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Analysis of FM/FM/1queuing System with Pentagon Fuzzy Numbers and Using DSW Algorithm

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Abstract: This paper intends a method to construct the membership functions using DSW algorithms for the performance measures in queuing systems where the arrival rate and service rate are fuzzified. The performance measures of FM/FM/1 are analyzed mathematically using pentagon fuzzy numbers, and by applying DSW (Dong, Shah& Wong) algorithm where the arrival rate and service rate are fuzzy numbers. To determine the validity of the proposed approach numerical example is illustrated.

Keywords: Queuing Theory, α-cuts, Pentagon Fuzzy Numbers, Performance Measure, DSW Algorithm.

INTRODUCTION

Queuing models has much application including communication, Traffic Engineering, and computing. All probability queuing models have assumed poison input and exponential service times. In various real-world situations, conventions may be rather limiting, especially the assumption regarding service times being distributed exponentially. The fuzzy queues are much more truthful than the normally used crisp queues. In practice the arrival rate, service rate are commonly defined by a linguistic term such as fast, slow or moderate which can be best described by the fuzzy sets. Clearly, when the arrival and service rate is fuzzy, the performance measure of the queue also is fuzzy as well. Queuing models in Fuzzy have been described by researchers like Li and Lee [10], Buckley [1], Negi and Lee [6], Kao et al & Chan [2, 3], they have analyzed fuzzy queues using Zadeh's extraction principle, Zadeh [8].

Recently Chan [2, 3] developed (FM/FM/1) & (FM/FM/K) based on Zadeh's extension principle, analyzed results for two fuzzy queuing systems derived by [10] Li and Lee. But the performance measures are not completely described. Srinivasan [9] discussed DSW approach for M/M/1 model. This paper intends to follow α-cut approach to decompose a fuzzy queue into a family of crisp queues. When α various the DSW algorithm is used to describe the family of crisp queues. The solutions from the DSW algorithm derive the membership functions of the crisp queues.

FUZZY SET THEORY

The fuzzy set theory provides us not only with the meaningful and powerful representation measurement of uncertainties but also with a meaningful representation measurement of vague concepts expressed in natural languages. Because every crisp set is fuzzy set but not conversely, the mathematical embedding of conventional set theory into fuzzy sets is as natural as the idea of embedding the real numbers, into the complex plane. Thus the idea of fuzziness is one of enrichment, not of replacement.

Definition 2.1

Let z denote a universe of discourse. Then a fuzzy set $\tilde{A}$ in z is characterized by a membership function as follows. $\mu_{\tilde{A}} : z \to [0,1]$, which assigns to each element x in z, a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1]. Thus the function value of $\mu_{\tilde{A}}(x)$ is termed as the grade of membership of x in $\tilde{A}$.

Definition 2.2

If a fuzzy set $\tilde{A}$ is defined on x, for any $\alpha \in [0,1]$, the α-cuts of the fuzzy set $\tilde{A}$ is represented by

$\tilde{A}_\alpha = \left\{x / \mu_{\tilde{A}}(x) \geq \alpha, x \in Z\right\}$ = $\{l_{\tilde{A}}(\alpha), U_{\tilde{A}}(\alpha)\}$, $\tilde{A}_\alpha$ is a non-empty bounded interval contained in Z, $l_{\tilde{A}}(\alpha)$ and $U_{\tilde{A}}(\alpha)$ represent the lower and upper bound of α-cut of Ã respectively.

Definition 2.3

A fuzzy set Ã is convex subset of Z if and only if

$\mu_{\tilde{A}}(\lambda x_1+(1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1),\mu_{\tilde{A}}(x_2))$ for all $x_1, x_2 \in z$ and $\lambda \in [0,1]$.

TRAPEZOIDAL FUZZY NUMBER

The trapezoidal fuzzy number is usually defined as $\tilde{A} = [a_1, a_2, a_3, a_4]$ and its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \end{cases}$$

PENTAGON FUZZY NUMBER

A fuzzy number $\tilde{A} = [a_1, a_2, a_3, a_4, a_5]$ where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ is said to be a pentagon fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x-a_1}{a_2-a_1} & a_1 \leq x \leq a_2 \\ \dfrac{x-a_2}{a_3-a_2} & a_2 \leq x \leq a_3 \\ 1 & x = a_3 \\ \dfrac{a_4-x}{a_4-a_3} & a_3 \leq x \leq a_4 \\ \dfrac{a_5-x}{a_5-a_4} & a_4 \leq x \leq a_5 \\ 0 & x > a_5 \end{cases}$$

SOLUTION PROCEDURE

Let us consider a single server FM/FM/1 queuing system with first come first served discipline. The interval time Ã and service time $\tilde{S}$ are described by the following fuzzy sets.

