Control of position and swing angle of overhead crane using different controllers

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ABSTRACT

A crane is a machine equipped with a hoist which is mainly used both to lift and lower materials and move them horizontally to different places. However, the underactuated structure of the crane makes difficult to control. In the practical situations, however, the achievement of both transporting trolley to target position and reducing payload sway is not always easy. The main objective is to evaluate different types of controllers for controlling the trolley position and swing motion by conducting a study on the position and swing angle characteristics of different controllers. The various controllers used for the analysis are conventional PID and PD controller, fuzzy-tuned PID and PD controller, LQR, and LQG. The analysis shows that the LQG controller has the best performance even in the presence of a disturbance.

Keywords: Linear quadratic regulator (LQR), Linear quadratic Gaussian (LQG), Crane.

1. INTRODUCTION

The main purpose of control of an overhead crane is to transport the load to the desired location with less swing. However, most of the common overhead crane results in a swing movement when the payload is suddenly stopped after a fast motion [1]. The overhead crane is used mainly in the industries, hazardous environments like nuclear power plant due to its low cost, easy assembly, precise positioning of load and less maintenance. The swing motion can be reduced by adjusting the speed of the motor but it will be a time-consuming process. A skillful operator is needed to control both the position and swing manually. The inefficiency of controlling the crane also might cause problems and create harm to the people and the surroundings.

Various methods were done on cranes system based on an open loop system. Earlier open loop time optimal strategies were applied to the overhead crane by many researchers such as discussed in [2],[3]. The results showed that open loop system is sensitive to the system parameters and could not compensate for wind disturbances. Another importance of open loop strategy is the input shaping introduced by Karnopp [4], Teo [5] and Singhose [6]. However, the input shaping method is still an open-loop approach. Hubbel et al. [7] used an open-loop method to control the motion of the crane. In this open loop control method, the input control profile was determined in such a way that unwanted oscillations and residual pendulations were eliminated. However their approach was applicable, but the open loop control scheme is not robust to disturbances and parameter uncertainties [8]. Moreover, a feedback PID anti-swing controller is developed in [9] to control of an overhead crane. Ahmad et al. [10] used a hybrid input-shaping method to control of the crane. Wahyudi and Jalani [11] introduced fuzzy logic feedback technique to control the crane. They also presented an optimal control method is used in [12] to control the dynamic motion of the crane. Here, minimum energy of the system and also the integrated absolute error of payload angle are assumed as their optimization criterion. Zhao and Gao [13] studied the control of the overhead crane. They proposed a fuzzy control method to control the input delay and actuator saturation of the system. Nazemizadeh et al. [14] studied tracking control of the crane. Furthermore, Nazemizadeh [15] presented a PID tuning method for tracking control of a crane.

In this paper, an overhead crane is provided with a Linear Quadratic Regulator to control both swing and position of the crane and the results are compared with fuzzy-tuned PID and PD controller and conventional PD and PID control. It is then provided with a Linear Quadratic Gaussian to nullify the external disturbance. The structure of this paper is as follows. The modeling of the crane is explained in section 2. Section 3 highlights the development of model-based soft sensor. Section 4 discusses the crane with conventional PD and PID controller. Section 5 explains crane control with fuzzy-tuned PID and PD controller. Section 6 discusses
the crane with Linear Quadratic Regulator. Section 7 discusses the crane with Linear Quadratic Gaussian. Simulation results are given in section 8. Conclusion and future scope is discussed in section 9 and 10 respectively.

2. MODELING OF CRANE

Fig. 1 shows a schematic diagram of the crane considered in this paper. Due to the fact that only planar motion of crane is considered in this paper, there are two independent coordinates namely $y$ and $\theta_y$ to describe the trolley position and the swing angle of the payload respectively.

Since the mass of the rope is small enough as compared to the payload mass $m_p$, it is considered as massless. The non-linear dynamic model of overhead crane prototype is derived using Lagrange equations.

\[
\left( m_t + m_p \right) \ddot{y} = F_y + m_p \left( \ddot{\theta}_y \sin \theta_y - \dot{\theta}_y \cos \theta_y \right)
\]

\[
\cos \theta_y \ddot{y} + i \dot{\theta}_y = -g \sin \theta_y
\]

By assuming small motion of $\theta_y$, the following linearized model of the crane is obtained.

\[
\left( m_p + m_t \right) \ddot{y} + m_p \left( \ddot{\theta}_y \right) = F_y
\]

\[
\dot{\theta}_y + g \theta_y = 0
\]

