INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact factor: 4.295
(Volume3, Issue6)
Available online at www.ijariit.com

Shape Optimization of Planar Mechanisms

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Abstract: This paper presents an optimization method to find link shapes for a dynamically balanced planar four-bar mechanism. The shaking force and shaking moment developed in the mechanism due to inertia are minimized by optimally distributing the link masses. The link shapes are then found using cubic B-spline curves and an optimization problem is formulated to minimize the percentage error in resulting links inertia values in which the control points of the B-spline curve are taken as the design variables. The effectiveness of the proposed method is shown by applying it to a numerical problem available in the literature.

Keywords: Dynamic Balancing; Equimomental System; Four-bar Mechanism; Genetic Algorithm; Shape Optimization.

1. INTRODUCTION

The four-bar mechanism is the basic mechanism used in the machines. When operated with high-speeds, these mechanisms transmit forces and moments to the ground known as shaking forces and shaking moments. These are defined as the vector sum of inertia forces and moments of all moving links of the mechanism. The dynamic performance of mechanism is adversely affected by vibrations, wear and noise associated with these forces and moments and several methods are presented in the literature [1-3] to reduce them. The complete force balancing in a mechanism is achieved by making the mass center of moving links stationary either by mass redistribution [4] or by adding counterweights [5].

The complete balancing of force increases the shaking moment, driving torque and bearing forces in joints of the mechanism [6]. The shaking moment is reduced along with the full force balancing using inertia or disk counterweights [7] and duplicate mechanism [8]. However, these methods are not preferred as they increase the complexity and mass in the mechanism. Some other methods are developed to minimize shaking force and shaking moment simultaneously through optimization [9-11]. These methods find the optimal distribution of link masses to reduce shaking force and shaking moment as they depend on link masses, their locations of CGs and moment of inertias. The mechanism balancing problem is presented as a multi-objective optimization problem and solved using evolutionary optimization techniques like particle swarm optimization (PSO) and genetic algorithm (GA) considering appropriate design constraints [12, 13].

Very few methods deal with finding the optimum shape of links corresponding to the balanced mechanism. The small element superposing method is used to form link shape considering the link as a combination of several small rectangular elements [14]. A shape optimization problem for mechanism links is formulated using external work done by a given external force as the objective function to maximize and volume of all the links is used as the constraint function [15]. The shape of a compliant mechanism is generated by topology optimization method using Cubic Bezier curves [16]. However, the dynamic balance for mechanisms is not considered in these methods. Also, these methods require an initial shape or design domain to start the procedure.

This paper presents an optimization problem formulation for balancing of the planar four-bar mechanism. The rigid links of the mechanism are presented as a dynamically equivalent system of point-masses, known as an *equimomental system* [11, 17]. This problem is presented as a multi-objective optimization problem to minimize both shaking force and shaking moment and solved using a genetic algorithm. For the resulting optimum inertial properties for the balanced mechanism, shapes of mechanism links are then found using cubic B-spline curves. The inertial properties of resulting link shapes are presented as constraints to keep the same as that of the balanced mechanism. The percentage error in resulting links inertia values is presented as the objective function which is minimized by taking the control points of the B-spline curve as design variables.

The structure of this paper is as follows. Section 2 presents definitions of shaking force and shaking moment for the mechanism. The procedure for link shape formation is presented in Section 3. The optimization problems for dynamic balancing and shape formation are formulated in Section 4. In Section 5, a numerical example is solved using the proposed method. Finally, conclusions are given in Section 6.

