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A study on N-policy FM/FG/1 vacation queuing system with server timeout in triangular, trapezoidal and pentagon fuzzy numbers using α-cuts

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ABSTRACT

This paper studied and investigated about the N-policy queueing system with server timeout, using Triangular Trapezoidal and Pentagon Fuzzy numbers with the application of fuzzy set. We consider single server vacation queueing system that operates in the following manner; when the system is empty, the server waits for fixed time c. At this time if n-customers are waiting in queue then server return to the system and do the service. At the expiration of the time, if no customer arrives server goes for vacation. Similarly in vacation also if n-customers are waiting in queue serve return to the system and commence service exhaustively. By the approach of DSW algorithm we develop the membership function of the system performance are of fuzzy natured. Based on α-cut approach the fuzzy queues are reduced to a family of crisp set. The numerical examples are also studied to this model.

Keywords: *N-policy server timeout, membership function of (Triangular, Trapezoidal & Pentagon) Fuzzy numbers, α-cuts, DSW algorithm and Interval Analysis.*

1. INTRODUCTION

Queueing theory is mostly used in our daily life. A vacation queuing system is one in which a server may become unavailable for a random period of time is called a vacation. Vacation queuing theory is also studies about expected system length and number of customers in the system. Queueing system with server vacations idea was first discussed by Levy and Yechiali [6] they introduced the utilization of idle time in an M/G/1 queueing system.

When the system is empty the server will wait for fixed time known as server timeout. Oliver C.Ibe [12],[13] derived an expression for mean waiting time of a vacation queueing system in which server does not take frequently another vacation upon returning from a vacation and finding system empty, as in the multiple vacation scheme or wait indefinitely for a customer to arrive and also he extended this model for N-policy. E.Ramesh Kumar and Y. Praby Loit [3] derived an expression for mean waiting time of a vacation queueing system in which server does not take frequently another vacation upon returning from a vacation and finding system empty, as in the multiple vacation scheme or wait indefinitely for a customer to arrive. Y.Saritha, K.Satish Kumar and K.Chandan derived the expected system length using different bulk size distribution for $M^x/G/1$ Vacation Queueing System with Server Timeout [15]. Y.Saritha, K.Satish Kumar, V.N.RamaDevi and K.Chandan derived the expected system length for M/G/1 Vacation Queueing System with breakdown, repair and Server Timeout [16].

But in fuzzy queuing theory it is best described about arrival rate and service rates through linguistic terms of very low, low, moderate, high and very high. A. Karfmann, [1] Introduced the Theory of Fuzzy Subsets and this models are formulated to calculate the lower and upper limits and can be splits into eleven distinct points through the α-cuts. Here DSW algorithm is used to define a membership function of the performance measure for fuzzy queues, where '**F**' denotes Fuzzy time and '**FM**' denotes Fuzzified time. Fuzzy queues are developed by many authors such as J.J.Buckely [4], D.S.Negi and E.S.Lee [2] and latest authors are R. Srinivasan [7], S.Shanmuugasundaram and BB. Venkatesh [8], S. Thamotharan [9], S.Bark and M.S. Fallahnezhad [11] are did many experiments on fuzzy models and S.P.Chen [10], by using fuzzy set theory hence the classical queuing model with priority discipline will have more applications to expand the fuzzy models, recently developed on (FM/FM/1):(infinite/FCFS). K.Usha Madhuri and K.Chandan [5] derived FM/FM/1 queueing system with pentagon fuzzy Numbers using α cuts. V.Ashok Kumar [14] derived FM/FM/1 queueing system with pentagon fuzzy Numbers using DSW algorithm.

Our aim of this paper is FM/FG/1 N-policy queuing system with server timeout by using triangular, trapezoidal and pentagon fuzzy numbers. Under DSW algorithm the fuzzy set decomposes into eleven distinct points through α cut method, by using membership function the performance measures is obtained for expected number of customers E(T) of triangular, trapezoidal and pentagon fuzzy numbers.

