INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact factor: 4.295
(Volume3, Issue4)
Available online at www.ijariit.com

Between Closed Sets and ω-Closed Sets in Topological Spaces

C. Rajan
Vivekananda College
rajanc2012@gmail.com

***Abstract:** Sheik John [24] (Veera Kumar [26]) introduced the notion of ω-closed sets (=$\hat{g}$-closed sets). Many variations of ω-closed sets were introduced and investigated. In this paper, we introduce the notion of mω-closed sets and obtain the unified characterizations for certain families of subsets between closed sets and ω-closed sets.*

***Keywords:** ω-closed set, m-structure, m-space and mω-closed. AMS Mathematics Subject Classification: 54A05, 54D15, 54D30.*

1. INTRODUCTION

In 1970, Levine [10] introduced the notion of generalized closed (g-closed) sets in topological spaces. Recently, many variations of g-closed sets are introduced and investigated. In this paper, we introduce the notion of *m*ω-closed sets and obtain the basic properties, characterizations and preservation properties. In the last section, we define several new subsets which lie between closed sets and *m*ω-closed sets.

2. PRELIMINARIES

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively. A subset A of a space (X, τ) is an α-open [19] (resp. preopen [15]) set if $A \subset$ int(cl(int(A))) (resp. $A \subset$ int(cl(A))). The family of all α-open sets in (X, τ), denoted by τ^{α} , is a topology on X finer than τ . The closure of a subset A in (X, τ^{α}) is denoted by $cl_{\alpha}(A)$.

Definition 2.1 A subset A of a topological space (X, τ) is said to be

(a) semi open [11] if A$\subset$cl(int (A)).

(b) semi preopen [5] if A$\subset$cl(int(cl (A))).

The complement of semi-open (resp. semi preopen) set is said to be semi closed (resp. semi-preclosed). The family of all semi open (resp. semi preopen) sets in X is denoted by SO(X) (resp . SPO(X)). The semi closure of A [3] (resp. semi preclosure of A [5]), denoted by scl(A) (resp. spcl(A)), is defined by

scl(A) = ∩{ F: A$\subset$F, X – F ϵ SO(X)},

spcl(A) = ∩{ F: A$\subset$F, X – F ϵ SPO(X)}.

Definition 2.2. A subset A of a topological space (X, τ) is said to be g-closed set [10] if cl(A)$\subset$U whenever A$\subset$U and U is open in X.

Definition 2.3. A subset A of a topological space (X, τ) is said to be ω-closed set [24] (or $\hat{g}$-closed set [30]) if cl(A)$\subset$U whenever A$\subset$U and U is semi-open in X. The complement of ω-closed set is said to be ω-open in X.

Definition 2.4. A subset A of a topological space (X, τ) is said to be*g-closed set [28] if $cl(A) \subset U$ whenever $A \subset U$ and U is ω-open in X. The complement of *g-closed set is said to be *g-open in X.

Definition 2.5. A subset A of a topological space (X, τ) is said to be# gs-closed set [27] if $scl(A) \subset U$ whenever $A \subset U$ and U is *g-open in X. The complement of # gs-closed set is said to be # gs-open in X.

Definition 2.6. A subset A of a topological space (X, τ) is said to be spg-closed set [29] if $spcl(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

Definition 2.7. A subset A of a topological space (X, τ) is said to be $\tilde{g}$ -closed set [7] if $cl(A) \subset U$ whenever $A \subset U$ and U is # gs-open in X.

Definition 2.8. A subset A of a topological space (X, τ) is said to be $\tilde{g}$ s-closed set [20] if $scl(A) \subset U$ whenever $A \subset U$ and U is # gs-open in X.

Definition 2.9. A subset A of a topological space (X, τ) is said to be sg-closed set [2] if $scl(A) \subset U$ whenever $A \subset U$ and U is semi-open in X.

Definition 2.10. A subset A of a topological space (X, τ) is said to be gs-closed set [1] if $scl(A) \subset U$ whenever $A \subset U$ and U is open in X.

Definition 2.11. A subset A of a topological space (X, τ) is said to be gsp-closed set [5] if $spcl(A) \subset U$ whenever $A \subset U$ and U is open in X.

Throughout the present paper (X, τ) and (Y, σ) always denote topological spaces and $f : (X, \tau) \to (Y, \sigma)$ presents a function.