2. SHAKING FORCE AND SHAKING MOMENT

Figure 1 shows a planar four-bar mechanism where the fixed link is detached from the moving links to show the reactions. The shaking force is defined as the reaction of the vector sum of all the inertia forces whereas the shaking moment is the reaction of the resultant of the inertia moment and the moment of the inertia forces about a fixed point. Once all the joint reactions are determined, the shaking force and shaking moment at and about joint 1 are obtained as [11]:

$$\mathbf{f}_{sh} = -(\mathbf{f}_{01} + \mathbf{f}_{03}) \text{ and } n_{sh} = -(n_1^e + \mathbf{a}_0 \text{ x } \mathbf{f}_{03}) \quad (1)$$

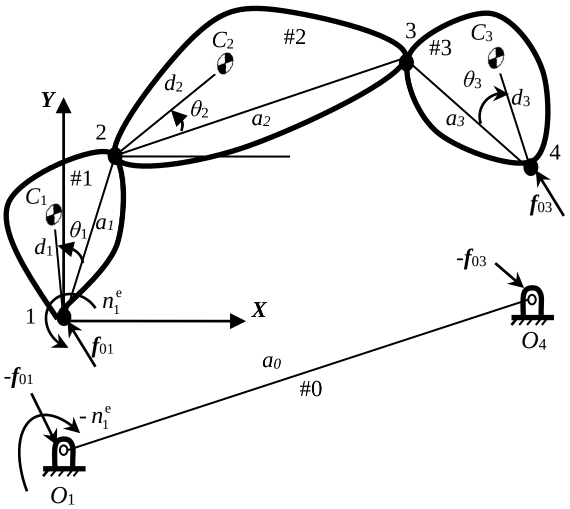


Fig. 1. Four-bar mechanism detached from its frame

In Eq. (1), $\mathbf{f}_{01}$ and $\mathbf{f}_{03}$ are the reaction forces of the frame on the links #1 and #3, respectively. The driving torque applied at joint #1 is represented by n_1^e while $\mathbf{a}_0$ represents the vector from O_1 to O_4.

3. LINK SHAPE FORMATION

The link shape is synthesized using parametric closed cubic B-spline curve which interpolates or approximates a set of n+1 control points, P_0, P_1,…, P_n [18] and defined in Eq. (2).

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(u), \quad 0 \le u \le u_{max} \quad (2)$$

In Eq. (2), the parameters k, $N_{i,k}(u)$ and u are defined as the degree of curve, B-spline blending function, and parametric knots, respectively. The cubic B-spline curve is a composite sequence of curve segments connected with C^2 continuity which blends two curve segments with the same curvature. The coordinates of any point on the ith segment of the curve is given as:

$$x_i(u) = \frac{\alpha_1 x_{i-1} + \alpha_2 x_i + \alpha_3 x_{i+1} + \alpha_4 x_{i+2}}{6} \quad (3)$$

$$y_i(u) = \frac{\alpha_1 y_{i-1} + \alpha_2 y_i + \alpha_3 y_{i+1} + \alpha_4 y_{i+2}}{6} \quad (4)$$

Where

$$\alpha_1 = -u^3 + 3u^2 i - 3ui^2 + i^3 \quad (5)$$

$$\alpha_2 = 3u^3 + u^2(3 - 9i) + u(-3 + 9i^2 - 6i) - 3i^3 + 3i^2 + 3i + 1 \quad (6)$$

$$\alpha_3 = -3u^3 + u^2(-6 + 9i) + u(-9i^2 + 12i) + 3i^3 - 6i^2 + 4 \quad (7)$$

$$\alpha_4 = u^3 + u^2(3 - 3i) + u(3 + 3i^2 - 6i) - i^3 + 3i^2 - 3i + 4 \quad (8)$$