$\tilde{A} = \{(a, \mu_{\tilde{A}}(a) / a \in x\}$

$\tilde{S} = \{(s, \mu_{\tilde{S}}(s) / s \in y\}$

Where x is the set of inter arrival time, y is the set of service time, $\mu_{\tilde{A}}(a)$ is membership function of inter arrival time and $\mu_{\tilde{S}}(s)$ is the membership function of service time,

The α-cuts of Ã and $\tilde{S}$ are

$A_\alpha = \{a \in x / \mu_{\tilde{A}}(a) \geq \alpha)$

$S_\alpha = \{s \in y / \mu_{\tilde{S}}(s) \geq \alpha)$

Using these α-cuts, we shall define the membership function of $p(\tilde{A}, \tilde{S})$ as follows.

This queue adopts a first-come-first served discipline and considers an infinite source population where both the arrival time and the service times follows poison and exponential distributions with parameters λ and μ respectively which are fuzzy variables rather than crisp values.

The expected number of customers in the system $N_s = \dfrac{\lambda}{\mu-\lambda}$

The expected waiting time in the system $W_s = \dfrac{1}{\mu-\lambda}$

The expected waiting time in the queue $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

When both inter-arrival time and service time are fuzzy numbers, based on Zadeh's extension principle, the membership function of the performance measure p(x,y) is defined as $\mu_{p(\tilde{A},\tilde{S})}(z) = \sup_{\substack{x\in x \\ y\in y}} \left\{ \min \mu_{\tilde{A}}(x), \mu_{\tilde{S}}(y) / z = p(x,y) \right\}$.

$l_{p(\alpha)} = \min p(x,y)$ Where $l_{\tilde{A}(\alpha)} \le x \le U_{A(\alpha)}$ and $l_{s(\alpha)} \le y \le U_{s(\alpha)}$.

$U_{p(\alpha)} = \max p(x,y)$ Where $l_{A(\alpha)} \le x \le U_{A(\alpha)}$ and $l_{s(\alpha)} \le y \le U_{s(\alpha)}$.

If both $l_{p(\alpha)}$ and $U_{p(\alpha)}$ are invertible with respect to α, then a left shape function L(z) = $l^{-1}_{p(\alpha)}$ and a right shape function R(z) = $U^{-1}_{p(\alpha)}$ can be obtained from which the membership function $\mu_{p(\tilde{A},\tilde{S})}$ is constructed.

$$\mu_{p(\tilde{A},\tilde{S})}(z) = \begin{cases} L(z), & z_1 \le z_2 \le z_3 \\ R(z), & z_3 \le z_4 \le z_5 \end{cases}$$

INTERVAL ANALYSIS ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

I_1 = [a, b], a < b
I_2 = [c, d], c < d

Define a general arithmetic property with a symbol∗, where ∗ = [+, -, ×, ÷] symbolically, The operation $I_1 * I_2$ = [a, b] ∗ [c, d] represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d.

$I_1 + I_2$ = [a + c, b + d]
$I_1 - I_2$ = [a - c, b - d]
$I_1 * I_2$ = [min (ac, ad, bc, bd), max (ac, ad, bc, bd)]
$I_1 \div I_2$ = [a,b]. $\left[\frac{1}{d}, \frac{1}{c}\right]$ provide dc ≠ 0

$$\alpha[a, b] = \begin{cases} \alpha a, \alpha b & \alpha > 0 \\ \alpha b, \alpha a & \alpha < 0 \end{cases}$$

DSW ALGORITHM

DSW (Dong, Shah, and Wong) algorithm is one of the appropriate methods to make use of intervals at various α-cut levels in defining membership functions. The DSW algorithm streamlines the manipulation of the extension principle for continuous-valued fuzzy variables. Such a fuzzy numbers defined on the real line. It avoids the irregularity in the output membership function due to the application of the discrimination each of the domain of the fuzzy variable, it can prevent the spreading of resulting functional expression by conventional interval analysis method. Any membership function which is continuous curve of α-cut of N terms α = 0 to α = 1. Suppose we have single input mapping given by y = f(x) that is to be extended for membership function for the selected α-cut level.

