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Nomograms for isotropically reinforced polygonal slab

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ABSTRACT

Slabs comprise a maximum area of the building which comes up to 20 to 25 percent of total cost. Thus the economical design of the building can be achieved to great extent by designing the slab economically.

The yield line method is a method which allows the redistribution of force that will take place after the yielding of slab reinforcement. Using this method, slabs that are easy and quick to design and to construct can be generated. The resulting slab is thin and has the very low amount of reinforcement in regular arrangements. This makes it easy to detail. Above all, this design generates concrete slab which is very economic, because features at the ultimate limit state are considered.

The aim of this study is to apply the yield line theory for polygonal slab of N equal sides by using the virtual work principle and obtain a general relationship between the ultimate load and desired arrangement of reinforcement in a slab for any given radius of circle in which the slab is inscribed and to generate nomograms for frequently used slabs.

Keywords: Slabs, Yield line, Ultimate load, Ultimate moment, Virtual work.

1. INTRODUCTION

Buildings are made of different components like beams, columns, slabs, walls, foundation etc. Each of these components plays their role in transferring and distributing the load coming on the structure to the soil. Slabs comprise a major portion of the building and are used as floors and ceilings. Vital care must be taken in the design of slabs as a large part of imposed loads like the movement of people, furniture, ceiling fans, and dead load like floor finishes etc. first act on the slab which is further transferred to the foundation through beams and columns.

1.1. Yield Line Theory

Ingerslev first proposed the theory of yield line analysis for slabs. This was greatly extended by Johansen. Early publications were mainly in Danish but after Hognestad's English language summary of Johansen's work, the method received wide attention. Since then, a number of important publications on the method have appeared.

A yield line is a crack in a reinforced concrete slab across which the reinforcing bars have yielded and along which plastic rotation occurs. When a slab is loaded to failure, yield lines are formed in highly stressed areas and are developed into continuous plastic hinges. These plastic hinges develop to form a mechanism resulting in a yield line pattern. The Yield lines divide the slab into individual regions, which pivot about their axes of rotation. The ultimate load of the slab system is estimated by postulating a collapse mechanism that is compatible with the boundary conditions. The moments at the plastic hinge lines are the ultimate moments of resistance of the sections, and the ultimate load is determined using the principle of virtual work.

The slab region between the yields lines is not examined because these moments do not exceed the ultimate moments of resistance of the sections. The ultimate moments of resistance between the yield lines will be exceeded only if incorrect collapse mechanism is used. Thus all the possible collapse mechanisms of the slab must be examined to ensure that the load carrying capacity of the slab is not overestimated.

1.2. Yield Line Patterns

The important properties of yield line patterns are:

- 1) Generally, the Axes of rotation lies along the lines of support and pass alongside any columns.
- 2) Yield lines are straight.
- 3) Between adjacent rigid regions, the yield lines must pass through the point of intersection of the axes of rotation of those regions.
- 4) Yield lines must end at a slab boundary.
- 5) Continuous supports attract negative or hogging yield line and the positive or sagging yield lines are attracted by simple supports.

1.3. The objective of the Study

The present work aims to prepare nomograms for frequently used polygonal slabs from which additional reserves of strength of existing slabs can be analyzed and when used for design reinforcement detail is obtained.

2. PROJECT METHODOLOGY

2.1. Determination of Collapse Load by Principle of Virtual Work

The principle of virtual work states that, if a system is in equilibrium under a system of forces and undergoes a deformation, the work done by the external forces will be equal to the work done by the internal stresses due to those forces.

To analyze a slab by virtual work method a yield line pattern is postulated for the slab at the ultimate load. The segments of the yield line pattern may be regarded as rigid bodies because the slab deformation with further deflection occurs only at yield lines. The segments of the slab are in equilibrium under external loading and the bending and the torsional moments and shears along the yield lines. A point within a slab is conveniently chosen and a small virtual displacement Δ in the direction of the load is given. Then the resulting displacements at all points of the slab, $\Delta(x,y)$, and the rotations of the slab segments about the yield lines may be found in terms of Δ and the dimensions of the slab segments. Work will be done by the external loads and by the internal actions along the yield line. The internal work done on the slab is given by sum of the rotations in the yield lines multiplied by the resisting ultimate moments, while the external loss of work will be the sum of the loads multiplied by their respective deflections. When the internal and external work is equated, the relation between the ultimate resistance moments in the slab and the ultimate load will be obtained.

