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Radio Mean D-distance Number of Banana Tree, Thorn Star and Cone Graph

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Abstract: A Radio Mean D-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to ℕ such that for two distinct vertices u and v of G, $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G)$, where $d^D(u, v)$ denotes the D-distance between u and v and $diam^D(G)$ denotes the D-diameter of G. The radio mean D-distance number of f, $rmn^D(f)$ is the maximum label assigned to any vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the minimum value of $rmn^D(f)$ taken over all radio mean D-distance labeling f of G. In this paper we find the radio mean D-distance number of banana tree, thorn star and cone graph.

Keywords. D-distance, Radio D-distance number, Radio mean D-distance, Radio mean D-distance number.

INTRODUCTION

By a graph G = (V, E) we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively.

Let G be a connected graph of diameter d and let k an integer such that $1 \leq k \leq d$. A radio k-coloring of G is an assignment f of colors (positive integers) to the vertices of G such that $d(u, v) + |f(u) - f(v)| \geq 1 + k$ for every two distinct vertices u, v of G. The radio k-coloring number $rc_k(f)$ of a radio k-coloring f of G is the maximum color assigned to a vertex of G. The radio k-chromatic number $rc_k(G)$ is $\min\{rc_k(f)\}$ over all radio k-colorings f of G. A radio k-coloring f of G is a minimum radio k-coloring if $rc_k(f) = rc_k(G)$. A set S of positive integers is a radio k-coloring set if the elements of S are used in a radio k-coloring of some graph G and S is a minimum radio k-coloring set if S is a radio k-coloring set of a minimum radio k-coloring of some graph G. The radio 1-chromatic number $rc_1(G)$ is then the chromatic number $\chi(G)$. When k = Diam (G), the resulting radio k-coloring is called radio coloring of G. The radio number of G is defined as the minimum span of a radio coloring of G and is denoted as rn(G).

Radio labelling (multi-level distance labelling) can be regarded as an extension of distance-two labelling which is motivated by the channel assignment problem introduced by Hale [6]. Chartrand et al. [2] introduced the concept of radio labelling of the graph. Chartrand et al. [3] gave the upper bound for the radio number of Path. The exact value for the radio number of Path and Cycle was given by Liu and Zhu [10]. However, Chartrand et al. [2] obtained different values than Liu and Zhu [10]. They found the lower and upper bound for the radio number of Cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. M. M. Rivera et al. [21] gave the radio number of $C_n \times C_n$, the cartesian product of C_n. In [4] C. Fernandez et al. found the radio number for complete graph, star graph, complete bipartite graph, wheel graph and gear graph. M. T. Rahim and I. Tomescu [16] investigated the radio number of Helm Graph. The radio number for the generalized prism graphs were presented by Paul Martinez et.al. in [11].

The concept of D-distance was introduced by D. Reddy Babu et al. [17, 18, 19].If u, v are vertices of a connected graph *G*, the *D*-length of a connected u-v path *s* is defined as $\ell^D(s) = \ell(s) + \deg(v) + \deg(u) + \sum \deg(w)$ where the sum runs over all intermediate vertices *w* of *s* and $\ell(s)$ is the length of the path. The *D*-distance, $d^D(u, v)$ between two vertices u, v of a connected graph G is defined a $d^D(u, v) = \min\{\ell^D(s)\}$where the minimum is taken over all u-v paths *s* in *G*. In other words, $d^D(u, v) = \min\{\ell(s) + \deg(v) + \deg(u) + \sum \deg(w)\}$where the sum runs over all intermediate vertices *w* in *s* and minimum is taken over all u-v paths *s* in *G*.

In [12], we introduced the concept of Radio D-distance. The radio D-distance coloring is a function f: V(G) → ℕ ∪{0} such that $d^D(u, v) + |f(u) - f(v)| \geq diam^D(G) + 1$. It is denoted by $rn^D(G)$. A radio D-distance coloring f of G is a minimum radio D-distance coloring if $rn^D(f) = rn^D(G)$, where $rn^D(G)$ is called radio D-distance number.

