INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X

Impact factor: 4.295

(Volume 4, Issue 2)

Available online at: www.ijariit.com

A Comparative Study of the Methods of Solving Intuitionistic Fuzzy Linear Programming Problem

Dr. AL. Nachammai
nachusenthilnathan@gmail.com
Kongu Engineering College
Perudurai, Tamil Nadu

Dr. M. Dhavamani
md@kongu.ac.in
Kong Engineering College
Perudurai, Tamil Nadu

Dr. B. Usha
usha_b@kongu.ac.in
Kongu Engineering College
Perudurai, Tamil Nadu

ABSTRACT

Ranking Intuitionistic Fuzzy Numbers plays an important role in decision making and information systems. Many researchers have proposed different ranking functions, score functions and signed functions for ranking Intuitionistic Fuzzy Numbers but unfortunately every method produces some anti-intuitive results in certain places. In this paper we compare different methods of ranking of Intuitionistic Fuzzy Number to solve Intuitionistic Fuzzy Linear Programming.

Keywords: *Fuzzy Linear Programming, Intuitionistic Fuzzy Linear Programming, Triangular Intuitionistic Fuzzy Number, Ranking Methods.*

1. INTRODUCTION

Linear programming is a technique used for determine an optimum schedule of interdependent activities based on the available resources. In many practical situations, the decision maker may not be in a position to specify the objective function and the constraints functions precisely in crisp environment but rather can specify them in intuitionistic fuzzy sense. Intuitionistic fuzzy linear programming, a new concept and method of uncertainty linear programming is put forward on the basis of IFS, which is a development of fuzzy linear programming. Since this method can consider both the degree of acceptance and rejection of objectives and constraints, it acts as the richest apparatus for solving linear programming problem, In addition, it makes fuzzy process for objective and constraint more meticulous and avoids high rank uncertain factors.

Fuzzy set theory has been pioneered by Zadeh (1965). Fuzzy linear programming, an offspring of fuzzy set, is first formulated by Zimmermann (1978). Atanassov (1983, 1986) has generalized the notion of Zadeh's fuzzy set to the concept of IFS which includes the degree of membership, non-membership and hesitation degree of an element in a set. Mahapatra & Roy (2009) proposed the arithmetic operations for intuitionistic fuzzy numbers. Many authors such as S.K.Bharati and A.Nagoorgani have studied Intuitionistic fuzzy linear programming. The author AL.Nachammai have proposed many methods to solve Intuitionistic fuzzy linear programming. The work present in this paper is based on a comparative study of methods of solving Intuitionistic Fuzzy Linear Programming.

2. INTUITIONISTIC FUZZY LINEAR PROGRAMMING

Definition 2.1

Intuitionistic fuzzy linear programming is defined as

$$\text{Max} \sum_{j=1}^{n} \tilde{c}_j^I \tilde{x}_j^I \qquad \text{Subject to the constraints}$$

$$\sum_{j=i}^{n} \tilde{a}_{ij}^I \tilde{x}_j^I = \tilde{b}_i^I \qquad i = 1,2,\ldots,m$$

$$\tilde{x}_j^I \geq 0$$

where

$$\tilde{c}_j^I = \left\{\left(c_{1j}, c_{2j}, c_{3j}, c_{4j};\; c'_{1j}, c_{2j}, c_{3j}, c'_{4j} \right)\right\}$$

$$\tilde{b}_i^I = \left(b_{1i}, b_{2i}, b_{3i}, b_{4i};\ \ b'_{1i}, b_{2i}, b_{3i}, b'_{4i} \right)$$

$$\tilde{a}_{ij}^I = \left(a_{1ij}, a_{2ij}, a_{3ij}, a_{4ij};\ \ a'_{1ij}, a_{2ij}, a_{3ij}, a'_{4ij} \right)$$

i=1………… m, j=1-------n are intuitionistic fuzzy numbers.

Definition 2.2

A triangular intuitionistic fuzzy number $\tilde{a}^I = \langle (a_1, a, a_2); \omega_{\tilde{a}}, \mu_{\tilde{a}} \rangle$ is a special IFS on a real number set R, whose membership function and non-membership function are defined as below.

$$\vartheta_{\tilde{a}^I}(x) = \begin{cases} (x-a_1)\omega_{\tilde{a}} / (a-a_1) & if\ a_1 \le x < a \\ \omega_{\tilde{a}} & if\ x = a \\ (a_2 - x)\ \omega_{\tilde{a}} / (a_2 - a) & if\ a < x \le a_2 \\ 0 & if\ x \le a_1\ or\ x > a_3 \end{cases}$$

and

$$\nu_{\tilde{a}^I}(x) = \begin{cases} [a - x + \mu_{\tilde{a}}\ (x - a_1\ )]/(a - a_1) & if\ a_1 \le x < a \\ \mu_{\tilde{a}} & if\ x = a \\ [x - a + \mu_{\tilde{a}}\ \ (a - x)]/\, a_2 - a & if\ a < x \le a_2 \\ 1 & \end{cases}$$