3. M-STRUCTURES

Definition 3.1. A subfamily $m_x \subset P(X)$ is said to be a minimal structure [19] (briefly *m*-structure) on X if ϕ, $X \in m_x$. The pair (X, m_x) is called a minimal space (*m* -space). Each member of m_x is said to be *m*-open and the complement of an *m*-open set is said to be *m*-closed.

Remark 3.2. Let (X, τ) be a topological space. Then $m_x = \tau$, SO(X) and SPO(X) are minimal structures on X.

Definition 3.3. Let (X, m_x) be an *m*-space. For a subset A of X, the m_x-closure of A and the m_x-interior of A are defined in [14] as follows:

(1) $m\text{-cl}(A) = \cap \{F : A \subset F, F^c \in m_x\}$,

(2) $m\text{-int}(A) = \cup \{U : U \subset A, U \in m_x\}$.

*4. M*Ω-CLOSED SETS

In this section, let (X, τ) be a topological space and m_x an *m*-structure on X. We obtain several basic properties of *m*ω-closed sets.

Definition 4.1. Let (X, τ) be a topological space and m_x an *m*-structure on X. A subset A of X is said to be *m*-semi open [14] if $A \subset m\text{-cl}(m\text{-int}(A))$. The family of all *m*-semi open sets in X is denoted by *m*SO(X). The complement of *m*-semi open set is said to be *m*-semi closed.

Definition 4.2. Let (X, τ) be a topological space and m_x an *m*-structure on X. For a subset A of X, the *m*-semi closure of A [14] and the *m*-semi interior of A, denoted by *m*-scl(A) and *m*-sint(A), respectively are defined as follows:

(a) $m\text{-scl}(A) = \cup \{F : A \subset F, F$ *is* *m*-semi closed in X$\}$,
(b) $m\text{-sint}(A) = \cap \{U : U \subset A, U$ is *m*-semi open in X$\}$.

Definition 4.3. Let (X, τ) be a topological space and m_x an *m*-structure on X. A subset A of X is said to be *m*-space. A subset A of X is said to be

(a) mω-closed if $cl(A) \subset U$ whenever $A \subset U$ and U is *m*-semi-open,

(b) *m*ω-open if its complement is *m*-ω-closed.

Remark 4.4. Let (X, τ) be a topological space and A a subset of X. If *m*SO(X) = SO(X) (resp. τ) and A is *m*ω-closed , then A is ω-closed (g-closed).

Theorem 4.5. Let (X, *m*SO(X)) be an *m*-space and A a subset of X. Then x $\in$ *m*-scl(A) if and only if U $\cap$ A $\neq \phi$ for every *m*-semi open set U containing x.

Proof. Suppose there exists *m*-semi open set U containing x such that U $\cap$ A = ϕ. Then A $\subset$ X –U and X– (X–U)=U $\in$ *m*SO(X). Then by definition 4.2, *m*-scl(A) $\subset$ X –U. Since x $\in$ U, we have x $\notin$ *m*-scl(A). Conversely, suppose that x $\notin$ *m*-scl(A). There exists a subset F of X such that X – F $\in$ *m*SO(X), A $\subset$ F and x $\notin$ F. Then there exists *m*-semi open set X– F containing x such that (X– F) $\cap$ A = ϕ.

Definition 4.6. An *m*-structure m_x on a nonempty set X is said to have property **C** [19] if the union of any family of subsets belonging to m_x belongs to m_x.

Example 4.7. Let X = {a, b, c, d}, m_x = { ϕ, X, {a,b}, {a,c}, {b,d}}, τ ={ ϕ, X, {a}, {d}, {a,d},{a,b,d}}.Then $m\hat{g}$ -open sets are ϕ, X, {a}, {b}, {d}, {a,b}, {a,d} and {a,b,d}}. It is shown that $m\hat{g}$ O(X) does not have property **C.**

Remark 4.8. Let (X, τ) be a topological space. Then the families SO(X) and τ^{α} are all *m*-structure with property **C.**

Lemma 4.9. Let X be a nonempty set and *m*SO(X) an *m*-structure on X satisfying property **C.** For a subset A of X , the following properties hold:

(a) A $\in$ *m*SO(X) if and only if *m*-sint(A) =A,
(b) A is *m*-semi closed if and only if *m*-scl(A) =A,
(c) *m*-sint(A) $\in$ *m*SO(X) and *m*-scl(A) is *m*- semi closed.