Radio means labeling was introduced by R. Ponraj et al [13, 14, 15]. A radio mean labelling is a one to one mapping f from V(G) to ℕ satisfying the condition

$$d(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(G). \quad (1.1)$$

For every u, v ∈V(G). The span of a labelling f is the maximum integer that f maps to a vertex of G. The radio mean number of G, rmn (G) is the lowest span taken over all radio mean labelling's of the graph G. The condition (1.1) is called radio mean condition.

In [13], we introduce the concept of radio mean D-distance number. A radio mean D-distance labelling is a one to one mapping f from V (G) to ℕ satisfying the condition

$$d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam^D(G). \quad (1.2)$$

For every u, v ∈V(G). The span of a labelling f is the maximum integer that f maps to a vertex of G. The radio mean D-distance number of G, $rmn^D(G)$ is the lowest span taken over all radio mean D-distance labelling of the graph G. The condition (1.2) is called radio mean D-distance condition. In this paper we determine the radio mean D-distance number of banana tree thorn star and cone graph. The function f: V(G) → ℕ always represents injective map unless otherwise stated.

2. MAIN RESULTS

A banana tree is a tree obtained by connecting a new vertex to one leaf of each of any number of stars (x is not in any of the stars). The banana tree is denoted by $B(n, t_1, t_2, \ldots, t_n)$.

A uniform banana tree is a banana tree where $t_1 = t_2 = \ldots = t_n = t$, which is denoted by B(n, t), where $n \geq 2$ and $t \geq 2$. Hence, vertex set $V(B(n, t)) = \{x, x_i, u_{i,j} / i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, t\}$ and $E(B(n, t)) = \{x_i u_{i,j} / i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, t\} \cup \{x u_{i,t} / i = 1, 2, \ldots, n\}$.

Theorem 2.1. $rmn^D\, B(2, t) \leq \begin{cases} 18 \text{ if } n = 2, t = 2. \\ 3t + 11 \text{ if } n = 2, t \geq 3. \end{cases}$

Proof.

It is obvious that $diam^D\{B(2, t)\} = 2t + 14$. The vertices of $B(2, t) = \{x, x_i, u_{i,j} / i = 1, 2 \text{ and } j = 1, 2, \ldots, t\}$. Define the function f as $f(x) = 3t + 11$, $f(x_i) = 3t + 11 - i$, $f(u_{i,t}) = t + 11 - i$, $f(u_{i,j}) = 3t + 8 + i - 2j$. We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(G) + 1 = 2t + 15$, for every pair of vertices (u, v) where u ≠ v.

For $(x, u_{i,t-1})$, $d^D(x, u_{i,t-1}) + \left\lceil \frac{f(x)+f(u_{i,t-1})}{2} \right\rceil \geq t + 8 + \left\lceil \frac{3t+11+t+10+i}{2} \right\rceil \geq 2t + 15$.

For $(x, u_{i,t})$, $d^D(x, u_{i,t}) + \left\lceil \frac{f(x)+f(u_{i,t})}{2} \right\rceil \geq 5 + \left\lceil \frac{3t+11+t+11+i}{2} \right\rceil \geq 2t + 15$.

For any pair (x, x_i), $d^D(x, x_i) + \left\lceil \frac{f(x)+f(x_i)}{2} \right\rceil \geq t + 6 + \left\lceil \frac{3t+11+3t+11-i}{2} \right\rceil \geq 2t + 15$.

For any pair $(u_{1,t-1}, u_{1,t})$, $d^D(u_{1,t-1}, u_{1,t}) + \left\lceil \frac{f(u_{1,t-1})+f(u_{1,t})}{2} \right\rceil \geq t + 5 + \left\lceil \frac{t+11+t+10}{2} \right\rceil \geq 2t + 15$.

For $u_{1,t-1}$ and $u_{2,t-1}$, $d^D(u_{1,t-1}, u_{2,t-1}) + \left\lceil \frac{f(u_{1,t-1})+f(u_{2,t-1})}{2} \right\rceil \geq 2t + 14 + \left\lceil \frac{t+11+t+12}{2} \right\rceil \geq 2t + 15$.

