Channel and Clipping Level Estimation for OFDM in IoT –Based Networks: A Review

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Abstract: Internet of Things (IoT) is the idea to connect all devices to the internet. To implement such systems, we need to design low cost and less complex transmitters and make the receiver side complex. Now days OFDM is mainly used for communication due to its great advantages. But it faces the main problem such as PAPR due to the non-linear performance of High power amplifiers. There are so many methods are available to reduce the effect of PAPR in OFDM transmission, among this clipping is the simplest one. In this paper, we propose two algorithms to find the clipping level as well as the channel estimation. The efficiency of these algorithms is evaluated by using CLRBS calculation.

Keywords: OFDM, PAPR, Clipping, Channel, Estimation.

INTRODUCTION

Orthogonal Frequency Division Multiplexing or OFDM is a modulation technique that is used for many wireless and telecommunications standards. It is a form of the multicarrier modulation method. An OFDM signal consists of a number of closely spaced modulated carriers. These carriers are mutually orthogonal. When modulation of any form - voice, data, etc. is applied to a carrier, then sidebands spread out either side. It is necessary for a receiver to be able to receive the whole signal to be able to successfully demodulate the data. As a result, when signals are transmitted close to one another they must be spaced so that the receiver can separate them using a filter and there must be a guard band between them. This is not the case with OFDM. Although the sidebands from each carrier overlap, they can still be received without the interference that might be expected because they are orthogonal to each another. This is achieved by having the carrier spacing equal to the reciprocal of the symbol period.

But this system suffers from envelope fluctuations producing a problem called Peak-to-Average Power Ratio (PAPR). This leads to an unwelcome tradeoff between the linearity of the transmitted signal and the cost of the High Power Amplifier (HPA). A high PAPR causes the HPA to work in the saturation region which introduces nonlinear distortion at the transmitter output.

PAPR (Peak-to-Average Power Ratio)

When the independently modulated subcarriers are added coherently, the instantaneous power will be more than the average power. High PAPR degrades the performance of OFDM signals by forcing the amplifier to work in a nonlinear manner, distorting this way the signal and making the amplifier to consume more power. The PAPR for the continuous-time signal $x(t)$ is the ratio of the maximum instantaneous power to the average power. For the discrete-time version $x[n]$, PAPR is expressed as,

$$\text{PAPR}(x[n]) = \max_{0 \leq n \leq N} \frac{|x(n)|^2}{E[|x(n)|^2]} \quad \text{or} \quad \frac{\max{|x(n)|^2}}{E[|x(n)|^2]}$$
The performance of a PAPR reduction scheme is usually described by three main factors: the complementary cumulative distributive function (CCDF), bit error rate (BER), and spectral spreading. There are also other factors to be considered such as transmitted signal power, computational complexity, and bandwidth expansion and data rate loss.

PAPR reduction techniques can be broadly classified into three main categories:
- Signal distortion techniques,
- Multiple signaling and probabilistic techniques and
- Coding techniques

**System Model**

Consider an OFDM system with N number of subcarriers, in which $S = [s_0, s_1, s_2, ..., s_{N-1}]^T$ is the frequency domain symbol vector selected from a constellation such as QAM. The time-domain symbols are obtained by taking the Inverse Discrete Fourier Transform (IDFT) from the frequency-domain symbols as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k e^{-j2\pi mn/N}, \ n = 0, 1, 2, ..., N-1 \quad (1)$$

Rewrite this equation in matrix form as:

$$x = F^H s \quad (2)$$
Where ‘F’ is the $N \times N$ unitary discrete Fourier transform matrix. The channel is slow fading with L+1 taps (L<<N), denoted as $h = [h_0, h_1, h_2, \ldots, h_L]^T$, $g(\cdot; A)$ is the limiter non linearity with the following amplitude modulation and phase modulation conversion characteristics:

$$F(r) = \begin{cases} r, & r \leq A \\ A, & A > A \end{cases}$$

$$\emptyset(r) = 0$$ (4)

Where $r$ is the magnitude of the limiter input signal, and $A$ is the CA. Combining AM/AM and AM/PM conversion characteristics, $g(u; A)$ as a function of complex scalar $u$ (and parameterized by $A$) can be written as,

$$g(u; A) = \begin{cases} u, & |u| \leq A \\ Ae^{j\arg(u)}, & |u| > A \end{cases}$$ (5)