In the above equation, $m_t$ is the mass of the trolley, $m_p$ is mass of the payload, $l$ is the length of the rope, $y$ is the position of the trolley, $\theta_y$ is the swing angle and $F_y$ is the force provided by the dc motor. Thus the state space model of the overhead crane can be obtained as

\[
\dot{X} = Ax + Bu
\]

\[
Y = Cx + Du
\]

Where

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & m_p g & 0 \\
0 & 0 & m_p & 0 \\
0 & 0 & \frac{(m_t + m_p) g}{m_t l} & 1
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
1/m_t \\
0 \\
-1/m_t l
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
D = [0]
\]

The translational motion of the trolley is driven by DC motor. Therefore, to obtain the entire model of the crane, the motor dynamic is modeled according to equivalent DC motor circuit. The equivalent circuit of DC motor has armature resistance $R$, inductance $L$, motor inertia $J$, torque constant $k_T$, the input voltage to the dc motor $V$, armature current $I$ and damping coefficient $B$. The rotational motion is converted to translational motion through the mechanical part (pulley or gear) with radii of $r$. The dynamics of dc motor circuit is given by the following equations

\[
V = RI + k_T \frac{dI}{dt} + k_b \delta
\]

\[
T = k_T \delta
\]

\[
J \ddot{\delta} + B (\dot{\delta}) = T
\]
3. DEVELOPMENT OF MODEL-BASED SENSOR

Most of the cranes use controllers for controlling both the position of the trolley and swing angle of the crane payload. Therefore two sensors are needed to measure the trolley position \( Y(s) \) and swing angle \( \theta_y(s) \). The latter is usually installed on the load side. A model-based soft sensor is proposed to provide output estimation of the plant as the use of sensors practically on the load side is very difficult. The dynamic information from trolley position \( Y(s) \) is given to the developed soft sensor. It produces an output of the payload motion that will be used for the feedback signal to the controller.

A model-based soft sensor is adopted in the proposed method. The proposed model based soft sensor is used to estimate the swing angle of the payload based on the position of the trolley. So the model is used as a model-based soft sensor. According to the equations, the swing angle of the payload is calculated by using the following:

\[
\frac{\theta_y(s)}{Y(s)} = \frac{s^2}{g + ls^2}
\]

where \( \theta_y(s) \) and \( Y(s) \) are estimated swing motion of the payload and trolley motion in Laplace domain respectively. It is shown that the proposed model-based soft sensor is easily and practically implemented since it has a simple structure and depends only on the length of the rope \( l \) and acceleration due to gravity \( g \) which are easily known.

4. CRANE WITH CONVENTIONAL PID AND PD CONTROLLER

The conventional PID and PD controllers are used to evaluate the ability of the proposed model-based soft sensor. The function of the controller is to control the payload position \( Y(s) \) so that it moves to the desired position \( Y_r(s) \) as fast as possible without excessive swing angle \( \theta_y(s) \). A PID controller is adopted to control the position of the crane, while a PD controller is used for control of swing angle. The controller gains for PID and PD are designed and optimized with simulation model by using Simulink response optimization library block. It is mainly a numerical time domain optimizer developed under MATLAB/Simulink environment. Hence the response obtained by the Simulink optimization library block assists in time-domain-based control design by setting the required value of overshoot, settling time and steady-state error.

<table>
<thead>
<tr>
<th>Table-1: Crane and Motor Parameters</th>
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<tbody>
<tr>
<td>Symbol</td>
</tr>
<tr>
<td>( m_p )</td>
</tr>
<tr>
<td>( m_t )</td>
</tr>
<tr>
<td>( L )</td>
</tr>
<tr>
<td>( G )</td>
</tr>
<tr>
<td>( r )</td>
</tr>
<tr>
<td>( R )</td>
</tr>
<tr>
<td>( L )</td>
</tr>
<tr>
<td>( J )</td>
</tr>
<tr>
<td>( B )</td>
</tr>
<tr>
<td>( k_B )</td>
</tr>
<tr>
<td>( k_t )</td>
</tr>
</tbody>
</table>

In order to realize motion of the crane quickly with low value for overshoot, the PID controller is optimized. Moreover, in order to suppress the swing angle quickly, the PD controller is optimized. Thus there are five parameters to be optimized in order to have satisfactory control performance. The parameters, \( K_p, K_i, K_d, K_{ps} \) and \( K_{ds} \) which are the proportional, integral and derivative gains for the position control and proportional, derivative gains for the anti-swing control. The optimization to obtain PID+PD gains for position and anti-swing crane control was done using Ziegler Nichols tuning method. Based on the result, the gains of \( K_p, K_i, K_d, K_{ps} \), and \( K_{ds} \) are shown in Table 2.