The DSW algorithm consists of the following steps.

1. Choose the value of the α cut in [0, 1].
2. Find the intervals in the input membership function that corresponds to this α.
3. Using standard binary interval operations, calculate the interval for the output membership function for the selected α-cut level.
4. Continue the steps 1-3 for various values of α to complete α-cut representation of the solution.

NUMERICAL EXAMPLE

Consider an integrated system in which the service consists of two phase's arrival and service. Both the arrival rate and service rate are pentagon fuzzy numbers denoted by $\tilde{\lambda}$ = [1,2,3,4,5] and $\tilde{\mu}$ = [7,8,9,10,11] per minute respectively. The confidence interval at α are [1 + 2α, 5 - 2α] and [7 + 2α, 11 - 2α].

The expected number of customers in system $N_s = \frac{x}{y-x}$

The expected waiting time in the system $W_s = \frac{1}{y-x}$

The expected waiting time in queue $W_s = \frac{x}{y(y-x)}$

Where x = [1 + 2α, 5 - 2α] and y = [7 + 2α, 11 - 2α].

MEMBERSHIP FUNCTIONS

Membership Function of N_s

$l_{N_s}(\alpha) = \min \frac{x}{y-x}$ such that

$1 + 2\alpha \le x \le 5 - 2\alpha,\ 7 + 2\alpha \le y \le 11 - 2\alpha$ ... (1)

$U_{N_s}(\alpha) = \max \frac{x}{y-x}$ such that

$1 + 2\alpha \le x\ 5 - 2\alpha,\ 7 + 2\alpha \le y \le 11 - 2\alpha$ ... (2)

When x reaches minimum and y reaches maximum $\frac{x}{y-x}$ attains minimum, on the other hand it attains maximum.

Consequently the optimal solution for

(1) is $l_{N_s}(\alpha) = \frac{1+2\alpha}{10-4\alpha}$ and

(2) is $U_{N_s}(\alpha) = \frac{5-2\alpha}{2+4\alpha}$

$l_{N_s}(\alpha)$ is invertible

Let Z = $\frac{1+2\alpha}{10-4\alpha}$

$\alpha = \frac{10Z-1}{4Z+2}$

$\alpha \ge 0 \Rightarrow Z \ge 0.1$

$\alpha \le 1 \Rightarrow Z \le 0.5$

$U_{N_s}(\alpha)$ is invertible

Let Z = $\frac{5-2\alpha}{2+4\alpha}$

$\alpha = \frac{5-2Z}{2+4Z}$

$\alpha \ge 0 \Rightarrow Z \le 2.5$

$\alpha \le 1 \Rightarrow Z \ge 0.5$

So $\mu_{N_s}(Z) = \begin{cases} \frac{10Z-1}{4Z+2} & 0.1 \le Z \le 0.5 \\ \frac{5-2Z}{2+4Z} & 0.5 \le Z \le 2.5 \end{cases}$

Membership Function of W_s

$l_{W_s}(\alpha) = \min \frac{1}{y-x}$ such that

$1 + 2\alpha \le x \le 5 - 2\alpha,\ 7 + 2\alpha \le y \le 11 - 2\alpha$ ... (3)

$U_{W_s}(\alpha) = \max \frac{1}{y-x}$ such that

$1 + 2\alpha \le x \le 5 - 2\alpha,\ 7 + 2\alpha \le y \le 11 - 2\alpha$ ... (4)

When x reaches minimum and y reaches maximum $\frac{1}{y-x}$ attains minimum, on the other hand it attains maximum.

Consequently the optimal solution for

(3) is $l_{W_s}(\alpha) = \frac{1}{10-4\alpha}$ and

(4) is $U_{W_s}(\alpha) = \frac{1}{2+4\alpha}$

$l_{W_s}(\alpha)$ is invertible

Let Z = $\frac{1}{10-4\alpha}$

$$\alpha = \frac{10Z - 1}{4Z}$$

$\alpha \geq 0 \Rightarrow Z \geq 0.1$

$\alpha \leq 1 \Rightarrow Z \leq 0.1666$

$U_{W_s}(\alpha)$ is invertible

Let Z = $\frac{1}{2 + 4\alpha}$

$$\alpha = \frac{1 - 2Z}{4Z}$$

$\alpha \geq 0 \Rightarrow Z \leq 0.5$

$\alpha \leq 1 \Rightarrow Z \geq 0.1666$

$$\text{So } \mu_{W_s}(Z) = \begin{cases} \frac{10Z - 1}{4Z} & 0.1 \leq Z \leq 0.1666 \\ \frac{1 - 2Z}{4Z} & 0.1666 \leq Z \leq 0.5 \end{cases}$$