The output of the limiter is $z = g(x; A)$, in which $g(\cdot; A)$ is taken element-wise. However, it is difficult to directly work with the output of the limiter, where the output of the limiter can be represented in a linear fashion by introducing $N$ augmented binary variables $c_n$ for $n \in \{0, 1, \ldots, N-1\}$ indicating whether the sample at time $n$ has been clipped ($c_n = 1$) or not ($c_n = 0$), i.e.

$$c_n = \begin{cases} 1, & r_n > A \\ 0, & r_n \leq A \end{cases}$$ (6)

Which lead to,

$$z_n = (1 - c_n) x_n + A c_n e^{j \theta_n}$$ (7)

It can be represented in vector form

$$z = (1 - c) \odot x + A c \odot e^{j \theta}$$ (8)

To remove the Inter-Symbol Interference (ISI), a Cyclic Prefix (CP) with a length Lcp ($\geq L$) is pre-added to the time-domain symbols at the transmitter and is removed at the receiver. After the process of adding and removing CP, the matrix form time-domain representation of the OFDM transmission can be written as:

$$u = Hz + w$$ (9)

Where $H$ is an $N \times N$ circulant matrix whose first column which represents the circular convolution operator, and ‘$w$’ is a zero-mean circularly symmetric complex Gaussian noise vector, Taking the DFT from both sides of (9), we obtain the frequency-domain representation of the OFDM transmission as:

$$y = Fu = \sqrt{N} D_H Fz + \tilde{w}$$ (10)

Where $D_H$ is a diagonal matrix with the N-point DFT of $h$ as its diagonal elements, $\tilde{w}$ is the DFT of time-domain noise vector. **Joint Channel and Clipping Amplitude Estimation**

Here propose an alternating optimization algorithm to jointly estimate the channel and CA. To do this, we use frequency-domain block-type training symbols. Once we have the estimates, we can use them to detect the transmitted symbols in the subsequent OFDM blocks. The intuition behind this algorithm is that CA is a slowly time-varying parameter (much slower than channel variations). Moreover, wireless channels are usually slow time varying, so the block-fading channel model, which remains the same during the transmission of several OFDM blocks is reasonable. Using frequency-domain block-type training symbols, the problem
of joint Maximum-Likelihood (ML) estimation of channel and CA can be formulated as the following Least- Squares (LS) using (10):

\[
\min_{A>0} \| y - \sqrt{N} D_H F z \|^2
\]  

(11)

Where \( \| . \| \) denotes the Euclidean norm.

A. Clipping Amplitude Estimation Given the Channel

First, solve (11) for A given h. To do so, we first sort the elements in \( r \) and construct a new vector denoted \( \phi^r \). Note that, \( \phi^r = Ps r \), in which \( Ps \) is the sorting permutation matrix. Using the sorted vectors, the estimate of CA at the \( i^{th} \) iteration can be written as,

\[
A = \operatorname{arg \ min}_{A>0} \| y - \sqrt{N} D_H F P_s \phi^r \|^2
\]  

(12)

Where

\[
z = x \odot (1 - \hat{e}) + A e^{j \phi} \odot \hat{e}
\]  

(13)

In which \( x = P_S x \), \( e = P_S e \), and \( \phi = P_S \phi \). Note that \( e \) is constant within each interval \( \hat{r}_k - 1 \leq A \leq \hat{r}_k \), \( k = 0 \ldots N - 1 \), consequently (13) is piecewise affine in A, therefore the minimization problem (12) can be written as

\[
\hat{A} = \operatorname{arg \ min}_{A>0} J(A)
\]  

(14)

Where

\[
J(A) = \begin{cases} 
J_0(A), & 0 \leq A \leq \hat{r}_0 \\
J_1(A), & \hat{r}_0 \leq A \leq \hat{r}_1 \\
J_{N-1}(A), & r_{N-2} \leq \hat{A} < r_{N-1}
\end{cases}
\]  

(15)

In which

\[
J_k(A) = \| y - \sqrt{N} D_H F P_s \phi^r \|^2
\]

\[
= \| B(C_k \odot e^{j \phi}) \|^2 A^2 - 2Re \left\{ y - B(\hat{x} \odot \bar{C}_k) \right\} \end{align}

(16)