The values $\omega_{\tilde{a}}$ and $\mu_{\tilde{a}}$ represent the maximum degree of membership and the minimum degree of membership respectively and satisfy the condition

$$0 \le \omega_{\tilde{a}} \le 1\ ,\ \ 0 \le \mu_{\tilde{a}} \le 1 \text{ and } 0 \le \omega_{\tilde{a}} + \mu_{\tilde{a}} \le 1.$$

Let $\pi_{\tilde{a}}(x) = 1 - \omega_{\tilde{a}}(x) - \mu_{\tilde{a}}(x)$ is called hesitancy of x in $\tilde{a}$.

Definition 2.2

Let $\tilde{a}^I = \langle (a_1, a, a_2); \omega_{\tilde{a}}, \mu_{\tilde{a}} \rangle$ and $\tilde{b}^I = \langle (b_1, b, b_2); \omega_{\tilde{b}}, \mu_{\tilde{b}} \rangle$ be two TIFNs and λ be a real number. The arithmetical operations are defined below.

$$\tilde{a}^I + \tilde{b}^I = \langle (a_1 + b_1, a + b, a_2 + b_2); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle$$

$$\tilde{a}^I - \tilde{b}^I = \langle (a_1 - b_2, a - b, a_2 - b_1); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle$$

$$\tilde{a}^I\, \tilde{b}^I = \begin{cases} \langle \langle (a_1 b_1, ab, a_2 b_2); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle if\ a_2 > 0 and\ b_2 > 0 \rangle \\ \langle \langle (a_1 b_2, ab, a_2 b_1); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle if\ a_2 < 0 and\ b_2 > 0 \rangle \\ \langle \langle (a_2 b_2, ab, a_1 b_1); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle if\ a_2 < 0 and\ b_2 < 0 \rangle \end{cases}$$

$$\tilde{a}^I / \tilde{b}^I = \begin{cases} \langle\langle (a_1/b_2, a/b, a_2/b_1); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle if\ a_2 > 0 and\ b_2 > 0 \rangle \\ \langle\langle (a_2/b_2, ab, a_1/b_1); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle if\ a_2 < 0 and\ b_2 > 0 \rangle \\ \langle\langle (a_2/b_1, ab, a_1/b_2); \min(\omega_{\tilde{a}}, \varpi_{\tilde{b}}), \max(\mu_{\tilde{a}}, \mu_{\tilde{b}}) \rangle if\ a_2 < 0 and\ b_2 < 0 \rangle \end{cases}$$

$$(\tilde{a}^I)^{-1} = \langle (1/a_2, 1/a, 1/a_1); \omega_{\tilde{a}}, \mu_{\tilde{a}} \rangle.$$

3. RATIO RANKING METHOD (Dr. AL. Nachammai)

Definition 3.1

Let $\tilde{a}^I = \langle (a_1, a, a_2); \omega_{\tilde{a}}, \mu_{\tilde{a}} \rangle$ be TrIFN. The value and ambiguity of $\tilde{a}^I$ are defined below.

The value of the membership function of $\tilde{a}^I$ is $v_\mu(\tilde{a}^I) = (a_1 + 4a + a_2)\omega_{\tilde{a}}/6$ and the value of non-membership of $\tilde{a}^I$ is $v_\gamma(\tilde{a}^I) = (a_1 + 4a + a_2)(1 - \mu_{\tilde{a}})/6$.

The ambiguity of the membership function of $\tilde{a}^I$ is $A_\mu(\tilde{a}^I) = \dfrac{(a_2 - a_1)}{3}\omega_{\tilde{a}}$

The ambiguity of the non-membership function of $\tilde{a}^I$ is $A_\gamma(\tilde{a}^I) = \dfrac{(a_2 - a_1)}{3}(1 - \mu_{\tilde{a}})$.

It is noted that $0 \le \omega_{\tilde{a}} + \mu_{\tilde{a}} \le 1$. Obviously $A_\mu(\tilde{a}^I) \le A_\gamma(\tilde{a}^I)$.

Definition 3.2

Let $\tilde{a}^I = \langle (a_1, a, a_2); \omega_{\tilde{a}}, \mu_{\tilde{a}} \rangle$ be a TIFN.

