INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact factor: 4.295
(Volume3, Issue4)
Available online at www.ijariit.com

Study on FM/FM/1 queuing system with Pentagon Fuzzy Number using α cuts

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Abstract: This paper Studies on FM/FM/1 queueing system with Pentagon fuzzy numbers using α cut method. The arrival rate and service rate are fuzzy natures and also analyzed the performance measure in Pentagon fuzzy numbers. The numerical example is illustrated that it shows the efficiency of the system.

Keywords: Membership Function, Pentagon Fuzzy Number, α cuts, Interval Analysis and DSW Algorithm.

INTRODUCTION

Queueing models have great extent applications in service organizations as well as manufacturing firms. Various customers get service by different types of serve's according to specific queue discipline within the context of queuing theory, the inter arrival time and service times are required to follow certain distributions. Fuzzy Logic was initiated in 1965 by Zadeh[4]. Fuzzy queuing model was first introduced by R.J.Lie and E.S.Lee[5] in 1989, further developed this model by many authors namely J.J.Buckely[1] in 1990, R.S.Negi and E.S.Lee[6] in 1992, S.P.Chen in 2005 and R. Srinivasan[7]in 2014, S. Shanmugasundaram and BB. Venkatesh[8] in 2015.

Also fuzzy queuing models have been described by such researchers like Chen[2,3] analyzed bulk arrival queuing model with fuzzy parameters and varying batch sizes using Zadeh's extension principle and recently he developed (FM/FM/1): (α/FCFS) and (FM/FM(k)/1): (α/FCFS) queueing models. S. Thamotharan[9] studied multi server queueing model in triangular and trapezoidal fuzzy numbers using α cuts in 2016. In practical, the queuing model, the input data arrival rate, service rates are uncertainly known. Uncertainly is resolved by using fuzzy set theory. Hence the classical queuing model has more applications if it is expanded using fuzzy models.

Here the parameters, fuzzy arrival rate and fuzzy service rate are best described by linguistic terms very high, high, low, very low and moderate.

FUZZY SET THEORY

Fuzzy set theory provides us not only with the meaningful and powerful representation of measurement of uncertainties, but also with a meaningful representation of measurement of vague concepts expressed in natural languages. Because every crisp set is fuzzy set but not conversely, the mathematical embedding of conventional set theory into fuzzy sets is as natural as the idea of embedding the real number, into complex plane. Thus the idea of fuzziness is one of enrichment, not of replacement.

2.1 Fuzzy set

A fuzzy set A in X is characterized by its membership function A: X→[0, 1]. Here X is a non-empty set.

2.2 α- cut

An α- cut of a fuzzy set $\tilde{A}$ is a crisp set A_α that contains all the elements of the universal set X that have a membership grade in A greater than or equal to the specified value of α. Thus

$A_\alpha = \{x \in X : \mu_{\tilde{A}}(x) \geq \alpha$,0≤α≤1.

2.3 Pentagon Fuzzy Number with Membership Function: A fuzzy number $\tilde{A} = (a,b,c,d,e)$ where a ≥ b ≥ c ≥ d ≥ e is said to be a Pentagon fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & for\ \ x < a \\ \dfrac{x-a}{b-a}, & for\ \ a \le x \le b \\ \dfrac{x-b}{c-b}, & for\, b \le x \le c \\ 1, & x = c \\ \dfrac{d-x}{d-c}, & for\ \ c \le x \le d \\ \dfrac{e-x}{e-d}, & for\ d \le x \le e \\ 0, & if\ \ x > e \end{cases}$$

SOLUTION PROCEDURE

Let us consider a single server FM/FM/1 queuing system first come first served discipline. The inter arrival time A and service times S are described by the following fuzzy sets:

$$A = \{(a, \tilde{\mu}_A(a) / a \in X\}$$

$$S = \{(s, \tilde{\mu}_S(s) / s \in Y\}$$

Here X is the set of the inter arrival time and Y is the set of the service time.

$\tilde{\mu}_A(a)$ is membership function of the inter arrival time

$\tilde{\mu}_S(S)$ is membership function of the service time

The α cuts of inter arrival time, service time are represented as

$$A(\alpha) = \{(a \in X /, \tilde{\mu}_A(\alpha) \ge \alpha\}$$

$$S(\alpha) = \{(s \in Y /, \tilde{\mu}_S(S) \ge \alpha\}$$

Using these α cuts we have to define the membership function P(A, S) as follows:

$$\mu_{P(A,S)}(x) = \begin{cases} 0, & for\ \ x < a \\ \dfrac{x-a}{b-a}, & for\ \ a \le x \le b \\ \dfrac{x-b}{c-b}, & for\, b \le x \le c \\ 1, & x = c \\ \dfrac{d-x}{d-c}, & for\ \ c \le x \le d \\ \dfrac{e-x}{e-d}, & for\ d \le x \le e \\ 0, & if\ \ x > e \end{cases}$$

This queue adopts a first-come first served discipline and consider an infinite source population where both the arrival time and the service times follows Poisson and exponential distributions with parameters λ and μ respectively, which are fuzzy variables rather than crisp values.