2. FUZZY SET THEORY

2.1 Set

By a set we understand every collection made into a whole of definite distinct made a whole of definite distinct objects of our intuition (or) of our thoughts ("Georg cantor")

For a set in cantors sense the following properties

1. $x \neq \{x\}$.
2. If $x \in x$ and $x \in y$ then $x \notin y$.
3. The set of all subsets of x is denoted as 2^X.
4. ϕ is the empty set and thus very important.

2.2 Fuzzy set

A fuzzy set A in X is characterized by its membership function X: A → [1, 0]. Here X is a non – empty set.

2.3 Triangular fuzzy number

A triangular fuzzy number $\tilde{A}$ is defined by (a_1, a_2, a_3), where $a_i \in$ R and $a_1 \leq a_2 \leq a_3$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \ for x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} \ for a_1 \leq x \leq a_2 \\ \dfrac{a_3-x}{a_3-a_2} \ for a_2 \leq x \leq a_3 \\ 0, \ for a_3 > 0 \end{cases}$$

2.4 Trapezoidal Fuzzy Number

A trapezoidal fuzzy number $\tilde{A}$ is defined by (a1, a2, a3, a4) where $a_i \in$ R and a1≤a2≤ a3≤a4.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \ for x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} \ for a_1 \leq x \leq a_2 \\ 1, \ for a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3} \ for a_3 \leq x \leq a_4 \\ 0, \ for a_4 > 0 \end{cases}$$

2.5 Pentagon Fuzzy Number

A Pentagon fuzzy number $\tilde{A}$ is defined by (a_1, a_2, a_3, a_4, a_5) where $a_i \in$ R and $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \ for x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} \ for a_1 \leq x \leq a_2 \\ \dfrac{x-a_2}{a_3-a_2} \ for a_2 \leq x \leq a_3 \\ 1, if x = a_3 \\ \dfrac{a_4-x}{a_4-a_3} \ for a_3 \leq x \leq a_4 \\ \dfrac{a_5-x}{a_5-a_4} \ for a_4 \leq x \leq a_5 \\ 0, if a_5 > 0 \end{cases}$$

2.6 Membership Function

Fuzzy set is any set that allows its members to have different degree of membership called membership function in the interval [0,1].For a set A, we define a membership function μ_A such as

$\mu_A(x)$ =1 if and only if x € A

0 if and only if x $\notin$ A.

2.7 Crisp Set

The operational characteristic of union, intersection, and complement set is known as crisp set.

Eg: A $\cup$ B=B $\cup$ A

(A $\cup$ B) $\cup$ C=A $\cup$ (B $\cup$ C

2.7 α-Cut

An α-Cut of a fuzzy set $\mathring{A}$ is a crisp set A_α that contains all the elements of the universal set X that have a membership grade in A. A_α= {x ∈ X: $\mu_{\mathring{A}} \geq \alpha, 0 \leq \alpha \leq 1$}

3. SOLUTION PROCEDURE

In single server queueing system customers are arriving into the system according to Poisson process with rate λ. When the system is empty, the server waits for fixed time c named as server timeout. At this time if n-customers are waiting in queue then server return to the system and do the service till N-customer arrive. At the expiration of the time, if no customer arrives server goes for vacation. Similarly in vacation also if n-customers are waiting in queue server return to the system and commence service exhaustively up to N-customers. Let Wq denote the expected waiting time in the System and let the utilization factor be defined by ρ=λ/μ then the mean waiting time is taken from [Ref 11].

$$E(w_q) = \frac{N-1}{2\lambda} + \frac{\lambda E(X^2)}{2(1-\rho)} - \sum_{n=1}^{N-1} \frac{(N-n)(\lambda c)^{n-1}}{(n-1)!} e^{-\lambda c} \qquad (1)$$

Here fuzzy model is applying for the above waiting queue to derive expected number of customers denoted by E (T) for triangular, trapezoidal and pentagon fuzzy numbers including α-cuts intervals. Let us consider FM/FG/1 N-policy queuing system with server timeout. An infinite calling source of first come first served discipline. The inter arrival time A, service times S, and vacation time V are described by the following fuzzy sets:

A = {a, $\tilde{\mu}_A(a)$ / a Є X}

S = {s, $\tilde{\mu}_S(s) \geq$ α / s Є Y_1}

V = {v, $\tilde{\mu}_V(v) \geq$ α / v Є Y_2}

Here X is the set of the inter arrival time and Y_1 is the set of the service time, and Y_2 is the set of vacation time.