For x_1 and x_2, $d^D(x_1, x_2) + \left\lceil \frac{f(x_1)+f(x_2)}{2} \right\rceil \geq 2t + 10 + \left\lceil \frac{3t+10+3t+9}{2} \right\rceil \geq 2t + 15$.

Therefore, $f(x) = 3t + 11$ is the largest label.

$$\text{rmn}^D\ B(2, t) \leq \begin{cases} 18\ if\ n = 2, t = 2. \\ 3t + 11\ if\ n = 2, t \geq 3. \end{cases} \quad ■$$

Theorem 2.2. rmn^D B(n, t) ≤ (n + 1)t + n + 8 if n ≥ 3,t ≥ 2.

Proof.
It is obvious that diam^D {B(n, t)}= 2t + n + 12. The vertices of B(n, t) = {x, x_i, $u_{i,j}$ / i = 1, 2, . . . , n and j = 1, 2, . . . , t}. Define the function f as f(x) = (n + 1)t + n + 8, f(x_i) = (n + 1)t + n + 8 – i, f($u_{i,t}$) = t + n + 8 – i, f($u_{i,j}$) = (n + 1)t + i + 7 – n(j – 1) . We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq \text{diam}^D(G) + 1 = 2t + n + 13$, for every pair of vertices (u, v) where u ≠ v.

For (x, $u_{i,t-1}$), $d^D(x, u_{i,t-1}) + \left\lceil \frac{f(x)+f(u_{i,t-1})}{2} \right\rceil \geq n + t + 6 + \left\lceil \frac{(n+1)t+n+8+2n+t+i+7}{2} \right\rceil \geq 2t + n + 13$.

For (x, $u_{i,t}$), $d^D(x, u_{i,t}) + \left\lceil \frac{f(x)+f(u_{i,t})}{2} \right\rceil \geq n + 3 + \left\lceil \frac{(n+1)t+n+8+t+n+8-i}{2} \right\rceil \geq 2t + n + 13$.

For any pair (x, x_i), $d^D(x, x_i) + \left\lceil \frac{f(x)+f(x_i)}{2} \right\rceil \geq n + t + 4 + \left\lceil \frac{(n+1)t+n+8+(n+1)t+n+8-i}{2} \right\rceil \geq 2t + n + 13$.

For any pair ($u_{1,t-1}$, $u_{1,t}$),
$d^D(u_{1,t-1}, u_{1,t}) + \left\lceil \frac{f(u_{1,t-1})+f(u_{1,t})}{2} \right\rceil \geq t + 5 + \left\lceil \frac{t+2n+8+t+n+7}{2} \right\rceil \geq 2t + n + 13$.

For $u_{1,t-1}$ and $u_{2,t-1}$, $d^D(u_{1,t-1}, u_{2,t-1}) + \left\lceil \frac{f(u_{1,t-1})+f(u_{2,t-1})}{2} \right\rceil \geq 2t + n + 12 + \left\lceil \frac{t+2n+8+t+2n+9}{2} \right\rceil \geq 2t + n + 13$.

For x_1 and x_2, $d^D(x_1, x_2) + \left\lceil \frac{f(x_1)+f(x_2)}{2} \right\rceil \geq 2t + n + 8 + \left\lceil \frac{(n+1)t+n+7+(n+1)t+n+6}{2} \right\rceil \geq 2t + n + 13$.

Therefore, f(x) = (n + 1)t + n + 8 is the largest label.
rmn^D B(n, t) ≤ (n + 1)t + n + 8 if n ≥ 2,t ≥ 2. ■

- Thorn stars are graphs obtained from a *n*-arm star by attaching *k*-1 terminal vertices to each of the star arms. (x is not in any of the stars). Thorn star is denoted by S(n, k). Hence, vertex set V{S(n, k)} = {x, x_i, $u_{i,j}$ / i = 1, 2, …, n and j = 1, 2, …, k} and E {S(n, k)} ={$x_iu_{i,j}$ / i = 1, 2, …, n and j = 1, 2, …, k} ∪ {xx_i / i = 1, 2, …, n}

Theorem 2.3. rmn^D S(n, k) ≤ (n + 1)k +2n + 1 if n ≥ 2, k ≥ 2.