The external work can be written as

$$\text{External work} = \sum \int \omega \Delta dA \quad \dots\dots\dots \text{Eqn. 1.1}$$

ω = Ultimate uniform distributed load/unit area of the slab

Δ = Deflection on the element

dA = Unit area ($dx \times dy$)

The integral is necessary because the load need not be constant over the rigid region.

The internal work is the work absorbed by rotation of yield lines and it is generally expressed as

$$\text{Internal work} = \sum \int m \theta ds \quad \dots\dots\dots \text{Eqn. 1.2}$$

Where,

m = normal moment per unit length at a point in yield line.

θ = Total rotation of yield line.

ds = Short length of yield line at that point.

The integral is necessary because the moment across the yield line can vary along the length, due to change in thickness of slab or in reinforcement spacing.

Therefore the general expression is

$$\sum \int \omega \Delta dA = \sum \int m \theta ds .$$

The reactions at the supports will not contribute to the work, as they do not undergo displacement. The work done by the internal actions at the yield lines will be only due to the bending moments because the work done by the torsional moments and shear forces is zero when summed over the whole slab. This follows because the actions on each side of the yield line are equal and opposite as

shown in Fig 1 and for any displacement of the yield line pattern there is no relative movement between the sides of the yield line corresponding to the torsional moments and shear forces.

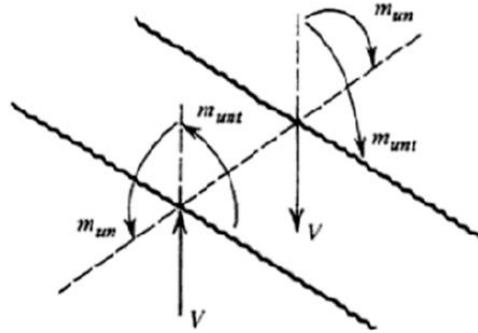


Figure 1 Actions at the yield line

However, there is relative movement corresponding to the bending moments since there is a relative rotation between the two sides of the yield line. Thus the work done at the yield lines is due to only the ultimate bending moments.

2.2. Polygonal Slab Supported on its Edges

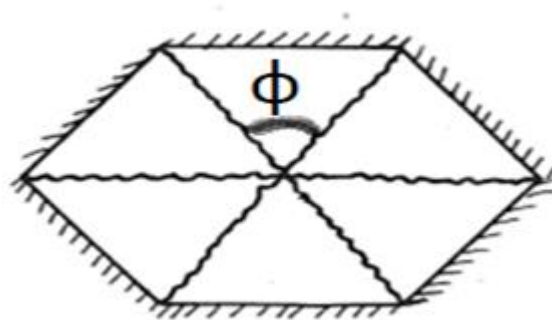


Figure 2 Polygonal slab of six equal edges

A general polygonal slab of 'N' number of edges of equal dimension and subjected to uniformly distributed load all over the slab is taken into consideration. If the polygonal slab is assumed to be inscribed in a circle of radius R, then the general formula for the ultimate load carrying capacity of the slab in terms of R and N is given by

$$W = 24 m \tan^2(\pi/N)(1+k) / 4R^2 \sin^2(\pi/N).$$

Where,

W is the ultimate load-bearing capacity of the slab

μ is the ratio of moments

k is the constant of fixity. k is equal to 1 if the edge is fixed and is equal to 0 when the edge is simply supported.

3. RESULT AND DISCUSSION

A nomogram is plotted which represents the variation of ultimate load carrying capacity of the slab with respect to the number of edges and the radius of the circle for different spacing of reinforcement.