The value index is defined as $V(\tilde{a}^I, \lambda) = V_\mu(\tilde{a}^I) + \lambda(V_\gamma(\tilde{a}^I) - V_\mu(\tilde{a}^I))$ and the ambiguity index is defined as $A(\tilde{a}^I, \lambda) = A_\gamma(\tilde{a}^I) - \lambda(A_\gamma(\tilde{a}^I) - A_\mu(\tilde{a}^I))$

where $\lambda \in [0,1]$ is a weight which represents the decision maker's choice.

The decision maker prefers negative feeling or uncertainty when $\lambda \in [0, 1/2]$ and prefers positive feeling or certainty when $\lambda \in [1/2, 1]$. In this, $\lambda = 1/2$ shows that the decision maker is indifferent between positive feeling and negative feeling, so $\lambda = 1/2$ is taken for the entire study.

Definition 3.3

A ratio of the value index to the ambiguity index for a TIFN is defined below:

$$R(\tilde{a}^I, \lambda) = \frac{V(\tilde{a}^I, \lambda)}{1 + A(\tilde{a}^I, \lambda)}$$

Illustration 3.4

Max z = (6, 9, 12; 0.5, 0.4) $\tilde{x}_1^I$ + (2, 4, 8; 0.2, 0.7) $\tilde{x}_2^I$

Subject to $(2, 3.5, 5; 0.1, 0.6)\tilde{x}_1^I + (3, 5, 9;\ 0.2, 0.7)\tilde{x}_2^I \le (4, 8, 11; 0.3, 0.4)$

$$(6,9,14;0.2,0.5)\tilde{x}_1^I + (7,8,11;0.4,0.5)\tilde{x}_2^I \le (9,12,15;0.7,0.2) \text{ and } \tilde{x}_1^I, \tilde{x}_2^I \ge 0$$

The intuitioistic fuzzy optimum feasible solution is

Max $\tilde{z}^I$ = (4.8, 20.7, 66; 0.1, 0.6) at $\tilde{x}_1^I$ = (0.8, 2.3, 5.5; 0.1, 0.6) and $\tilde{x}_2^I$ = 0.

4. SIGN DISTANCE METHOD (S. K. Bharathi)

Intuitionistic fuzzy origin 4.1

When $a_1 = a_2 = a_3 = b_1 = b_2 = b_2 = b3 \in R$, Then $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, b_2, b_3)\}$ is called intuitionistic fuzzy origin denoted by $\tilde{o}^I$

Sign distance Method 4.2

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (b_1, b_2, b_3)\}$ be Triangular Intuitionistic fuzzy number. Then sign distance of $\tilde{A}^I$ can be calculated as

$$D^S(\tilde{A}^I, \tilde{o}^I) = \frac{1}{4}\left[\int_0^1 (\tilde{A}^{I+}_L(\alpha)d\alpha + \int_0^1 (\tilde{A}^{I+}_U(\alpha)d\alpha + \int_0^1 (\tilde{A}^{I-}_L(\alpha)d\alpha + \int_0^1 (\tilde{A}^{I-}_U(\alpha)d\alpha\right]$$

Where $\tilde{A}^{I+}_L$, $\tilde{A}^{I+}_U$, $\tilde{A}^{I-}_L$, $\tilde{A}^{I-}_U$ are parametric triangular intuitionistic fuzzy number .

Illustration 4.3

Max z = {(8,16,24), (0,16,24)} $\tilde{x}_1^I$ +{(16,24,32),(8,24,40)} $\tilde{x}_2^I$

Subject to {(1,2,3), (0.5,2,4)} $\tilde{x}_1^I$ +{(2,3,4),(1,3,5)} $\tilde{x}_2^I$ = {(3,9,25), (1,9,60)}

{(1,2,3), (0.5,2,4)} $\tilde{x}_1^I$ +{(1,2,3),(0.5,2,4)} $\tilde{x}_2^I$ = {(3,8,24), (1,8,55)}

$\tilde{x}_1^I, \tilde{x}_2^I \ge 0$

The optimal solutions of decision variables are:

Max $\tilde{z}^I$ = {(24,72,200),(0,72,280)} at $\tilde{x}_1^I$ = {(3,7,7),(2,3,8.8)), $\tilde{x}_2^I$ = {(0,1,1),(0,1,5)}

5. SCORE FUNCTION AND ACCURACY FUNCTION (A. Nagoorgani)

Definition 5.1

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (a_1', a_2, a_3')\}$ be a Triangular Intuitionistic fuzzy number.