Proof.
It is obvious that diam^D {S(n, k)}= $2\left(\frac{n}{2}\right)$ + k + 10(if n is an even) and diam^D {S(n, k)}= $2\left(\frac{n-1}{2}\right)$+ k + 11(if n is an odd) The vertices of S(n, k) = {x, x_i, $u_{i,j}$ / i = 1, 2, . . . , n and j = 1, 2, . . . , k}. S(n, k) has n(k + 1) + 1 vertices. Define the function f as f(x) = k + 4, f(x_i) = (n+ 1)k+ 2(n+1) – i, f($u_{i,j}$) = ik + 4 + j.

If n is an even, We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq \text{diam}^D(G) + 1 = 2\left(\frac{n}{2}\right) + k + 11$, for every pair of vertices (u, v) where u ≠ v.

For (x, $u_{i,j}$), $d^D(x, u_{i,j}) + \left\lceil \frac{f(x)+f(u_{i,j})}{2} \right\rceil \geq k + n + 4 + \left\lceil \frac{k+4+ik+4+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

For any pair (x, x_i), $d^D(x, x_i) + \left\lceil \frac{f(x)+f(x_i)}{2} \right\rceil \geq k + n + 2 + \left\lceil \frac{k+4+(n+1)k+2(n+1)-i}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

If x_i and $u_{i,j}$ are adjacent $d^D(x_i, u_{i,j}) + \left\lceil \frac{f(x_i)+f(u_{i,j})}{2} \right\rceil \geq k + 3 + \left\lceil \frac{(n+1)k+2(n+1)-i+ik+4+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

If x_i and $u_{i,j}$ are not adjacent
$d^D(x_i, u_{i,j}) + \left\lceil \frac{f(x_i)+f(u_{i,j})}{2} \right\rceil \geq 2k + n + 6 + \left\lceil \frac{(n+1)k+2(n+1)-i+ik+4+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

For any pair ($u_{1,j}$, $u_{1,j+1}$),
$d^D(u_{1,j}, u_{1,j+1}) + \left\lceil \frac{f(u_{1,j})+f(u_{1,j+1})}{2} \right\rceil \geq k + 5 + \left\lceil \frac{k+4+j+k+5+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

For $u_{1,j}$ and $u_{2,j}$, $d^D(u_{1,j}, u_{2,j}) + \left\lceil \frac{f(u_{1,j})+f(u_{2,j})}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11 + \left\lceil \frac{k+4+j+2k+4+j}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

For x_i and x_{i+1},
$d^D(x_i, x_{i+1}) + \left\lceil \frac{f(x_i)+f(x_{i+1})}{2} \right\rceil \geq 2k + n + 6 + \left\lceil \frac{(n+1)k+2(n+1)-i+(n+1)k+2(n+1)-i+1}{2} \right\rceil \geq 2\left(\frac{n}{2}\right) + k + 11$.

If n is an even, We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq \text{diam}^D(G) + 1 = 2\left(\frac{n-1}{2}\right) + k + 12$, for every pair of vertices (u, v) where u ≠ v.

For $(x, u_{i,j})$, $d^D(x, u_{i,j}) + \left\lceil \frac{f(x)+f(u_{i,j})}{2} \right\rceil \geq k + n + 4 + \left\lceil \frac{k+4+ik+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

For any pair (x, x_i), $d^D(x, x_i) + \left\lceil \frac{f(x)+f(x_i)}{2} \right\rceil \geq k + n + 2 + \left\lceil \frac{k+4+(n+1)k+2(n+1)-i}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

If x_i and $u_{i,j}$ are adjacent

$d^D(x_i, u_{i,j}) + \left\lceil \frac{f(x_i)+f(u_{i,j})}{2} \right\rceil \geq k + 3 + \left\lceil \frac{(n+1)k+2(n+1)-i+ik+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

If x_i and $u_{i,j}$ are not adjacent

$d^D(x_i, u_{i,j}) + \left\lceil \frac{f(x_i)+f(u_{i,j})}{2} \right\rceil \geq 2k + n + 6 + \left\lceil \frac{(n+1)k+2(n+1)-i+ik+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