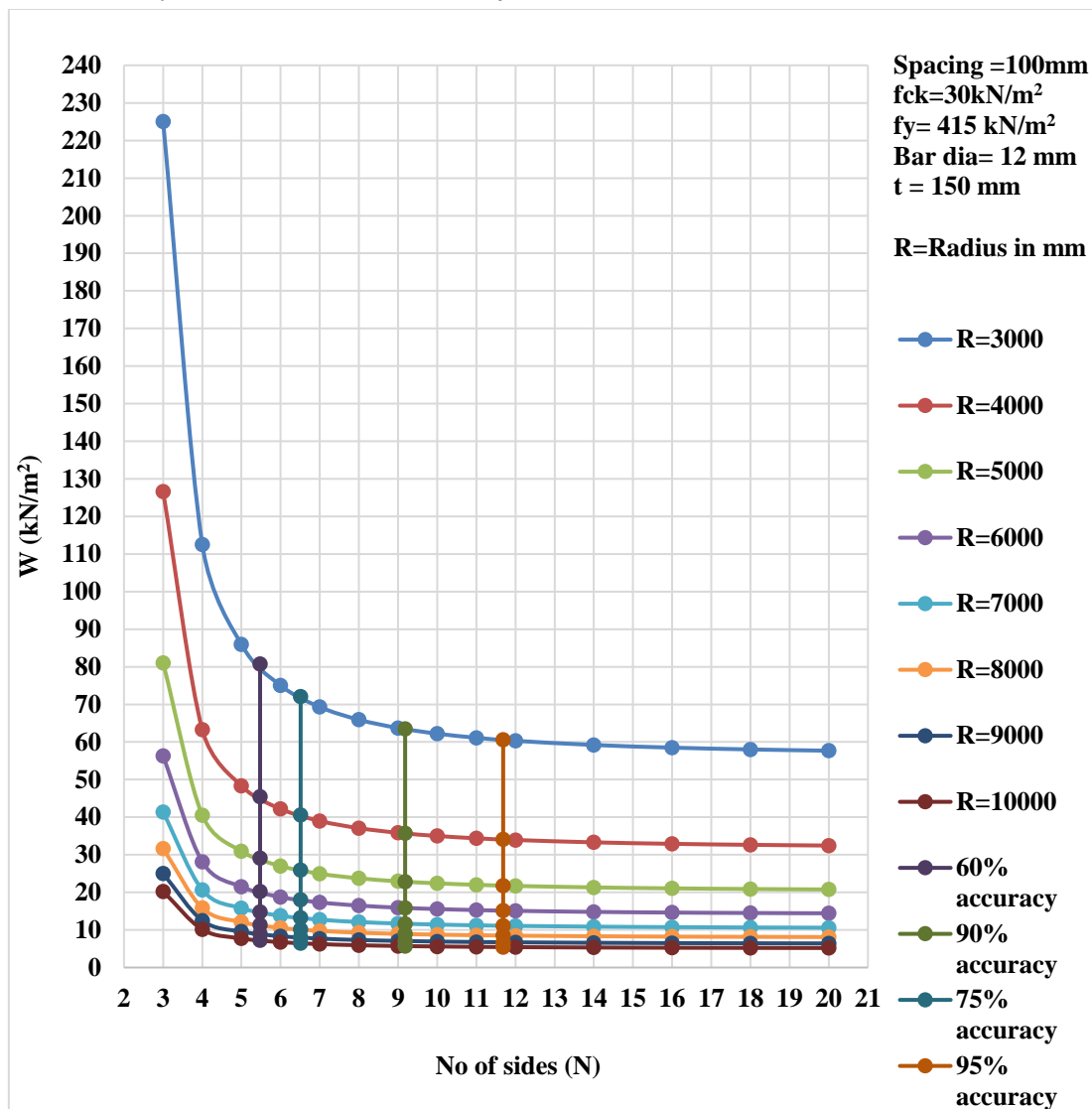


Figure 3 Nomogram representing the variation of the ultimate capacity for spacing of reinforcement = 100 mm

From the above nomogram it is clear that for a particular spacing of reinforcement of the slab and the radius of the circle in which the slab is inscribed, the ultimate load carrying capacity of the slab decreases with increase in the number of edges up to a particular point after which it remains nearly constant. This shows that with an increased number of edges, the capacity of the slab becomes more accurate. Also as the number of edges increases the slab tends to behave like a circular slab of radius R.

In the above nomogram, we can see that to obtain 90% accurate value of the capacity of the slab, it is enough to provide 9 edges. Similarly, for 60% accurate value of the capacity of the slab, nearly 6 edges are to be provided.