<table>
<thead>
<tr>
<th>Table-2: Optimised PID and PD position and antiswing gains</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
</tr>
<tr>
<td>140.4</td>
</tr>
</tbody>
</table>
5. CRANE WITH FUZZY TUNED PID AND PD CONTROLLER

To control the position of trolley crane, a fuzzy-tuned PID is used, while anti-swing control is implemented using fuzzy-tuned PD control. Initially, mandani fuzzy inference system is designed as a fuzzy tuner. It has error and error rate as inputs and tuned gain of PID as the output. The fuzzy-tuned PID and PD control designed to control the position and swing angle of the crane. The fuzzy system for the output is triangular membership function. The fuzzy system for the input has three Gaussian membership functions with certain width according to the determined range or universe of discourse.

Fuzzy logic strategies are also effective but are hard to tune. It can mimic human behavior accurately. Here parameters can be determined on-line based on the error signal and its time derivatives. Instead of fixed PID and PD gains, the gains are determined directly by means of a fuzzy inference system. As the nature of fuzzy control system requires expert knowledge to tune the parameters which are often difficult and time-consuming. They are hard to tune.

Table-3: Rules for fuzzy tuned PID and PD controllers

<table>
<thead>
<tr>
<th>IF-THEN RULE FOR FUZZY TUNER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error rate</td>
</tr>
<tr>
<td>N</td>
</tr>
<tr>
<td>small</td>
</tr>
<tr>
<td>Z</td>
</tr>
<tr>
<td>P</td>
</tr>
</tbody>
</table>

6. LINEAR QUADRATIC REGULATOR (LQR)

The stability is a major problem in an overhead crane system. If the state variables are known, then they can be utilized to design a feedback controller so that the input becomes $u = -Kx$. It is necessary to measure and utilize the state variables of the system in order to control the position and swing angle of the crane. The feedback control system is provided with a control signal $u(t)$ which is obtained as:

$$u(t) = -(K_1x_1 + K_2x_2 + K_3x_3 + K_4x_4)$$

(11)

Then the equation becomes,

$$x_1 = x_2$$

$$x_2 = x_4$$

$$x_4 = \frac{K_4}{\Delta I} \left( \frac{M_p + M_c}{M_1} \right) \frac{m_2 g}{m_1} x_3 - \frac{K_3}{\Delta I} \frac{m_2 g}{m_1} x_2$$

(12)

Arranging in matrix form, it is expressed as:

$$\dot{X} = Ax + Bu$$

(13)

$$Y = Cx + Du$$

(14)

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{K_2}{K_r m_2} & \frac{m_p g}{m_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{(m_c + m_p)g}{m_1} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{K_2}{K_r m_2} \\ 0 \\ -\frac{K_3}{K_r m_1} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D = [0]$$

$$X = [y \ y_\theta \ y_{\theta_\theta} \ y_{\theta_\theta}]^T$$

Which is of the form

$$\ddot{x} = Ax - kx = (A - Bk)x = Hx$$

(15)

To minimize the performance index $J$, consider the following two equations,

$$J = \int_0^\infty X^T X dt = X^T (0) PX (0)$$

(16)
This compensated system is considered to be an optimal system which results in a minimum value for the performance index. The main advantage of using the quadratic optimal control scheme is that the system designed will be stable, except in the case where the system is not controllable. The matrix 'P' is determined from the solution of the matrix Riccati equation. This optimal control is called the Linear Quadratic Regulator (LQR). The optimal feedback gain matrix k can be obtained by solving the following Riccati equation for a positive-definite matrix 'P'.

\[ A^T P + PA - PBR^{-1}B^T P + Q = 0 \]  

(18)

Let \( Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \)

(19)

\[ R = [0.01] \]

(20)

Tuning of Q and R matrix, P matrix is determined. So the optimal feedback gain is obtained as:

\[ k = R^{-1}B^T P \]

(21)

\[ u = -10x_1 - 13.4966x_2 + 31.2263x_3 + 7.8037x_4 \]

(22)

This control signal yields an optimal result for any initial state under the given performance index. In LQR the cost function which is to be minimized is given by

\[ J = \int_{0}^{\infty} (X^T Q X + U^T R U) \, dt \]

(23)

The two matrices Q and R are selected by design engineer by using Bryston rule. Selecting Q large means that, to keep J small. On the other hand, choosing R large means that the control input u must be smaller to keep J small. One should select Q to be positive semi-definite and R to be positive definite. This means that the scalar quantity \( X^T Q X \) is always positive or zero at each time t. The best value of the Q & R matrix is calculated by checking the step response of the system (with LQR).