For any pair $(u_{1,j}, u_{1,j+1})$,

$d^D(u_{1,j}, u_{1,j+1}) + \left\lceil \frac{f(u_{1,j})+f(u_{1,j+1})}{2} \right\rceil \geq k + 5 + \left\lceil \frac{k+4+j+k+5+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

For $u_{1,j}$ and $u_{2,j}$, $d^D(u_{1,j}, u_{2,j}) + \left\lceil \frac{f(u_{1,j})+f(u_{2,j})}{2} \right\rceil \geq 2k + 10 + \left\lceil \frac{k+4+j+2k+4+j}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

For x_i and x_{i+1},

$d^D(x_i, x_{i+1}) + \left\lceil \frac{f(x_i)+f(x_{i+1})}{2} \right\rceil \geq 2k + n + 4 + \left\lceil \frac{(n+1)k+2(n+1)-i+(n+1)k+2(n+1)-i+1}{2} \right\rceil \geq 2\left(\frac{n-1}{2}\right) + k + 12$.

Therefore, $f(x_1) = (n + 1)k + 2n + 1$ is the largest label.

$rmn^D S(n, k) \leq (n + 1)k + 2n + 1$ if $n \geq 2, k \geq 2$. ■

❖ A t-cone is a graph $C_n + tK_1$ obtained by attaching non-adjacent vertices, each with every vertex of the base cycle (rim).

Theorem 2.4. $rmn^D(C_n + tK_1) \leq 2n + t$ if $n \geq 3, t \geq 2$.

Proof.

It is obvious that $diam^D (C_n + tK_1) = t + 2n + 4$ (n is an odd) and $diam^D (C_n + tK_1) = 2t + 2n + 2$ (n is an even). The vertices of $(C_3 + tK_1) = \{ u_i, v_j / i = 1, 2, 3, \ldots, t$ and $j = 1, 2, 3, \ldots, n\}$. Define the function f as $f(u_i) = n + i$, $f(v_i) = t + n + i$.

If n is an odd, We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(C_3 + tK_1) + 1 = t + 2n + 5$, for every pair of vertices (u, v) where $u \neq v$.

For (u_i, u_j), $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 2n + t + 4 + \left\lceil \frac{n+i+n+j}{2} \right\rceil \geq t + 2n + 5$.

For (u_i, v_j), $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2n + t - 1 + \left\lceil \frac{3+i+t+3+j}{2} \right\rceil \geq t + 2n + 5$.

If v_i and v_j are adjacent

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 2t + 2 + \left\lceil \frac{t+n+i+t+n+j}{2} \right\rceil \geq t + 2n + 5$.

If v_i and v_j are not adjacent

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 6n + (n - 2)t - 14 + \left\lceil \frac{t+n+i+t+n+j}{2} \right\rceil \geq t + 2n + 5$.

If n is an even, We shall check the radio mean D-distance condition $d^D(u, v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq diam^D(C_3 + tK_1) + 1 = 2t + 2n + 3$, for every pair of vertices (u, v) where $u \neq v$.

For (u_i, u_j), $d^D(u_i, u_j) + \left\lceil \frac{f(u_i)+f(u_j)}{2} \right\rceil \geq 2n + 2t + 2 + \left\lceil \frac{n+i+n+j}{2} \right\rceil \geq 2t + 2n + 3$.

For (u_i, v_j), $d^D(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2n + t - 1 + \left\lceil \frac{n+i+t+n+j}{2} \right\rceil \geq 2t + 2n + 3$.

If v_i and v_j are adjacent

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq n + 2t + 2 + \left\lceil \frac{t+n+i+t+n+j}{2} \right\rceil \geq 2t + 2n + 3$.

If v_i and v_j are not adjacent

$d^D(v_i, v_j) + \left\lceil \frac{f(v_i)+f(v_j)}{2} \right\rceil \geq 6n + (n - 2)t - 14 + \left\lceil \frac{t+n+i+t+n+j}{2} \right\rceil \geq 2t + 2n + 3$.

Therefore, $f(v_n) = 2n + t$ is the largest label

$rmn^D(C_n + tK_1) \leq 2n + t$ if $n \geq 3, t \geq 2$. ■

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