As we can see, for each interval, (16) is a convex quadratic function of A. Consequently, for each interval, we can easily find the minimizer by taking the first derivative of (16) and setting it to zero, and check if the solution lies in that interval or not. If it lies in that interval we have already found the minimizer, otherwise, the minimum takes place at the interval’s end point with the smaller cost function. Mathematically, for each interval \([\hat{r}_k - 1, \hat{r}_k]\), the minimizer is

\[
\hat{A}_k = \begin{cases} 
\hat{A}_k, & r_{k-1} \leq \hat{A}_k \leq \hat{r}_k \\
\hat{r}_k - 1, & J_k(r_{k-1}) < J_k(\hat{r}_k) \\
\hat{r}_k, & otherwise.
\end{cases}
\]  

(17)

Where

\[
\hat{A}_k = \frac{Re \left\{ y - B(\hat{x} \odot \bar{C}_k) \right\} B(C_k \odot e^{j \phi})}{\| B(C_k \odot e^{j \phi}) \|^2}
\]  

(18)
To find the global minimizer, we find the minimizer for each interval using (17), and then find the minimizer among those minimizers. Since $J(A)$ is a continuous function, in each interval, we need just check the point at which the derivative is zero and the rightmost corner point. Therefore, we have

Algorithm 1. Estimation of Clipping Amplitude (A)

1. **Inputs:**
   - $y$, $r$ and $\exp(j\emptyset)$
   - $F$ and $D_H$
   - $c_k$ for $k = 0, 1, \ldots, N-1$

2. **Initialize:**
   - $\tilde{r}, P_s = \text{sort}(r)$
   - $e^{j\emptyset} = P_s e^{j\emptyset}$
   - $B = \sqrt{N} D_H FP_s^T$
   - $\tilde{r} - 1 = 0$

3. **for** $k = 0$ to $N-1$ **do**

4.  $\bar{A}_k = \frac{\|y - B(c_k \tilde{e}^{j\emptyset})^H B(c_k \tilde{e}^{j\emptyset})\|}{\|B(c_k \tilde{e}^{j\emptyset})\|}$

5. **if** $\tilde{r}_k - 1 < \bar{A}_k < \tilde{r}_k$ **then**

6.  $\hat{A}_k = \bar{A}_k$

7. **else**

8.  $\hat{A}_k = \tilde{r}_k$

9. **end if**

10. $j_k = J_k(\hat{A}_k)$

11. **end for**

12. $\hat{k} = \arg \min_{0 \leq k \leq N-1} j_k$

13. $\hat{A} = \hat{A}_k$

B. Channel Estimation gave Clipping Amplitude

Here, we solve (11) for $h$ given $A$. We can rewrite (11) as:

$$\min_h \|y - \sqrt{N} \text{diag}(Fz)\tilde{h}\|^2$$

(19)

Therefore, the LS channel estimate can be computed as:

$$\hat{h} = \frac{1}{\sqrt{N}} (\tilde{F}^H \text{diag}(Fz)^H \text{diag}(Fz)\tilde{F})^+ \tilde{F}^H \text{diag}(Fz)^H y$$

(20)

Where $(\cdot)^+$ denotes the Moore–Penrose pseudo-inverse.

C. Initialization of the Alternating Algorithm

Two alternative initialization strategies to the alternating optimization algorithm as follows:

1) Initializing by the Channel: Since the value of $A$ is unknown at the beginning, the channel can be estimated using the unclipped version of transmitted time-domain symbols. It is equivalent to putting $x$ instead of $z$ in (20), and using $s = Fx$. Therefore, the initializing channel estimate is:

$$\hat{h}^{(0)} = \frac{1}{\sqrt{N}} (\tilde{F}^H \text{diag}(s)^H \text{diag}(s)\tilde{F})^{-1} \tilde{F}^H \text{diag}(s)^H y$$

(21)
2) Initializing by the Clipping Amplitude: By substituting (20) into the cost function in (19), the resulting LS error for A is:

\[ \varepsilon(A) = y^H (I - T)y \]  

(22)

By minimizing (22) over A, the CA can be estimated. However, (22) is a complicated function of A and is not convex. Nevertheless, one can find its approximate solution by using grid search methods. Therefore, we can find the initializing CA by solving

\[ \hat{A}(0) = \arg \min_{A>0} \varepsilon(A) \]  

(23)

Due to the non-convexity of \( \varepsilon(A) \), it is highly possible that even more advanced optimization methods than grid search just find a local minimum.