Membership Function of W_q

$l_{W_q}(\alpha) = \min \frac{x}{y(y - x)}$ such that

$1 + 2\alpha \leq x \leq 5 - 2\alpha,\ 7 + 2\alpha \leq y \leq 11 - 2\alpha$ ... (5)

$U_{W_q}(\alpha) = \max \frac{x}{y(y - x)}$ such that

$1 + 2\alpha \leq x \leq 5 - 2\alpha,\ 7 + 2\alpha \leq y \leq 11 - 2\alpha$ ... (6)

When x reaches minimum and y reaches maximum $\frac{x}{y(y - x)}$ attains minimum, on the other hand it attains maximum.

Consequently the optimal solution for

(5) is $l_{W_q}(\alpha) = \frac{1 + 2\alpha}{8\alpha^2 - 64\alpha + 110}$ and

(6) is $U_{W_q}(\alpha) = \frac{5 - 2\alpha}{8\alpha^2 + 32\alpha + 14}$

$l_{W_q}(\alpha)$ is invertible

Let Z = $\frac{1 + 2\alpha}{8\alpha^2 - 64\alpha + 110}$ and

$$\alpha = \frac{(64Z + 2) \pm (576Z^2 + 288Z + 4)^{1/2}}{16Z}$$

$\alpha \geq 0 \Rightarrow Z \geq 0.0090$

$\alpha \leq 1 \Rightarrow Z \leq 0.0556$

$U_{W_q}(\alpha)$ is invertible

Let Z = $\frac{5 - 2\alpha}{8\alpha^2 + 32\alpha + 14}$ and

$$\alpha = \frac{-(32Z + 2) \pm (576Z^2 + 288Z + 4)^{1/2}}{16Z}$$

$\alpha \geq 0 \Rightarrow Z \leq 0.357$

$\alpha \leq 1 \Rightarrow Z \geq 0.0556$

$$\text{So } \mu_{W_q^z} = \begin{cases} \frac{(64Z + 2) \pm (576Z^2 + 288Z + 4)^{1/2}}{16Z} & 0.0090 \leq Z \leq 0.0555 \\ \frac{-(32Z + 2) \pm (576Z^2 + 288Z + 4)^{1/2}}{16Z} & 0.0555 \leq Z \leq 0.357 \end{cases}$$

Table 1: α-cuts of N_s, W_s, W_q at 11 distinct α-values

a	$x_a^{\ L}$	$x_a^{\ U}$	$y_a^{\ L}$	$y_a^{\ U}$	$(N_s)_a^L$	$(N_s)_a^U$	$(W_s)_a^L$	$(W_s)_a^U$	$(W_q)_a^L$	$(W_q)_a^U$
0	1.0	5	7.0	11.0	0.1000	2.500	0.1000	0.5000	0.0091	0.3571
0.1	1.2	4.8	7.2	10.8	0.1250	2.000	0.1042	0.4167	0.0116	0.2778
0.2	1.4	4.6	7.4	10.6	0.1522	0.6429	0.1471	0.3571	0.0144	0.2220
0.3	1.6	4.4	7.6	10.4	0.1818	1.3750	0.1493	0.3125	0.0175	0.1809
0.4	1.8	4.2	7.8	10.2	0.2143	1.1667	0.1515	0.2778	0.0210	0.1496
0.5	2.0	4.0	8.0	10.0	0.2500	1.0000	0.1538	0.2500	0.0250	0.1250
0.6	2.2	3.8	8.2	9.8	0.2895	0.8636	0.1563	0.2273	0.0295	0.1053
0.7	2.4	3.6	8.4	9.6	0.3333	0.7500	0.1587	0.2083	0.0347	0.0893
0.8	2.6	3.4	8.6	9.4	0.3824	0.6538	0.1613	0.1923	0.0407	0.0760
0.9	2.8	3.2	8.8	9.2	0.4375	0.5714	0.1639	0.1786	0.0476	0.0649
1.0	3.0	3.0	9.0	9.0	0.5000	0.5000	0.1667	0.1667	0.0556	0.0556