EXAMPLE 1

Consider a polygonal slab inscribed in a circle of radius $R = 3000 \text{ mm}$

If 9 edges are provided, then from the graph the capacity of the slab is 63.72 kN/m^2 . But the capacity with more discretization of the slab is 57.68 kN/m^2 . Thus the capacity of the slab is overestimated by 6.04 kN/m^2 and the slab is unsafe by 10.47 %.

Approximate capacity of the slab may be predicted as 85 to 90 percent of the capacity obtained from the calculation in 9 sides.

EXAMPLE 2

Consider a polygonal slab inscribed in a circle of radius $R = 6000 \text{ mm}$

If 6 edges are provided, then from the graph the capacity of the slab is 18.75 kN/m^2 . But the capacity with more discretization of the slab is 14.42 kN/m^2 . Thus the capacity of the slab is overestimated by 4.33 kN/m^2 and the slab is unsafe by 30.06 %.

Approximate capacity of the slab may be predicted as 65 to 70 percent of the capacity obtained from the calculation in 6 sides.

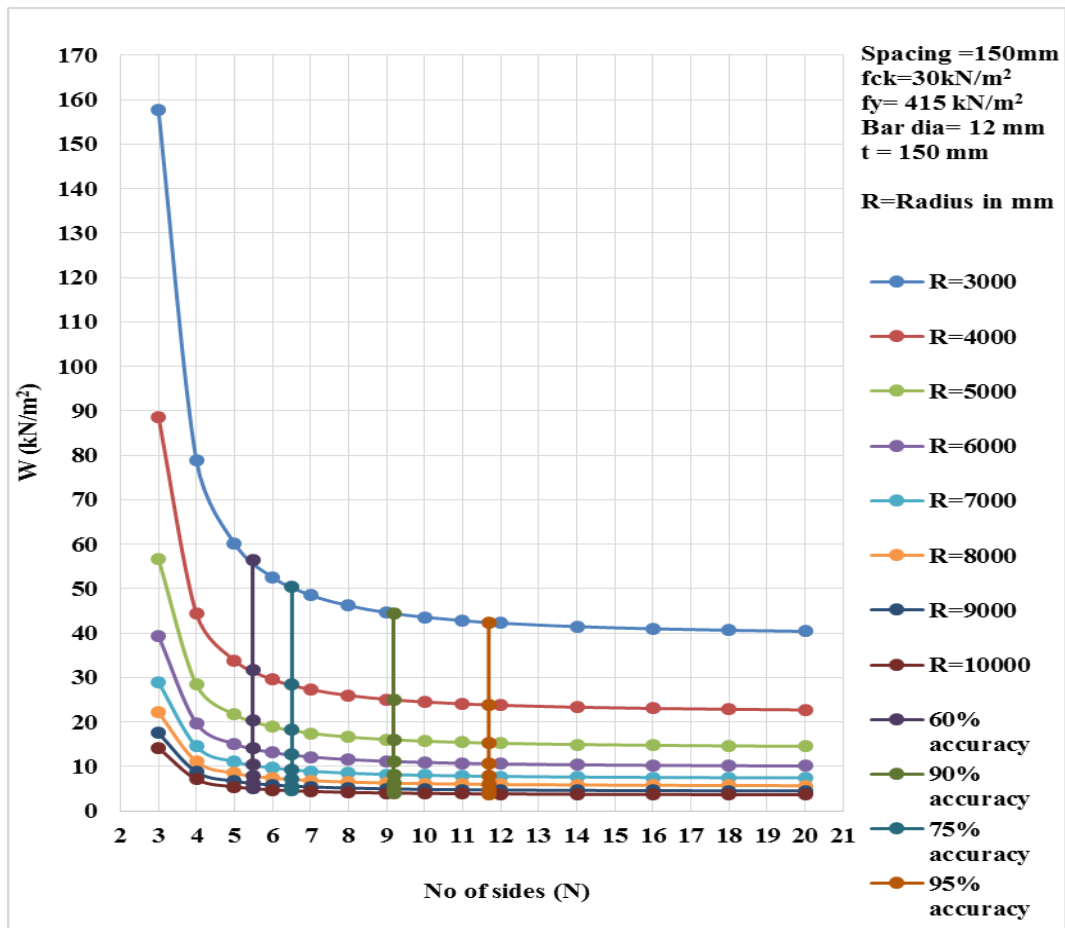


Figure 4 Nomogram representing the variation of ultimate capacity for spacing of reinforcement = 150 mm