7. LINEAR QUADRATIC GAUSSIAN (LQG)

The linear-quadratic-Gaussian (LQG) control is one of the most optimal control technique. It mainly deals with uncertain linear systems affected by additive white Gaussian noise, having incomplete state information and undergoing control subject to quadratic costs. Also, the solution is considered unique and constitutes a linear dynamic feedback control law that is easily computed and implemented. The LQG controller is the combination of a Kalman filter i.e. a linear-quadratic estimator (LQE) with a linear-quadratic regulator (LQR). The separation principle guarantees that these can be designed and computed separately. LQG control applied to both linear time-invariant systems as well as linear time-varying systems. LQR provides optimal state feedback which minimizes a quadratic cost about the states. Kalman filter provides optimal state estimation. In stochastic control, the above two give the Linear Quadratic Gaussian (LQG) controller. Thus the state space model of the overhead crane can be obtained as

\[ \dot{\hat{x}} = A\hat{x} + Bu + \xi(t) \]

\[ Y = C\hat{x} + Du + \phi(t) \]

(24)

(25)

In LQG design the disturbances are added to the system mainly the process and measurement noise respectively which are assumed to be zero mean with power spectral density matrices of W & V. The Kalman Gain can be found out by using the equation

\[ K_f = YC^TY^{-1} \]

(26)

where \( Y = Y^T \geq 0 \) is the unique positive-semi definite solution of the algebraic Riccati equation.

\[ YA^T + AY - YC^TV^{-1} + W = 0 \]

(27)

First, find \( \hat{x}(t) \) estimate the full state x(t) using the available information. After that, apply the LQR controller, using the estimation \( \hat{x}(t) \) in place of the true (now unknown) state x(t).

\[ W = \begin{bmatrix} Q & N_{\xi}^T \\ N_{\xi} & R \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} \xi & N_f^T \phi \end{bmatrix} \]

(28)

Where Q and R are weighing matrix of LQR and \( \xi \) and \( \phi \) are the covariance of plant noise \( \xi(t) \) and measurement noise \( \phi(t) \) and \( N_{\xi} \) & \( N_f \) are zero matrices.
8. SIMULATION RESULTS

MATLAB software package is used to determine the response of the system. The Simulink model of the system with optimized values of PD and PID controller is created in MATLAB. Tuning of the Q & R matrix is done by separate coding. The regulation of position and swing angle is determined with the tuning of Q & R matrix and the tracking of the crane also determined. The Simulink model of the system is developed. The figure 2, 3 and 4 show the position and swing angle control using optimized values for PD and PID controller and fuzzy tuned PID and PD controller. From figure 5, figure 6 and figure 7, the system is regulated at 2 sec, 3 sec, and 5 sec respectively and the system consists of a small number of overshoot and undershoots when compared to PD and PID controller. The system is precisely regulated at this condition. The figure 8 and 9 shows the position and swing control using LQG. LQG control the position and swing without undershoot in spite of the disturbances.

![Time Series Plot](image1.png)

**Figure-2 Position and swing angle control using PD and PID controller**

![Time Series Plot](image2.png)

**Figure-3 Position and swing angle control using fuzzy tuned PID controller**
Figure 4: Swing angle control using fuzzy tuned PD controller.

Figure 5: Voltage of the cart using LQR.

Figure 6: Swing angle of the crane using LQR.

Figure 7: Position control of the crane using LQR.
9. CONCLUSION

In this paper, an LQR system was designed for the crane which results in a minimum value for the performance index. Also, the control law given by equation (22) yields an optimal result for any initial state under the given performance index. Both the transient and steady-state response of the system is improved with LQR controller. This design based on the quadratic performance index yields a stable control system for the overhead crane. The system is then provided with external disturbances. LQG controls the position without undershoot in spite of the disturbances.

10. FUTURE SCOPE

A Linear Quadratic Gaussian controller which also eliminates position and swing due to the high value of disturbance which is not considered in this paper.

11. REFERENCES