Score function for membership and non-membership values are defined respectively as

$S\left(\tilde{A}^{I^\alpha}\right) = \dfrac{a_1 + 2a_2 + a_3}{4}$ & $S\left(\tilde{A}^{I^\beta}\right) = \dfrac{a_1' + 2a_2 + a_3'}{4}$. Then accuracy function is defined as

$$\left(\tilde{A}^I\right) = \frac{(a_1 + 2a_2 + a_3) + (a_1' + 2a_2 + a_3')}{8}$$

Ranking using Score function 5.2

Let $\tilde{A}^I = \{(a_1, a_2, a_3), (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3), (b_1', b_2, b_3')\}$ be two Intuitionistic fuzzy numbers and $S(\tilde{A}^{I^\alpha})$, $S(\tilde{A}^{I^\beta})$, $S(\tilde{B}^{I\alpha})$ and $S(\tilde{B}^{I^\beta})$ are the score functions. Then

i) $S(\tilde{A}^{I^\alpha}) \le S(\tilde{B}^{I\alpha})$ & $S(\tilde{A}^{I^\beta}) \le S(\tilde{B}^{I^\beta})$ then $\tilde{A}^I < \tilde{B}^I$

ii) $S(\tilde{A}^{I^\alpha}) \ge S(\tilde{B}^{I\alpha})$ & $S(\tilde{A}^{I^\beta}) \ge S(\tilde{B}^{I^\beta})$ then $\tilde{A}^I > \tilde{B}^I$

iii) $S(\tilde{A}^{I^\alpha}) = S(\tilde{B}^{I\alpha})$ & $S(\tilde{A}^{I^\beta}) = S(\tilde{B}^{I^\beta})$ then $\tilde{A}^I = \tilde{B}^I$

Illustration 5.3

Max $\tilde{z}^I = \tilde{5}^I \ \tilde{x}_1^{\ I} + \tilde{3}^I \tilde{x}_2^{\ I}$

subject to $\tilde{4}^I \tilde{x}_1^{\ I} + \tilde{3}^I \tilde{x}_2^{\ I} \le \tilde{12}^I$, $\tilde{1}^I \tilde{x}^I{}_1 + \tilde{3}^I \tilde{x}_2^{\ I} \le \tilde{6}^I$ and $\tilde{x}_1^{\ I}$, $\tilde{x}_2^{\ I} \ge 0$

The optimal feasible solution is Max $\tilde{Z}^I = \tilde{15}^I$ at $\tilde{x}_1^{\ I} = \tilde{3}^I$, $\tilde{x}_2^{\ I} = 0$, $\tilde{S}_1^{\ I} = 0$ and $\tilde{S}_2^{\ I} = \tilde{3}^I$

6. METRIC DISTANCE RANKING FOR IFNs (AL. Nachammai)

In order to rank the intuitionistic fuzzy numbers, let the intuitionistic fuzzy number $\tilde{B}^I = 0$, and the metric distance between $\tilde{A}^I$ and O is calculated as follows.

$$D(\tilde{A}^I, 0) = \left(\int_0^1 \left(g^L_{\tilde{A}^I}(y)\right)^2 dy + \int_0^1 \left(g^R_{\tilde{A}^I}(y)\right)^2 dy \right)^{1/2}$$

Definition 6.1

A triangular intuitionistic fuzzy number $\tilde{A}^I = [(a_1, a_2, a_3)(a_1', a_2, a_3')]$ can be approximated as a symmetry intuitionistic fuzzy number $S(\theta, \sigma)$ where θ denotes the mean of $\tilde{A}^I$, σ denotes the standard deviation of $\tilde{A}^I$ and the membership function of $\tilde{A}^I$ is defined as follows:

$$f_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x - (\theta - \sigma)}{\sigma} & \text{,if } \theta - \sigma \le x \le \theta \\ \dfrac{(\theta + \sigma) - x}{\sigma} & \text{,if } \theta \le x \le \theta + \sigma \end{cases}$$

where μ and σ are calculated in the following way.