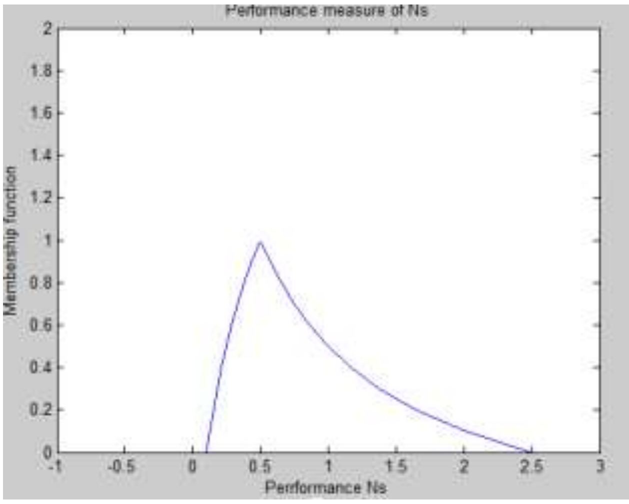

Fig. 1 Membership of N_s

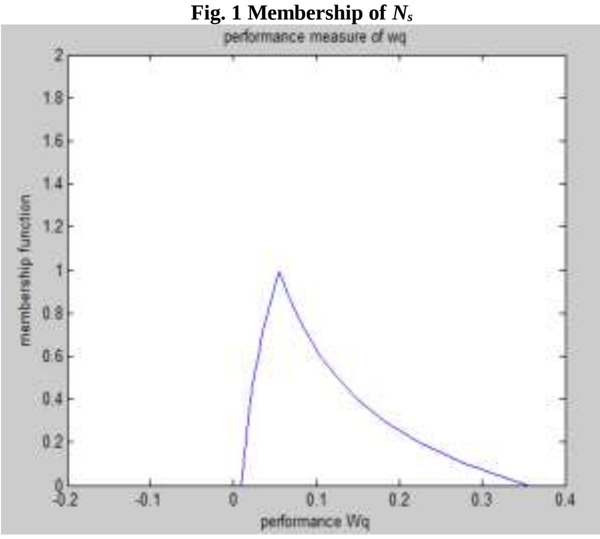

Fig. 2 Membership of W_q

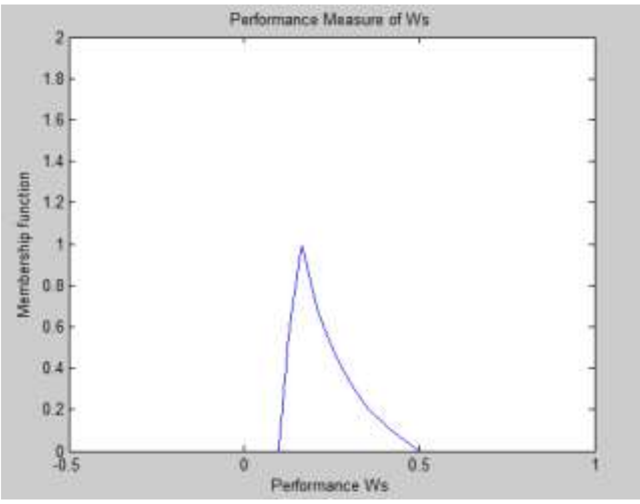

Fig. 3 Membership of W_s

With the help MATLAB software, we perform α-cuts of arrival rate, service rate, expected the number of jobs in the system, expected waiting time in the system and expected waiting time in queue at 11 distinct values of α, 0, 0.1, 0.2, ..., 1.0. Crisp intervals, for different performance measure at the different possibility α levels are presented in the table. The α-cut represents the possibility that these performance measures will lie in the associated range. Especially for $\alpha = 0$, the range the performance measures could appear and for $\alpha = 1$, the range the performance measures are likely to be. For example, while these performance measures are fuzzy, the most likely value of the expected numbers of jobs in the system N_s is 0.5000 and its value is impossible to fall outside the range [0.1000, 2.5000]. Similarly, for waiting time in system W_s, the most likely value is 0.1667 and it is impossible to fall outside the range [0.1000, 0.5000]. The above information will be highly useful for the designer for designing a queuing system and for their decision-making process.

CONCLUSION

The concept of fuzzy has been applied in the queuing system to provide a wider application in different areas. The inter-arrival time and service time are fuzzified. Analytical method and DSW algorithm are used to determine to be performance measures such as expected number of jobs in the system, waiting time in the system and waiting time in the queue which is also fuzzy. A numerical example is exemplified for the validity of the proposed approach.

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