. Generation of linear element stiffness matrix and mass matrix generation

c. Generation of non-linear stiffness matrix

d. Generation of overall stiffness and mass matrices as a single array (skyline form)

e. Fitting of boundary conditions

f. Solution for dominant eigenvalues and eigenvectors by using Corr and Jennings simultaneous iteration method [4]

g. Optimum utilization of memory by removing zeros from matrices

h. Use of symmetry of the plate so as to reduce the CPU time

4.1.1 Automatic mesh generation

The mesh division for the structures analyzed is generated automatically. The algorithm for this purpose is provided in the main program itself. The plate structure is divided into a number of elements by assigning the number of divisions in each direction. This information is given as input to the problem under consideration. The elements are numbered automatically moving from left to right and top to bottom as shown in the Fig.4.1. Each element has eight nodes which are numbered consecutively from left-hand top corner in a clockwise direction. For the whole structure, the nodes are numbered automatically, moving from left to right and from top to bottom

4.1.2 Input data

The input data required for the analysis are as follows:

i. Plate dimensions,
ii. Mesh division, number of nodes of the element, degrees of freedom at each node, number of Gauss points, number of loading conditions, the degree of freedom of the amplitude taken as reference for normalizing,

iii. Boundary conditions of the structure, iv. Material properties of the plate,

v. Number of amplitude levels and tolerance for convergence, and

vi. Amplitude data.

4.1.3 Output data

i. The input data,

ii. Linear frequency,

iii. Non-linear frequency, mode-shape error, frequency error and mode-shapes in each iteration, and number of iterations taken for convergence, and
iv. A number of iterations taken by the eigenvalue routine.

4.1.4 Program Flow

The program flow is hereby presented through the flow diagrams (Fig. 4.3-4.6). The entire program is controlled by the main program and several subroutines.

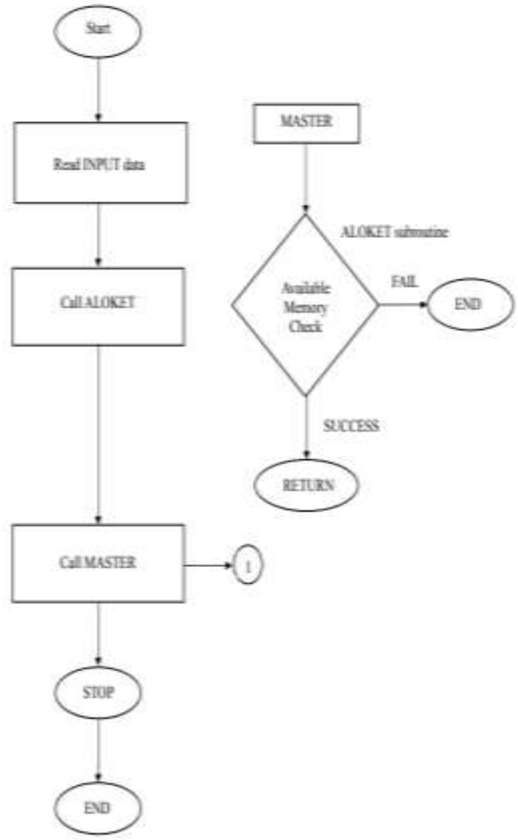

Figure-4.2(a)

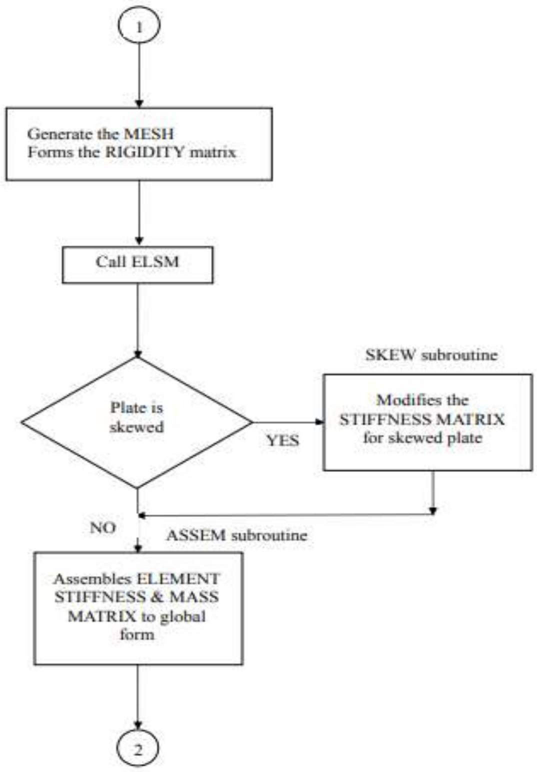

Figure-4.2(b)

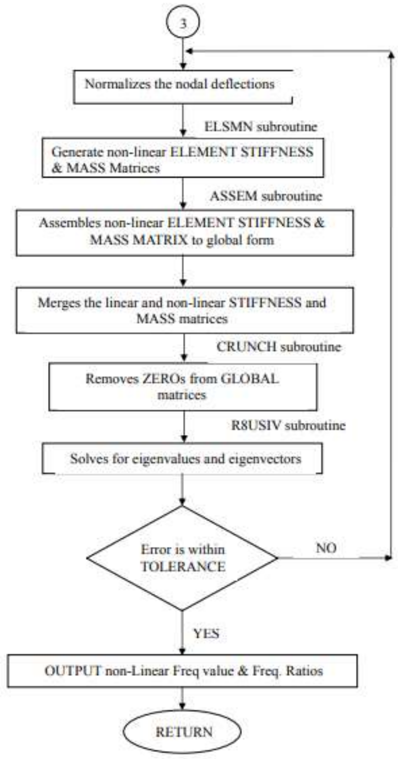

Figure-4.2(c)

Figure-4.2(d)

5. CONCLUSIONS

The following conclusions may be made from the present investigation of the large amplitude free vibration analysis for rectangular and skew plates:

- The boundary conditions of the plate directly affect the degree of the non-linearity assessed through the non-linear frequency ratio. More restrained boundaries tend to produce lower values of non-linear frequency.

- For the materials having higher Poisson's ratio, the non-linear frequency has been observed to be higher.
- At lower values of vibration amplitudes, the Poisson's ratio has no significant effect on the non-linear frequency of vibration. But as the amplitudes of vibration increase, there is a remarkable increase in the non-linear frequency ratio.
- There is no significant effect on the non-linear frequency ratio with the change in the thickness parameter.
- The non-linear frequency ratio varies directly with respect to the plate aspect ratio. The increase in the aspect ratio shifts the non-linear frequency towards the higher side.
- For a simply supported square plate, the non-linear frequency ratio first decreases slightly and then increases as the angle of skewness is increased.
- For a clamped-clamped square plate, the non-linear frequency ratio increases as the angle of skewness are increased

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