For membership function,

$$\sigma = \frac{a_3 - a_1}{2} \text{ and } \theta = \frac{a_1 + 2a_2 + a_3}{3}$$

and for non-membership function

$$\sigma = \frac{a_3' - a_1'}{2} \text{ and } \theta = \frac{a_1' + 2a_2 + a_3'}{3}$$

To rank the membership function of intuitionistic fuzzy numbers, the metric distance between $\tilde{A}^I$ and O is calculated in the following manner.

$$D_\mu\left(\tilde{A}^I, 0\right) = \left(\int_0^1 \left(g^L_{\tilde{A}^I_\mu}(y) \right)^2 dy + \int_0^1 \left(g^R_{\tilde{A}^I_\mu}(y) \right)^2 dy \right)^{1/2}$$

To rank the non-membership function of intuitionistic fuzzy numbers, the metric distance between $\tilde{A}^I$ and O is calculated as given below:

$$D_\nu\left(\tilde{A}^I, 0\right) = \left(\int_0^1 \left(g^L_{\tilde{A}^I_\nu}(y) \right)^2 dy + \int_0^1 \left(g^R_{\tilde{A}^I_\nu}(y) \right)^2 dy \right)^{1/2}$$

Definition 6.2

The rank is defined as $D\left(\tilde{A}^I, 0\right) = \dfrac{D_\mu\left(\tilde{A}^I, 0\right)}{1 + D_\nu\left(\tilde{A}^I, 0\right)}$

The inverse functions $g^L_{\tilde{A}^I}$ and $g^R_{\tilde{A}^I}$ of $f^L_{\tilde{A}^I}$ and $f^R_{\tilde{A}^I}$ respectively are shown below:

$$g^L_{\tilde{A}^I}(y) = (\theta - \sigma) + \sigma y \quad \text{and} \quad g^R_{\tilde{A}^I}(y) = (\theta + \sigma) - \sigma y$$

Illustration 6.3

Solve the following IFLPP:

Max Z = (6, 9, 11; 5, 9, 14) $\tilde{x}_1^I$ + (9, 10, 15; 7, 10, 17) $\tilde{x}_2^I$

Subject to the constraints

(2, 3, 6; 1, 3, 8) $\tilde{x}_1^I$ + (4, 6, 9; 2, 6, 15) $\tilde{x}_2^I$ $\leq$ (15, 18, 24; 11, 18, 27)

(4, 8, 9; 2, 8, 14) $\tilde{x}_1^I$ + (6, 7, 10; 5, 7, 13) $\tilde{x}_2^I$ $\leq$ (14, 20, 26; 10, 20, 29) and

$\tilde{x}_1^I, \tilde{x}_2^I \geq 0$

The intuitionistic fuzzy optimum basic feasible solution is

Maximum Z = (15.3, 30, 90; 4.9, 30, 229.5) at

$\tilde{x}_1^I$ = (0, 0, 0; 0, 0, 0) and $\tilde{x}_2^I$ = (1.7, 3, 6; 0.7, 3, 13.5)

7. CONCLUSION

In this paper different methods of ranking of Intuitionistic Fuzzy Number to solve Intuitionistic Fuzzy Linear Programming Problem is compared. Each method has its own role in viewing computational effort and time factor. In Ratio Ranking (AL.Nachammai), a special case of IFN is applied to solve Intuitionistic Fuzzy Linear Programming Problem. In sign distance method Triangular Intuitionistic Fuzzy Number is ranked and in other two methods same type of Triangular Intuitionistic Fuzzy Number is ranked. From the above comparison, we can conclude that the Metric Distance Ranking provides the accuracy in ranking the IFNs.

8. REFERENCES

[1] Atanassov, K 1983, 'Intuitionistic Fuzzy Sets', VII ITKR's session, Sofia, September.

[2] Atanassov, K 1986, 'Intuitionistic fuzzy sets', Fuzzy Sets and Systems, vol.20, pp.87-96.

[3] Bharathi, S. K 2015, ' A Note on Solving a Fully Intuitionistic Fuzzy Linear Programming Problem based on Sign Distance ', International Journal of Computer Applications , vol 119, No-23.

[4] Mahapatra, GS & Roy, TK 2009, 'Reliability evalution using triangular Intuitionistic fuzzy numbers arithmetic operations', Proceedings of World Academy of Science: Engineering & Technolog, vol.50, pp.578.

[5] Nachammai, AL & Thangaraj, P 2012, 'Solving Fuzzy Linear Fractional Programming Problem by using Metric Distance Ranking', Applied Mathematical Sciences, vol.6, no.26, pp.1275-1285.

[6]Nachammai, AL&Thangaraj, P 2013, 'Solving Intuitionistic Fuzzy Linear Programming by using Metric Distance Ranking'Researcher, vol.5, no.4, pp. 65-70.

[7] Nagoorgani, A, 2012, ' A New Approach on Solving Intuitionistic Fuzzy Linear Programming Problem', Applied Mathematical Sciences, vol.6, pp.3467-3474.

[8] Zadeh, LA 1965, 'Fuzzy sets', Information and control, vol.8.

[9] Zimmermann, HJ 1978, 'Fuzzy Programming and Linear Programming with several objective functions', Fuzzy Sets & Systems, vol.1.