In Eqs. (3-4), (x_{i-1} , y_{i-1}), (x_i , y_i), etc. are the coordinates of points P_{i-1}, P_i, etc., respectively. The inertial properties of the link synthesized using closed cubic B-spline curve are calculated using Green's theorem [19]. The area A, centroid ($\bar{x}$, $\bar{y}$) and area moment of inertia about centroidal axes (I_{xx}, I_{yy}, I_{zz}) of the closed curve made of n cubic B-spline segments are calculated as:

$$A = \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i(u) y_i'(u)\, du \quad (9)$$

$$\bar{x} = -\frac{1}{2A_i} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} y_i^2(u) x_i'(u)\, du \; ; \bar{y} = \frac{1}{2A_i} \sum_{i=1}^{n} \int_{u_{i-1}}^{u_i} x_i^2(u) y_i'(u)\, du \quad (10\text{-}11)$$

$$I_{xx}=-\frac{1}{3}\sum_{i=1}^{n}\int_{u_{i-1}}^{u_i} y_i^3(u)x_i'(u)\,du \quad ;I_{yy}=\frac{1}{3}\sum_{i=1}^{n}\int_{u_{i-1}}^{u_i} x_i^3(u)y_i'(u)\,du \quad ;I_{zz}=I_{xx}+I_{yy} \tag{12-14}$$

The first derivatives $x_i'(u)$ and $y_i'(u)$ of $x_i(u)$ and $y_i(u)$ w.r.t. *u*, respectively, in Eqs. (9-13) are given by:

$$x_i'(u)=\frac{\beta_1 x_{i-1}+\beta_2 x_i+\beta_3 x_{i+1}+\beta_4 x_{i+2}}{6} \tag{15}$$

$$y_i'(u)=\frac{\beta_1 y_{i-1}+\beta_2 y_i+\beta_3 y_{i+1}+\beta_4 y_{i+2}}{6} \tag{16}$$

Where

$$\beta_1=-3u^2+6ui-3i^2 \tag{17}$$

$$\beta_2=9u^2+2u(3-9i)-3+9i^2-6i \tag{18}$$

$$\beta_3=-9u^2+2u(-6+9i)-9i^2+12i \tag{19}$$

$$\beta_4=3u^2+2u(3-3i)+3+3i^2-6i \tag{20}$$

For geometric properties defined in Eqs. (9-14), the mass and mass moment of inertia of a link with shape represented by closed curve are calculated as:

$$m=At\rho \tag{21}$$

$$I=I_{zz}t\rho \tag{22}$$

Where t and ρ represent thickness and material density for the link, respectively.

4. OPTIMIZATION PROBLEM FORMULATION

4.1. Dynamic balancing

To dynamically balance the planar four-bar mechanism, an optimization problem is formulated to minimize the shaking force and shaking moment using the concept of the equimomental point-mass system. The links are systematically converted into a system of three equimomental point-masses and the point-mass parameters are taken as the design variables. A point mass is identified by three parameters, so 9-vector, x_i, of design variables for each link is defined as:

$$\mathbf{x}_i=[m_{i1}\quad l_{i1}\quad \theta_{i1}\quad m_{i2}\quad l_{i2}\quad \theta_{i2}\quad m_{i3}\quad l_{i3}\quad \theta_{i3}]^T \qquad \text{for } i=1, 2, 3 \tag{23}$$

Where m_{ij} is *j*th point mass of *i*th link, and l_{ij} and θ_{ij} are polar coordinates of it in the body-fixed frame. Hence, the design vector, **x**, for the mechanism is given by:

$$\mathbf{x}=[\mathbf{x}_1^{\mathrm{T}}\quad \mathbf{x}_2^{\mathrm{T}}\quad \mathbf{x}_3^{\mathrm{T}}]^T \tag{24}$$

Considering the RMS values of the magnitude of shaking force, $f_{\text{sh,rms}}$, and shaking moment, $n_{\text{sh,rms}}$, defined in Eq. (1), the optimization problem is posed as a weighted sum of the force and moment as:

$$\text{Minimize}\quad Z=w_1 f_{\text{sh,rms}}+w_2 n_{\text{sh,rms}} \tag{25}$$

$$\text{Subject to}\quad m_{\text{i,min}}\le\sum_j m_{ij}\le m_{\text{i,max}};\ \ I_{\text{i,min}}\le\sum_j m_{ij}l_{ij}^2 \qquad \text{for}\quad i=1, 2, 3 \text{ and } j=1, 2, 3 \tag{26}$$

Where w_1 and w_2 are the weighting factors used to assign weightage to shake force and shaking moment, respectively.