D. Alternating Optimization Algorithm

Depending on which initialization is used, we have two alternating algorithms denoted as Algorithms 2 and 3:

E. Convergence

Note that both algorithms of alternating optimization are guaranteed to converge to a local optimum because in every iteration we can find the unique optimal solution to the optimization problem. In particular, we have the following proposition:

Proposition 1: Both algorithms 2 and 3 converge to a local optimum point of the cost function given in (11) (or (12)). Proof: let us denote the cost function in (11) by \( J(h; A) \), then for Algorithm 2, we have

\[ \hat{\lambda}^{(i-1)} = \arg \min_{A>0} J(\hat{h}^{(i-1)}, A) \]  

(24)

Algorithm 2 Alternating Optimization with Initializing Channel

1. Inputs:
   - y, r and \( \exp(j\emptyset) \)
   - \( F \)
   - \( c_k \) for \( k = 0, 1, \ldots N-1 \)

2. Initialize:
   \[ [\hat{r}, \hat{P}_s] = \text{sort}(r) \]
   \[ e^{j\emptyset} = P_s e^{j\emptyset} \]
   \[ \hat{D}_H^{(0)} = \text{diag} \left( F\hat{h}^{(0)} \right), \hat{h}^{(0)} \text{ is given by (21) } \]
   \[ \hat{r} - 1 = 0 \]
   \[ i = 1 \]

3. while(convergence criteria not met ) do

4. \[ B = \sqrt{N} \hat{D}_H^{(i-1)} F P_s^T \]

5. calculate \( \hat{\lambda}^{(i-1)} \) and \( \hat{h}^{(i-1)} \) using algorithm 1

6. \[ \hat{z} = \hat{z} \odot (1 - \hat{c}_{k(i-1)}) + \hat{\lambda}^{(i-1)} e^{j\emptyset} \odot \hat{c}_{k(i-1)} \]

7. \[ V = \text{diag} \left( F P_s^T \hat{z} \right) F \]

8. \[ \hat{h}^{(i)} = (V^H V)^{-1} V^H \hat{z} \]

9. \[ \hat{D}_H^{(i)} = \text{diag} \left( F \hat{h}^{(i)} \right) \]

10. \( i = i + 1 \)

11. end while

Therefore

\[ J(\hat{h}^{(i-1)}, \hat{\lambda}^{(i-1)}) \leq J(\hat{h}^{(i-1)}, A), \forall A > 0 \]  

(25)

Also, note that,
\[ \hat{h}^{(i)} = \arg \min_h J(h, \hat{A}^{(i-1)}) \]  

(26)

Hence

\[ J(\hat{h}^{(i)}, \hat{A}^{(i-1)}) \leq h, \hat{A}^{(i-1)}, \forall h \in \mathcal{C}_{L+1} \]  

(27)

Algorithm 3 Alternating Optimization with Initializing CA

1. **Inputs:**
   - \( y, r \) and \( \exp(j\emptyset) \)
   - \( F \)
   - \( c_k \) for \( k = 0, 1, \ldots, N-1 \)

2. **Initialize:**
   - \([\hat{r}, \hat{P}_s] = \text{sort}(r)\)
   - \( e^{j\emptyset} = P_s e^{j\emptyset} \)
   - \( \hat{A}^{(i)} \) by solving (23) using a grid search
   - \( \hat{r} - 1 = 0 \)
   - \( i = 1 \)

3. **while (convergence criteria not met) do**

4. \( V = \text{diag}(FP_s^{T}z^{(i-1)}) \)

5. \( h^{(i-1)} = (V^H V)^{1/2} y \)

6. \( \hat{B}_H^{(i-1)} = \text{diag}(Fh^{(i-1)}) \)

7. \( B = \hat{B}_H^{(i-1)}FP_s^T \)

8. calculate \( \hat{A}^{(i)} \) and \( \hat{h}^{(i)} \) using algorithm 1

9. \( i = i + 1 \)

10. **end while**

Therefore, the objective function is decreasing at each iteration and eventually converges to a local minimum. The same argument is also valid for Algorithm 3. For Symbol Error Rate (SER) simulations use the iterative detection algorithm introduced in, but with the estimated channel and CA as follows:

Algorithm 4 Iterative Detection Algorithm

1. **Inputs:**
   - \( y, F \), and \( N_q \)
   - \( \hat{h} \) and \( \hat{A} \) calculated by Algorithm 2 (or 3)

2. **Initialize**
   - \( \hat{D}_H = \text{diag}(F \hat{h}) \)

   \[ \begin{align*} 
   k &= 1 - e^{-A^2} + \hat{A} \sqrt{\frac{n}{2}} \text{erfc}(\hat{A}) \\
   d^{(0)} &= 0 
   \end{align*} \]

3. **for** \( i = 1 \) to \( N_q \) **do**

4. \( \hat{s}^{(i)} = \left( \frac{1}{\hat{r}} (\hat{D}_H^{-1} y - d^{(i-1)}) \right) \)

5. \( \hat{\chi}^{(i)} = F^H \hat{s}^{(i)} \)

6. \( d^{(i)} = F(g(\hat{\chi}^{(i)}; \hat{A}) - \hat{\chi}^{(i)}) \)
7. the end of

Cramer-Rao Lower Bound
The Cramér Rao Lower Bound (CRLB) expresses a lower bound on the achievable variance of unbiased estimators. The ML estimator asymptotically achieves the CRLB under some regularity conditions. Here, the estimation parameters comprise of complex-valued channel taps and a real-valued CA. Therefore, to calculate the CRLB, we denote the parameter vector as \( \theta = [h^T, h^H, A]^T \), which is used to derive the complex Fisher Information Matrix (FIM). The logarithm of the Probability Density Function (PDF) of the frequency domain observation vector \( y \) given in (10) can be written as:

\[
\log p(y; \theta) = -\frac{1}{\sigma^2} \| y - \sqrt{N} D_H F z \|^2 + \text{constant}
\]

Where constant comprises the terms which are independent of the estimation parameter vector \( \theta \) the log PDF is a differentiable function of ‘h’ in the whole parameter space.

\[
CLRB(A) = \frac{1}{q - 2P_H C_1^{-1}P}
\]

And

\[
CLRB(h) = C_1^{-1} + \frac{1}{q - 2P_H C_1^{-1}P} C_1^{-1} P P_H C_1^{-H}
\]

RESULTS

NMSE performance of the CA estimation using Algorithms 2 and 3, when \( L + 1 = 7 \), \( N = 128 \) and \( CL = 1 \) dB.

NMSE performance of the channel estimation using Algorithms 2 and 3, when \( L + 1 = 7 \) \( N = 128 \) and \( CL = 1 \) dB.

NMSE performance of the CA estimation using Algorithms 2 and 3, when \( L + 1 = 7 \), \( N = 512 \) and \( CL = 3 \) dB.

NMSE performance of the channel estimation using Algorithms 2 and 3, when \( L + 1 = 7 \) \( N = 512 \) and \( CL = 3 \) dB.
SER performance of the iterative detection (Algorithm 4) vs. SNR, when \( L + 1 = 7 \)
\[ N = 128 \text{ and } CL = 1 \text{ dB}. \]

SER performance of the iterative detection (Algorithm 4) vs. SNR, when \( L + 1 = 7 \)
\[ N = 512 \text{ and } CL = 1 \text{ dB}. \]

SER performance of the iterative detection (Algorithm 4) vs. SNR, when \( L + 1 = 7 \)
\[ N = 256 \text{ and } CL = 3 \text{ dB}. \]

SER performance of the iterative detection (Algorithm 4) vs. CL, when \( L + 1 = 7 \)
\[ N = 128 \text{ and } SNR = 20 \text{ dB}. \]

**CONCLUSION**

In this, we studied joint maximum-likelihood estimation of channel and clipping level at the receiver side in future IoT-based OFDM networks, where there are lots of low-cost low-power nodes transmitting to and receiving from more complex nodes such as a BTS. In particular, we have proposed two alternating optimization algorithms, in which we have optimally solved a non-smooth non-convex optimization problem. And also computed the theoretical lower bounds (CRLB) on the performance of these estimators, and showed that they attain these lower bounds. Next, we have combined the channel and the CA estimates with the iterative detection method from to perform symbol detection at the receiver. Finally, we have shown by simulations that the performance of the iterative detection method using the proposed algorithms is almost the same as the one of the cases that the receiver has genie-aided knowledge of the channel and CA.

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