Proposition 4.10. Let SO(X) $\subset$ *m*SO(X). Then the following implications hold:

Closed $\rightarrow$ *m*ω-closed $\rightarrow$ ω-closed.

Proof. It is obvious that every closed set is *m*ω-closed. Suppose that A is an *m*ω-closed set. Let A $\subset$ U and U $\in$ SO(X). Since SO(X) $\subset$ *m*SO(X), cl(A) $\subset$ U and hence A is ω-closed.

Example 4.11. Let X = {a, b, c }, m_x = { ϕ, X, {c}} and τ ={ ϕ, X, {b}, {a,c}}. Then ω-closed sets are the power sets of X: *m*ω-closed are ϕ, X, {a}, {b}, {a,b} and {a,c} and closed sets are ϕ, X, {b} and {a,c}. It is clear that {b,c} is ω-closed but it is not *m*ω-closed and {a,b} is *m*ω-closed but it is not closed.

Proposition 4.12. If A and B are *m*ω-closed then A $\cup$ B is *m*ω-closed.

Proof. Let A $\cup$ B $\subset$ U and U $\in$ *m*SO(X), Then A $\subset$ U and B $\subset$ U. Since A and B are *m*ω-closed, we have cl(A $\cup$ B)=cl(A) $\cup$ cl(B) $\subset$ U. Therefore, A $\cup$ B is *m*ω-closed.

Proposition 4.13. If A is *m*ω-closed and *m*-semi open, then A is closed.

Proposition 4.14. If A is *m*ω-closed and A $\subset$ B $\subset$ cl(A), then B is *m*ω-closed.

Proof. Let B $\subset$ U and U $\in$ *m*SO(X). Then A $\subset$ U and A is *m*ω-closed. Hence cl(B) $\subset$ cl(A) $\subset$ U and B is *m*ω-closed.

Definition 4.15. Let (X, *m*SO(X)) be an *m*-space and A a subset of X. Then *m*Semi-Frontier of A, *m*S-Fr(A), is defined as follows: *m*S-Fr(A) = *m*-scl(A) $\cap$ *m*-scl(X– A).

Proposition 4.16. If A is *m*ω-closed and A $\subset$ U $\in$ *m*SO(X), then *m*S-Fr(U) $\subset$ int(X– A).

Proof. Let A be *m*ω-closed and A $\subset$ U $\in$ *m*SO(X). Then cl(A) $\subset$ U. Suppose that x $\in$ *m*S-Fr(U). Since U $\in$ *m*SO(X), *m*S-Fr(U) = *m*-scl(U) $\cap$ *m*-scl(X– U)= *m*-scl(U) $\cap$ (X–U)= *m*-scl(U) – U. Therefore, x $\notin$ U and x $\notin$ cl(A). This shows that x $\in$ int(X–A) and hence *m*S-Fr(U) $\subset$ int(X– A).

Proposition 4.17. A subset A of X is *m*ω-open if and only if F $\subset$ int(A) whenever F $\subset$ A and A is *m*-semi closed.

Proof. Suppose that A is *m*ω-open. Let F $\subset$ A and F be *m*-semi closed. Then X–A $\subset$ X– F $\in$ *m*SO(X) and X –A is *m*ω-closed. Therefore, we have X – int(A) = cl(X– A) $\subset$ X– F and hence F $\subset$ int(A). Conversely, let X –A $\subset$ G and G $\in$ *m*SO(X). Then X – G

$\subset$A and X – G is *m*-semi closed. By hypothesis, we have X – G $\subset$ int(A) and hence cl(X– A) = X – int(A) $\subset$ G. Therefore, X –A is *m*ω-closed and A is *m*ω-open.

Corollary 4.18. Let SO(X) $\subset$ *m*SO(X). Then the following properties hold:

(a) Every open set is *m*ω-open and every *m*ω-open set is ω-open,
(b) If A and B are *m*ω-open, then A $\cap$ B is *m*ω-open,
(c) If A is *m*ω-open and *m*-semi closed, then A is open,
(d) If A is *m*ω-open and int(A) $\subset$ B $\subset$ A, then B is *m*ω-open.