$\tilde{\mu}_A(a)$ Is a membership function of inter arrival time.

$\tilde{\mu}_S(s)$ Is a membership function of service time.

$\tilde{\mu}_V(v)$ Is a membership function of vacation time.

The α cuts of inter arrival time, service time, and vacation times are represented as

A (α) = {a Є X / $\tilde{\mu}_A(a) \geq$ α}

S (α) = {s Є Y_1 / $\tilde{\mu}_S(s) \geq$ α}

V (α) = {v Є Y_2 / $\tilde{\mu}_V(v) \geq$ α}

Using these α cuts we have to define the membership function P(A, S, V) as follows:

Triangular membership function $$\mu_{P(A,S,V)}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & for\ a_3 > 0 \end{cases} \qquad (2)$$

Trapezoidal membership function $$\mu_{P(A,S,V)}(x) = \begin{cases} 0 & for\ x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for\ a_1 \le x \le a_2 \\ 1 & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & for\ a_3 \le x \le a_4 \\ 0 & for\ a_4 > 0 \end{cases} \quad (3)$$

Pentagon membership function

$$\mu_{P(A,S,V)}(x) = \begin{cases} 0,\ for x < a_1 \\ \frac{x-a_1}{a_2-a_1}\ for a_1 \le x \le a_2 \\ \frac{x-a_2}{a_3-a_2}\ for a_2 \le x \le a_3 \\ 1, x = a_3 \\ \frac{a_4-x}{a_4-a_3}\ for a_3 \le x \le a_4 \\ \frac{a_5-x}{a_5-a_4}\ for a_4 \le x \le a_5 \\ 0, if a_5 > 0 \end{cases} \quad (4)$$

Now let us consider in single-server FM/FG/1 fuzzy N-policy queuing system of expected number of customers of E (T) by using little's formula [Ref 11].

$$E(T) = E(W_q) + E(X) \quad (5)$$

The arrival time follows Poison distribution and the service time, vacation time follows an exponential distribution.

4. INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered of real numbers with lower and upper bounds.

I_1= [a, b], a≤b, I_2=[c, d], c≤d,

Define a general arithmetic property with the symbol ∗= [+,-,÷×] symbolically the operation

$I_1 * I_2$ = [a, b]*[c, d]

Represents another interval. The interval calculation depends on the magnitudes and signs of the elements a, b, c, d

[a,b]+[c,d]=[a+c, b+d]

[a,b]-[c,d]=[a-d, b-c]

[a,b].[c,d]=[min(ac,ad,bc, bd), max(ac,ad,bc, bd)]

[a,b]÷ [c,d]=[a,b].[1/d,1/c] provided that 0∉[c,d]

$$\alpha\,[a,b] = \begin{cases} [\alpha a, \alpha b]\ for\ \alpha > 0 \\ [\alpha b, \alpha a]\ for\ \alpha < 0 \end{cases}$$

5. DSW ALGORITHM

The DSW algorithm consists of the following steps:

1) Select a α cut value where $0 \le \alpha \le 1$.

2) Find the intervals in the input membership functions that correspond to this α.

3) Using standard binary interval operations compute the interval for the output membership function for the selected α cut level.

4) Repeat steps 1 – 3 for different values of α to complete α cut representation of the solution.

6. NUMERICAL EXAMPLE

Consider an FM/FG/1 N-policy queuing system with server timeout, where the arrival rate, service rate, and vacation rate are of fuzzy natured for the equation (5) is

$$E(T) = E(W_q) + E(X)$$

By substituting the value of E (W_q) in equation (5), and E(X) is the mean of the X. Then expected number of customers is written as

$$E(T)=\frac{N-1}{2\lambda}+\frac{\lambda E(X^2)}{2(1-\rho)}-\sum_{n=1}^{N-1}\frac{(N-n)(\lambda c)^{n-1}}{(n-1)!}e^{-\lambda c}+\frac{1}{\mu} \qquad (6)$$

Hence substituting vacation time Y_2 = [N = 3, n = 1] in above equation (6), then the expression is