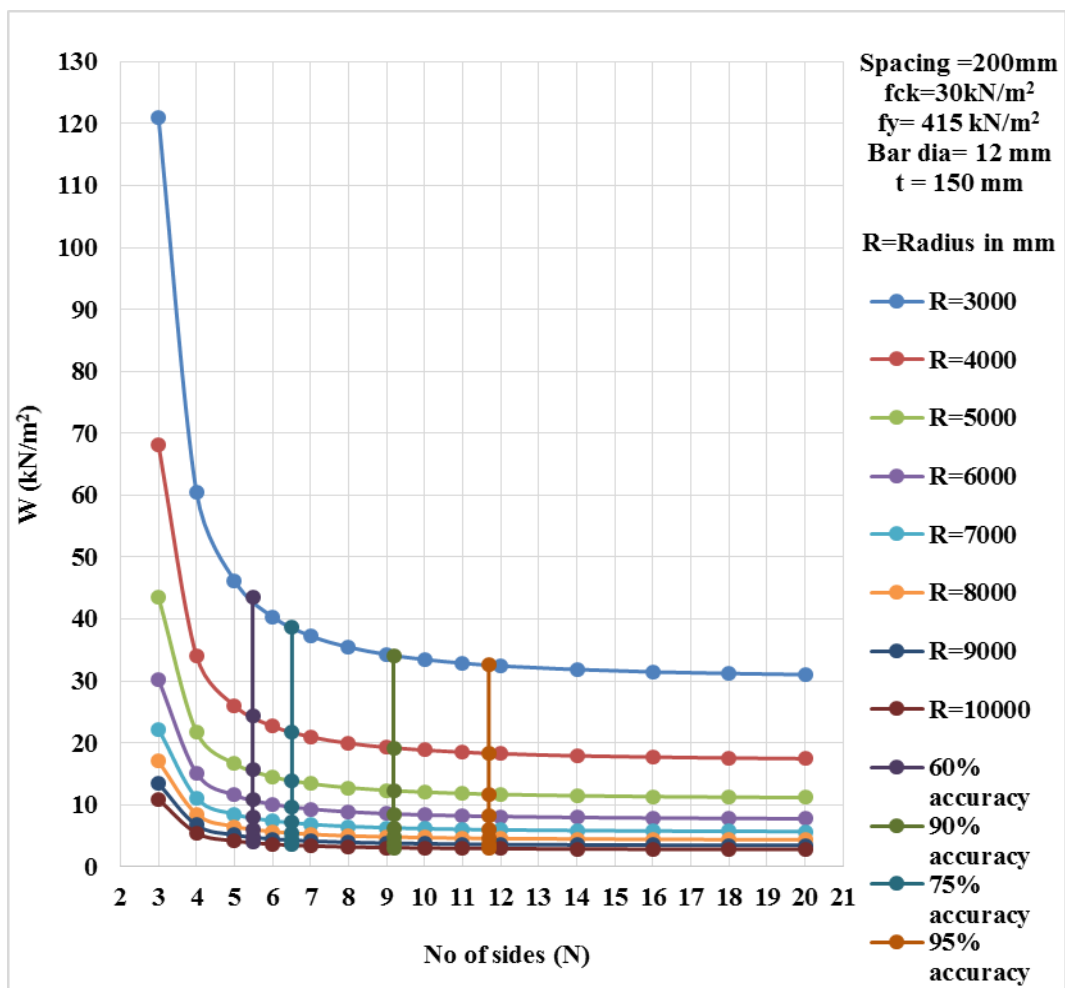


Figure 5 Nomogram representing the variation of ultimate capacity for spacing of reinforcement = 200 mm

4. CONCLUSION

The methodology has been formulated for preparation of nomograms for different characteristics of the slab, which can be effectively used for the determination of ultimate load carrying capacity of the slab.

These nomograms are the ready reckoner for the design of slabs using yield line analysis.

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