Proof. This follows from propositions 4.10, 4.12, 4.13 and 4.14.

5. CHARACTERIZATIONS OF *M*Ω-CLOSED SETS

In this section, let (X, τ) be a topological space and m_x an *m*-structure on X. We obtain some characterizations of *m*ω-closed sets.

Theorem 5.1. A subset A of X is *m*ω-closed if and only if cl(A) $\cap$ F= ϕ whenever A $\cap$ F= ϕ and F is *m*-semi closed.

Proof. Suppose that A is *m*ω-closed. Let A $\cap$ F= ϕ and F be *m*-semi closed. Then A $\subset$ X– F $\in$ *m*SO(X) and cl(A) $\subset$ X– F. Therefore, we have cl(A) $\cap$ F= ϕ . Conversely, let A $\subset$ U and U $\in$ *m*SO(X). Then A $\cap$ (X– U) = ϕ and X– U is *m*-semi closed. By the hypothesis, cl(A) $\cap$ (X– U) = ϕ and hence cl(A) $\subset$ U. Therefore, A is *m*ω-closed.

Theorem 5.2. Let SO(X) $\subset$ *m*SO(X) and *m*SO(X) have property **C.** A subset A of X is *m*ω-closed if and only if cl(A) –A contains no nonempty *m*-semi closed.

Proof. Suppose that A is *m*ω-closed. Let F $\subset$ cl(A) –A and F be *m*-semi closed. Then F $\subset$ cl(A) and F $\not\subset$ A and so A $\subset$ X– F $\in$ *m*SO(X) and hence cl(A) $\subset$ X – F. Therefore, we have F $\subset$ X –cl(A). Hence F= ϕ . Conversely, suppose that A is not *m*ω-closed. Then by Theorem 5.1, $\phi \neq$ cl(A) –U for some U $\in$ *m*SO(X) containing A. Since τ $\subset$ SO(X) $\subset$ *m*SO(X) and *m*SO(X) has property **C,** cl(A) –U is *m*-semi closed. Moreover, we have cl(A) –U $\subset$ cl(A) –A, a contradiction. Hence A is *m*ω-closed.

Theorem 5.3. Let SO(X) $\subset$ *m*SO(X) and *m*SO(X) have property **C.** A subset A of X is *m*ω-closed if and only if cl(A) –A is *m*ω-open.

Proof. Suppose that A is *m*ω-closed. Let F $\subset$ cl(A) –A and F be *m*-semi closed. By Theorem 5.2, we have F= ϕ and F $\subset$ It follows from proposition 4.16, cl(A) –A is *m*ω-open. Conversely, let A $\subset$ U and U $\in$ *m*SO(X). Then cl(A) $\cap$ (X– U) $\subset$ cl(A) –A and cl(A) –A is *m*ω-open. Since τ $\subset$ SO(X) $\subset$ *m*SO(X) and *m*SO(X) has property **C,** cl(A) $\cap$ (X– U) is *m*-semi closed and by proposition 4.17, cl(A) $\cap$ (X– U) $\subset$ int(cl(A) – A). Now int(cl(A) – A) = int(cl(A)) $\cap$ int(X– A) $\subset$ cl(A) $\cap$ int(X– A) = cl(A) $\cap$ (X– cl(A)) = ϕ Therefore, we have cl(A) $\cap$ (X– U)= ϕ and hence cl(A) $\subset$ U. This shows that A is *m*ω-closed.

Theorem 5.4. Let (X, *m*SO(X)) be an *m*-structure with property **C.** A subset A of X is *m*ω-closed if and only if *m*-scl({x}) $\cap$ A $\neq$ ϕ for each x $\in$ cl(A).

Proof. Suppose that A is *m*ω-closed and *m*-scl({x}) $\cap$ A = ϕ for some x $\in$ cl(A). By lemma 4.9, *m*-scl({x}) is *m*-semi closed and A $\subset$ X– (*m*-scl({x})) $\in$ *m*SO(X). Since A is *m*ω-closed, cl(A) $\subset$ X– (*m*-scl({x})) $\subset$ X– {x}, a contradiction, since x $\in$ cl(A). Conversely, suppose that A is not *m*ω-closed. Then by Theorem 5.1, $\phi \neq$ cl(A) –U for some U $\in$ *m*SO(X) containing A. There exists x $\in$ cl(A) –U. Since x $\notin$ U, by Theorem 4.5, *m*-scl({x}) $\cap$ U= ϕ and hence *m*-scl({x}) $\cap$ A $\subset$ *m*-scl({x}) $\cap$ U = ϕ . This shows that *m*-scl({x}) $\cap$ A = ϕ for some x $\in$ cl(A). Hence A is *m*ω-closed.