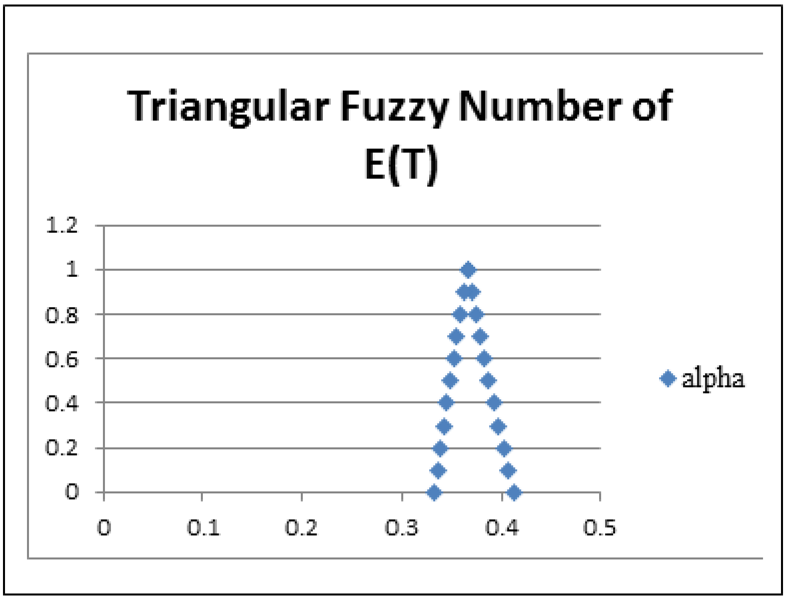
$$E(T)=\frac{1}{\lambda}+\frac{\lambda}{\mu(\mu-\lambda)}-e^{-\lambda c}\{2+\lambda c\}+\frac{1}{\mu} \qquad (7)$$

6.1 TRIANGULAR FUZZY NUMBERS WITH GRAPHICAL REPRESENTATION

Take the arrival rate, service rate and vacation rate of triangular fuzzy number represented for eqn (7) by $\tilde{\lambda}$ = [4,5,6], $\tilde{\mu}$ = [10,11,12], The interval of confidence limit at possibility level of α is [4+α , 6− α] and [10+ α , 12−α],Where x = [4+ α , 6− α], Y_1 = [10+ α , 12−α]. Non-parameter value c=2.

Table 1:

ALPHA	E(T) lower LIMIT	E(T) Upper
0	0.413317	0.333249
0.1	0.407774	0.336062
0.2	0.402432	0.338958
0.3	0.397274	0.342091
0.4	0.392319	0.345066
0.5	0.387536	0.348277
0.6	0.382927	0.351596
0.7	0.37849	0.355032
0.8	0.374223	0.358597
0.9	0.370095	0.362292
1	0.366123	0.366123



Graph 1:

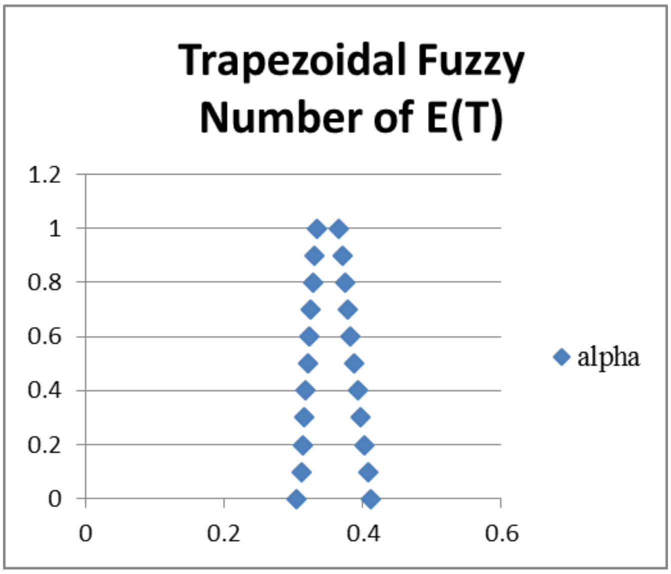
6.2 TRAPIZOIDAL FUZZY NUMBERS WITH GRAPHICAL REPRESENTATION

Take the arrival rate, service rate and vacation rate are trapezoidal fuzzy number represented for eqn (7) by $\tilde{\lambda}$ = [4,5,6,7], $\tilde{\mu}$ = [10,11,12,13], the The interval of confidence limit at possibility level of α is [4+ α , 7− α] and [10+ α , 13−α] Where x = [4+ α, 7− α], y = [10+ α, 13- α]. Non-parameter value c=2.