4.2. Shape optimization of mechanism links

After obtaining optimized inertial parameters of mechanism links, an optimization problem is formulated to find the corresponding link shapes in this section. The Cartesian coordinates of control points of the cubic B-spline curve are taken as design variables as shown in Fig. 2. The link length between joints origins O_i to O_{i+1} is divided into equal parts. To maintain symmetrical shape to have a product of inertia zero, *y* coordinates are taken as the design variables. The extensions of the link beyond joints origins O_i and O_{i+1} are controlled by P_0, P_1, P_{n-1} at right end and $P_{n/2-1}$, $P_{n/2}$, $P_{n/2+1}$ at left end. At right end, *x* coordinate of P_0, *y* coordinates of P_1 and P_{n-1} are chosen as the design variables and same is done at the left end. Finally, in this paper, the design vector is defined as:

$$\mathbf{x}=[x_0\ y_1\ldots y_{n/2-1}\ x_{n/2}\ y_{n/2+1}\ldots y_{n-1}]^{\mathrm{T}} \tag{27}$$

The objective function is formulated to minimize the percentage error in resulting links inertia values as:

$$\text{Minimize}\quad Z=\frac{(I_i^*-I_i)}{I_i^{o}}\,\text{x}\,100 \tag{28}$$

$$\text{Subject to}\quad m_i=m_i^*;\ \bar{x}_i=\bar{x}_i^*;\bar{y}_i=\bar{y}_i^* \qquad \text{for } i=1, 2, 3 \tag{29}$$

Here parameters with superscript '*' represent optimum parameters obtained in the previous section (4.1) and subscript '*i*' is used for the *i*th link of the mechanism.

5. NUMERICAL EXAMPLE

A numerical problem of planar four-bar mechanism [12] as shown in Fig. 1 is solved using the proposed method in this section (Table 1). As shaking force and shaking moment is of different units, these quantities need to be dimensionless for adding them in a single objective function. For this, the mechanism parameters are made dimensionless with respect to the parameters of the driving link, i.e., link #1.

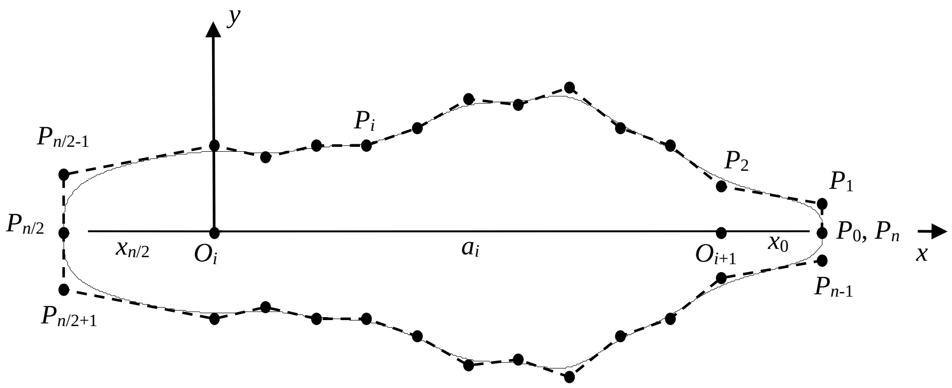


Fig. 2. Closed cubic B-spline curve representing link shape and its control points