Corollary 5.5. Let SO(X) $\subset$ *m*SO(X) and *m*SO(X) have property **C.** For a subset A of X, the following properties are equivalent:

(a) A is *m*ω-open,
(b) A– int(A) contains no nonempty *m*-semi closed set,
(c) A– int(A) is *m*ω-open,
(d) *m*-scl({x}) $\cap$ (X-A) $\neq$ ϕ for each x $\in$ A-int(A).

Proof. This follows from Theorems 5.2, 5.3 and 5.4.

6. PRESERVATION THEOREMS

Definition 6.1. A function f: (X, m_x) $\rightarrow$ (Y, m_y) is said to be

(a) M-semi continuous if $f^{-1}(V)$ is *m*-semi closed in (X, m_x) for every *m*-semi closed V in (Y, m_y),
(b) M-semi closed if for each *m*-semi closed set F of (X, m_x), f(F) is *m*-semi closed in (Y, m_y).

Theorem 6.2. Let *m*SO(X) be an m-structure with property **C**. Let f: (X, m_x) $\rightarrow$ (Y, m_y) be a function from a minimal space (X, m_x) into a minimal space (Y, m_y). Then the following are equivalent:

(a) f is M-semi continuous,
(b) $f^{-1}(V) \in m$SO(X) for every $V \in m$SO(Y).

Proof. Assume that f: (X, m_x) $\rightarrow$ (Y, m_y) is M-semi continuous. Let $V \in m$SO(Y). Then V^c is *m*-semi closed in (Y, m_y). Since f is M-semi continuous, $f^{-1}(V^c)$ is *m*-semi closed in (X, m_x). But $f^{-1}(V^c) = X - f^{-1}(V)$. Thus $X- f^{-1}(V)$ is *m*-semi closed in (X, m_x) and so $f^{-1}(V)$ is *m*-semi open in (X, m_x). Conversely, let for each $V \in m$SO(Y), $f^{-1}(V) \in m$SO(X). Let F be any *m*-semi closed in (Y, m_y). By assumption, $f^{-1}(F^c)$ is *m*-semi open in (X, m_x). But $f^{-1}(F^c) = X- f^{-1}(F)$. Thus $X- f^{-1}(F)$ is *m*-semi open in (X, m_x) and so $f^{-1}(F)$ is *m*-semi closed in (X, m_x), Hence f is M-semi continuous.

Lemma 6.3. A function f: (X, m_x) $\rightarrow$ (Y, m_y) is M-semi closed if and only if for each subset B of Y and each $U \in m$SO(X) containing $f^{-1}(B)$, there exists $V \in m$SO(Y) such that $B \subset V$ and $f^{-1}(V) \subset U$.

Proof. Suppose that f is M-semi closed. Let $B \subset Y$ and $U \in m$SO(X) containing $f^{-1}(B)$. Put V= Y – f(X –U). Then V is *m*-semi open in (Y, m_y) and $f^{-1}(V) \subset f^{-1}(Y) - (X- U) = X- (X- U) = U$. Also, since $f^{-1}(B) \subset U$, then $X- U \subset f^{-1}(Y- B)$ which implies $f(X- U) \subset Y- B$ and hence $B \subset V$. Hence we obtain $V \in m$SO(Y) such that $B \subset V$ and $f^{-1}(V) \subset U$. Conversely, let F be any *m*-semi closed of (X, m_x). Set f(F) = B, then $F \subset f^{-1}(B)$ and $f^{-1}(Y- B) \subset X- F \in m$SO(X). By the hypothesis, there exists $V \in m$SO(Y) such that $Y- B \subset V$ and $f^{-1}(V) \subset X- F$ and so $F \subset f^{-1}(Y- V)$. Therefore $f(F) \subset Y- V$. Hence, we obtain $Y- V \subset B = f(F) \subset Y- V$. Therefore f(F) = Y– V is *m*-semi closed in (Y, m_y). Hence f is M-semi closed.