The expected number of customers in the queue

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

INTERVAL ANALYSIS FOR ARITHMETIC

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

I_1=[a, b], a ≤ b; I_2=[c, d], c ≤ d.

Define a general arithmetic property with the symbol*, where *=[+,-,×,÷] symbolically the operation I_1*I_2=[a, b]*[c, d] represents another interval. The interval calculation depends on the magnitudes and signs of the element a, b, c, d

[a, b]+[c, d]=[a + c, b + d]

[a, b]-[c, d]=[a - d, b - c]

[a, b]*[c, d]=[min(ac, ad, bc, bd), max(ac, ad, bc, bd)]

[a, b]÷[c, d]=[a , b]*[1/d, 1/c] provided that 0 does not belong to [c,d]

α[a, b]=[α a, α b] for α>0 and [α b, α a] for α<0.

DSW ALGORITHM

DSW(Dong, Shah and Wong) is one of the approximate methods make use of intervals at various α- cut levels .The DSW algorithm consists of the following steps:

1) Select α cut value where 0 ≤ α ≤1.
2) Find the intervals in the input membership functions that correspond to this α.
3) Using standard binary interval operations compute the interval for the output membership function for the selected cut level.
4) Repeat steps 1-3 for different values of α to complete α cut representation of the solution.

6. Numerical example

Consider a FM/FM/1 queue where the both arrival rate and service rate are fuzzy numbers represented by $\lambda = [1\ \ 2\ 3\ \ 4\ \ 5]$ and $\mu = [15\ \ 16\ \ 17\ \ 18\ \ 19]$. The interval of confidence level α as [1+2α, 5-2α] and [15+2α, 19-2α].

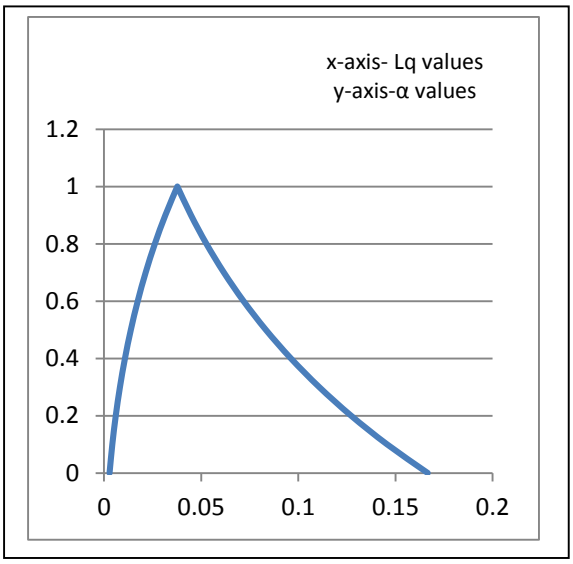
$$L_q = \frac{x^2}{y(y-x)}$$

Where x=[1+2α, 5-2α] and y=[15+2α, 19-2α]

By taking α values from 0, 0.1, . . . 1 the following results are obtained as shown in Table1.

Table1: The α cuts of L_q

α	Lq	Lq
0	0.002924	0.166667
0.1	0.004352	0.145749
0.2	0.006127	0.127225
0.3	0.008282	0.110806
0.4	0.010855	0.096246
0.5	0.013889	0.083333
0.6	0.01743	0.071884
0.7	0.021531	0.061738
0.8	0.02625	0.052756
0.9	0.031654	0.044818
1	0.037815	0.037815



We performed α cuts of arrival rate, service rate and fuzzy expected number of jobs in queue at eleven distinctα levels: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in Table-.

The α-cut represent the possibility that these of four presentation measure will lie in the associated range. Specially, α=0 the range, the presentation measures could appear and for α=1 the range. The most likely value of the expected queue length L_q falls at 0.0378 and its value is impossible to fall outside the range of 0.0029 and 0.1666.

These above results are very useful for defining a queuing system.

CONCLUSION

In this paper the performance of FM/FM/1 pentagon fuzzy number is studied. The interarrival time and service times are fuzzy nature. The performance of this system is also fuzzy nature. Numerical example shows that the efficiency of this system.

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