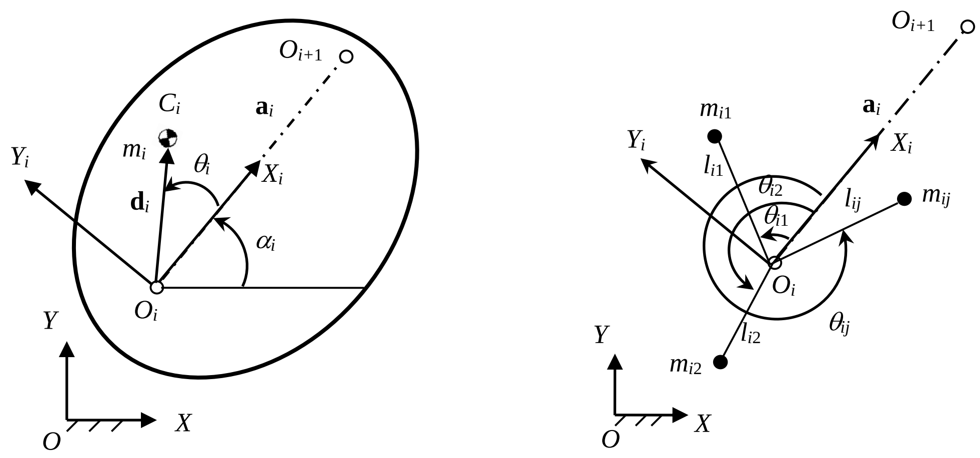
To reduce the dimension of the problem, out of nine variables, m_{ij}, l_{ij}, θ_{ij}, for j=1, 2, 3, for ith link as defined in Fig. 3, five parameters are chosen as:

$$\theta_{i1}=0;\ \theta_{i2}=2\pi/3;\ \theta_{i3}=4\pi/3 \text{ and } l_{i2}=l_{i3}=l_{i1} \qquad (30)$$

The other four point-mass parameters, namely, m_{i1}, m_{i2}, m_{i3}, l_{i1} are brought into the optimization scheme. Considering $m_{i,min} = 0.25 m_i^o$; $m_{i,max} = 5 m_i^o$ and $I_{i,min} = 0.25 I_i^o$ for ith link, the optimization problem as explained in Eqs. (25)-(26) is solved using "*ga*" function in *Genetic Algorithm and Direct Search Toolbox* of MATLAB. The superscript 'o' represents parameters for the original mechanism. The comparison of original values with optimum values of shaking force and shaking moment is presented in Table 2 and Fig. 4. The application of genetic algorithm results in a reduction of about 50% and 68% in the values of shaking force and shaking moment, respectively. The corresponding optimum parameters of the balanced mechanism are given in Table 1.

TABLE 1. PARAMETERS OF PLANAR FOUR-BAR MECHANISM

	Original mechanism					Balanced mechanism			
Link	Length a_i (m)	Mass m_i (kg)	Moment of inertia $I_{i,zz}$ (kg-m^2)	d_i (m)	θ_i (degree)	Mass m_i (kg)	Moment of inertia $I_{i,zz}$ (kg-m^2)	d_i (m)	θ_i (degree)
1	0.1	0.3925	0.0014	0.05	0	0.2459	5.2286e-4	0.0222	0
2	0.4	1.5700	0.0841	0.20	0	1.0440	0.0299	0.0594	0
3	0.3	1.1775	0.0356	0.15	0	0.6750	0.0115	0.0702	0
0	0.3	-	-						



(a) Rigid link **(b) Point-masses**

Fig. 3. Conversion of rigid link into equimomental system of point-masses

Table 2. The RMS values of dynamic quantities of standard and optimized mechanisms

	Shaking force	Shaking moment
Standard values	5.9604	10.7250
Optimized values	2.9620 (-50.31%)	3.4484 (-67.85%)