Theorem 6.4. If f: (X, m_x) $\rightarrow$ (Y, m_y) is closed and f: (X, m_x) $\rightarrow$ (Y, m_y) is M-semi continuous, where *m*SO(X) has property **C**, then f(A) is *m*ω-closed in (Y, m_y) for each *m*ω-closed set A of (X, m_x).

Proof. Let A be any *m*ω-closed set of (X, m_x) and $f(A) \subset V \in m$SO(Y). Then, by Theorem 6.2, $A \subset f^{-1}(V) \in m$SO(X). Since A is *m*ω-closed, $cl(A) \subset f^{-1}(V)$ and $f(cl(A)) \subset V$. Since f is closed, $cl(f(A)) \subset f(cl(A)) \subset V$. Hence f(A) is *m*ω-closed in (Y, m_y).

Theorem 6.5. If f: (X, m_x) $\rightarrow$ (Y, m_y) is continuous and f: (X, m_x) $\rightarrow$ (Y, m_y) is M-semi closed, then $f^{-1}(B)$ is *m*ω-closed in (X, m_x) for each *m*ω-closed set B of (Y, m_y).

Proof. Let B be any mω-closed set of (Y, m_y) and $f^{-1}(B) \subset U \in m$SO(X). Since f is M-semi closed, by Lemma 6.3, there exists $V \in m$SO(Y) such that $B \subset V$ and $f^{-1}(V) \subset U$. Since B is *m*ω-closed, $cl(B) \subset V$ and since f is continuous, $cl(f^{-1}(B)) \subset f^{-1}(cl(B)) \subset f^{-1}(V) \subset U$. Hence $f^{-1}(B)$ is *m*ω-closed in (X, m_x).

7. NEW FORMS OF CLOSED SETS IN TOPOLOGICAL SPACES

Definition 6.1. A subset A of X is called a $\tilde{g}$ -semi preclosed set ($\tilde{g}$ sp-closed set) if spcl (A) $\subset U$ whenever $A \subset U$ and U is #gs-open in X.

By SO(X) (resp. $\tilde{G}$ SO(X), SGO(X), GSO(X), SPO(X), $\tilde{G}$ SPO(X), SPGO(X), GSPO(X)) we denote the collection of all semi open (resp. #gs-open, sg-open, gs-open, semi preopen, $\tilde{g}$ sp-open, spg-open, gsp-open) sets of topological space (X, τ). If $m_x = \tau$, these collection are minimal structures on X.

By the definitions, we obtain the following diagram:

Diagram I

$$\begin{array}{ccccccc}
\text{Semi open} & \rightarrow & \tilde{g}\text{ s -open} & \rightarrow & \text{sg -open} & \rightarrow & \text{gs -open} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
\text{Semi preopen} & \rightarrow & \tilde{g}\text{ sp -open} & \rightarrow & \text{spg -open} & \rightarrow & \text{gsp -open}
\end{array}$$

For subsets of a topological space (X, τ), we can define new types of closed sets as follows:

Definition 6.2. A subset A of a topological space (X, τ) is said to be ω-closed (resp. $\tilde{g}$ sg -closed, sgg -closed, gsg -closed, sog -closed, $\tilde{g}$ spg -closed, spgg -closed, gspg -closed) if cl(A)$\subset$U whenever A$\subset$U and U is semi-open (resp. $\tilde{g}$ s -open, sg-open, gs-open, semi-preopen, $\tilde{g}$ sp -open, spg-open, gsp-open) in (X, τ).

By Diagram I and Definition 6.2, we have the following diagram:

Diagram II

$$\begin{tikzcd}
\omega\text{-closed} & \tilde{g}\,sg\text{-closed} \arrow[l] & sgg\text{-closed} \arrow[l] & gsg\text{-closed} \arrow[l] & \\
sog\text{-closed} \arrow[u] & \tilde{g}\,spg\text{-closed} \arrow[l] \arrow[u] & spgg\text{-closed} \arrow[l] \arrow[u] & gspg\text{-closed} \arrow[l] \arrow[u] & \text{closed} \arrow[l]
\end{tikzcd}$$