Table 2:

ALPHA	E(T) Lower	E(T) Upper
0	0.411921	0.304164
0.1	0.407774	0.311578
0.2	0.402432	0.31371
0.3	0.397274	0.315897
0.4	0.392319	0.318154
0.5	0.387536	0.320479
0.6	0.382927	0.322804
0.7	0.37849	0.325349
0.8	0.374223	0.327898
0.9	0.370095	0.33053
1	0.366123	0.333244

Graph 2:



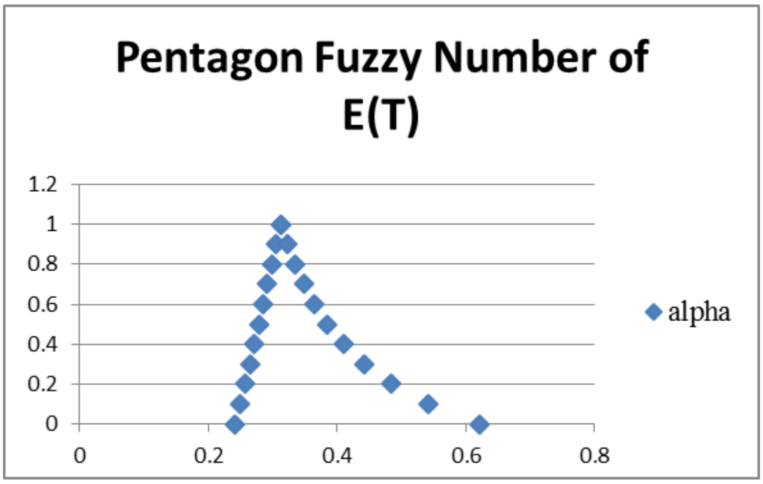
6.3 PENTAGON FUZZY NUMBERS WITH GRAPHICAL REPRESENTATION

Take the arrival rate, service rate and vacation rate are Pentagon fuzzy number represented for eqn (7) by $\tilde{\lambda}$ = [1,2,3,4,5], $\tilde{\mu}$ = [6,7,8,9,10], The interval of confidence limit at possibility level of α is [1+2 α , 5− 2α] and [6+2 α , 10−2α] Where x = [1+2α , 5−2α], y = [6+ 2α, 10-2α]. Non-parameter value c=1.

Table 3:

ALPHA	E(T) LOWER Limit	E(T) UPPER Limit
0	0.2402	0.6216
0.1	0.249262	0.540952
0.2	0.257248	0.483922
0.3	0.264368	0.441901
0.4	0.271621	0.409859
0.5	0.27789	0.384667
0.6	0.284287	0.364652
0.7	0.290774	0.347808
0.8	0.29827	0.334278
0.9	0.305264	0.323462
1	0.313333	0.313333

Graph 3:



We performed α cuts of arrival rate, service rate and vacation rate for expected number of customers in queue at eleven distinct α levels: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for fuzzy expected number of customers of E (T) at different possibility of α levels are presented in following:

From 6.1: Expected number of customer of E (T) in the system is 0.3661 and falls outside at [0.4133, 0.3332].
From 6.2: Expected number of customer of E (T) in the system is 0.3661 and 0.3332 and falls outside at [0.4119, 0.3041].
From 6.3: Expected number of customer of E (T) in the system is 0.3133 and falls outside at [0.2402, 0.6216].

7. CONCLUSION

In this paper the performance measures of single server FM/FG/1 N-policy queuing system with server timeout in triangular, trapezoidal and pentagon fuzzy numbers using α-cuts are studied and derived. The arrival time, service time, and vacation time are of fuzzy nature. The performances of this system are also fuzzy nature. Numerical example shows that the efficiency of this system.

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