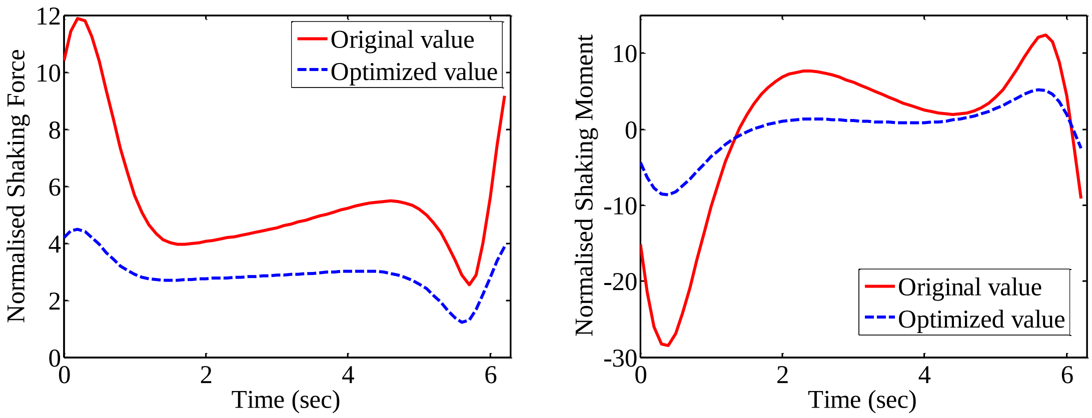


Fig. 4. Variations of shaking force and shaking moment for complete cycle

Next, the shape optimization problem formulated in Eqs. (28)-(29) is solved using "ga" function in Genetic Algorithm and Direct Search Toolbox of MATLAB. The resulting link shapes for cubic B-spline curve are shown in Fig. 5.

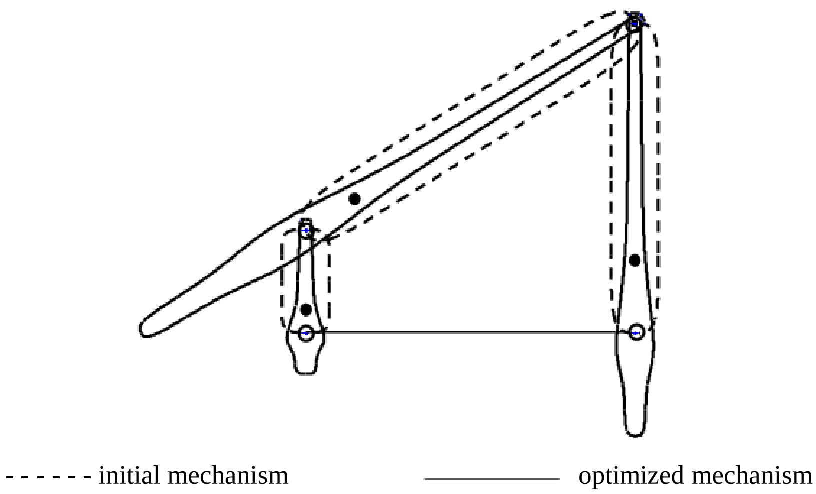


Fig. 5. Initial and optimized link shapes corresponding to balanced mechanism

The proposed method doesn't require any pre-defined shapes or design domain to start with. The percentage error of resulting inertia values was found within ± 5 percent. The resulting stresses for links of the balanced mechanism can be calculated at the weakest sections under external loads.

CONCLUSIONS

An optimization method for dynamic balancing and shape formation of a planar four-bar mechanism is presented in this paper. For dynamic balancing, the rigid links of the mechanism are represented by an equimomental system of point-masses. The shaking force and shaking moment are minimized through optimization of the mass distribution of link masses by taking point-mass parameters as the design variables. A reduction of 50% and 68% in shaking force and shaking moment is achieved by using the genetic algorithm as the solver. Then link shapes of the balanced mechanism are modeled as cubic B-spline curves. The boundary of the B-spline curve is optimized such that the link masses and inertias are the optimized values. For this purpose, control points act as the design variables. The constraints are defined to keep the resulting link parameters same as that of the balanced mechanism parameters. The methodology presented in this paper is simple and can be applied for multi-loop planar and spatial mechanisms also.

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