REFERENCES

[1] S. P. Arya and T. M. Noiri, Characterizations of s-normal spaces, Indian J. Pure Appl. Math., 21(1990), 717-719.
[2] P. Bhattacharyya and B. K. Lahiri, Semi-generalized closed sets in topology, Indian J. Math., 29(3)(1987), 375-382.
[3] S. G. Crossley and S. K. Hildrebrand, Semi-closure, Texas J. Sci., 22(1971), 99-112.
[4] J. Dontchev, M. Ganster and T. Noiri, Unified operation approach of generalized closed sets via topological ideals, Math. Japon., 49 (1999), 395-401.
[5] J. Dontchev, On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 16(1995), 35-48.
[6] T. R. Hamlett, D. Rose, and D. Jankovic, paracompactness with respect to an ideal, Internat.J. Math. Math. Sci., 20(1997), 433-442.
[7] S. Jafari, T. Noiri, N. Rajesh and M. L. Thivagar, Another generalization of closed sets, Kochi J. Math., 3(2008), 25-38.
[8] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly., 97 (1990), 295-310.
[9] K. Kuratowski, Topology, Vol. I, Academic Press (New York, 1966).
[10] N. Levine, Generalized closed sets in topology, Rend. Circ. Mat. Palermo., (2)19 (1970), 89-96.
[11] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly., 70(1963), 36-41..
[12] H. Maki, K. C. Rao and A. Nagoor Gani, On generalizing semi-open and preopen sets, Pure Appl. Math. Sci., 49(1999), 17-29.
[13] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, On precontinuous and weak precontinuous mappings, Proc. Math. Phy. Soc. Egypt., 53 (1982), 47–53.
[14] W. K. Min , m-semi-open sets and M-semicontinuous functions on spaces with minimal structures, Honam Mathematical Journal., 31(2)(2009), 239-245.
[15] M. Navaneethakrishnan and J. Paulraj Joseph, g-closed sets in ideal topological spaces, Acta Math Hung, 119(4)(2008), 365-371.
[16] O. Njastad, On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961–970.
[17] T. Noiri and V. Popa, Between closed sets and g-closed sets, Rend. Circ. Mat. Palermo (2), 55 (2006), 175-184.
[18] O. B. Ozbakir and E. D. Yildirim, On some closed sets in ideal minimal spaces, Acta Math Hung, 125(3), (2009), 227-235.
[19] V. Popa and T. Noiri, On M-continuous functions, Anal. Univ. "Dunarea de Jos" Galati, Ser. Mat. Fiz. Mec. Teor., (2), 18(23) (2000), 31-41.
[20] N. Rajesh, M. Lellis Thivagar, P. Sundaram, and Zbigniew Duszynski, $\tilde{g}$ -semi-closed sets in topological spaces, Mathematica Pannonica., 18(1) (2007), 51-61.
[21] O. Ravi, J. Antony Rex Rodrigo, A. Naliniramalatha , $\hat{g}$ -closed sets in ideal topological spaces, Methods of Functional Analysis and Topology., 17(3) (2011), 274-280.
[22] O. Ravi, R. G. Balamurugan, J. Antony Rex Rodrigo and K. Vijayalakshmi, A unified theory for modifications of $\hat{g}$ -closed sets, International Journal of Advances in Pure and Applied Mathematics, 1 (1) (2011), 53-67.
[23] O. Ravi, J. Antony Rex Rodrigo, S. Tharmar and K. Vijayalakshmi, Between closed sets and *g-closed sets, Fasciculi Mathematici, 49(2012), 127-136.
[24] M. Sheik John, A study on generalizations of closed sets and continuous maps in topological spaces and topological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 2002.
[25] R. Vaidyanathaswamy, Set Topology, Chelsea Publishing Company (1946).
[26] M.K.R.S. Veerakumar, $\hat{g}$ -closed sets in topological spaces, Bull. Allahabad Math. Soc., 18 (2003), 99-112.
[27] M.K.R.S. Veerakumar, # g-semi-closed sets in topological spaces, Antarctica J. Math., 2 (2) (2005), 201 – 222.
[28] M. K. R. S. Veerakumar, Between g*-closed sets and g-closed sets, Antarctica J. Math.,3(1) (2006), 43-65.
[29] M. K. R. S. Veerakumar, Semi-pre-generalized closed sets, Mem. Fac. Sci. Kochi Univ. (Japan) Scr. A. Math., 20(1999), 33-46.

