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## Wavelet Analytical Study of Sulphur Dioxide as an Air pollutant

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**Abstract:** Most of the sulphur dioxide in the atmosphere is an anthropogenic by-product. It is one of the basic causes of acid rain worldwide. NAAQS set the level of sulphur dioxide in the atmosphere for the safety of human health and environment. Breathing of Sulphur dioxide becomes the cause of many diseases concerned with a respiratory system like Bronchitis, Asthma, etc. In the perspective of human health and environment protection, continuous monitoring and analysis have become of great importance. Wavelet is a tool to analyze the non-stationary signal and hence wavelet transforms provide excellent analysis of non-stationary time series of SO<sub>2</sub> and extracts important information. Daubechies4 wavelet is orthogonal and compactly supportive and therefore, it is useful for multiresolution analysis of SO<sub>2</sub> data. Wavelet transforms provide simple and accurate framework for modeling the statistical behaviour of SO<sub>2</sub> variation in the interest of public health and environment protection.

**Keywords:** Sulphur dioxide, Air Pollution, Wavelet, Wavelet Transforms, Daubechies Wavelet.

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### I. INTRODUCTION

Sulphur dioxide (SO<sub>2</sub>) is a colourless, non-flammable gas with a suffocating and choking odour. It is produced naturally through geothermal activity. However, about 99% of sulphur dioxide in the atmosphere has been artificially made. It is emitted from fossil fuel combustion at power plants and other industrial facilities. Other sources of SO<sub>2</sub> include industrial processes such as extracting metal from ore and the burning of high sulphur fuels by locomotives, large ships, and non-road equipment. The National Ambient Air Quality Standards (NAAQS) set standards to protect human health, with an adequate margin of safety, including sensitive populations suffering from respiratory diseases. Limits established to protect human health are referred to as “primary standards”; limits established to prevent environmental damage are referred to as “secondary standards”. The primary NAAQS for SO<sub>2</sub> measured over a 1-hour period is set at 75 parts per billion parts of air. A secondary NAAQS for SO<sub>2</sub> measured over a 3-hour period is set at 0.5 parts per million parts of air [1].

Sulphur dioxide is one of the “criteria” pollutants and that are considered harmful to public health and the environment. It is a highly reactive gas having high potential to change in composition under certain conditions of pressure, temperature or light, or upon contact with another chemical. Sulphur dioxide released into the atmosphere dissolves in water vapour to form acid rain. Breathing SO<sub>2</sub> becomes the cause of many diseases concerned with respiratory systems such as breathing difficulty and Asthma. SO<sub>2</sub> is one of the primary contributors to acid rain, along with nitrogen oxides, which causes acidification of lakes and streams which damage trees and sensitive forest soils [2]. SO<sub>2</sub> contributes to the acceleration of the decay of building materials and paints throughout the country. Sulphur dioxide is widely monitored in urban areas because it is known to irritate mucous membranes, including the eyes and nasal passages, and cause broncho-constriction in sensitive individuals. This is due to its property of being readily soluble in water, forming sulphurous and sulphuric acids. Sulphuric acid in the air forms small particles around particulate matter by the process of nucleation. This leads to the formation of larger particles which, being strongly hygroscopic, bond readily to the walls of the air passages of the nose, trachea, and lungs. Sulphur dioxide and particulate matter may exert synergistic toxic effects. Respiration is the main route of exposure that leads to noticeable health effects from air pollution in general.

Emissions of sulphur dioxide can lead to the deposition of acid rain over large distances often more than 1000 kilometers from their source [3]. Acid rain can damage ecosystems on a regional scale. It is a major problem in the northern hemisphere, where entire forests have suffered defoliation and dieback, and lakes and watercourses have lost the ability to support life due to changing acidity and mobilisation of certain minerals. Acid rain is a worldwide problem; however, sulphur dioxide deposition can affect vegetation around large industrial discharges unless appropriate ambient concentrations are achieved through appropriate source management. Sulphur dioxide can form secondary particles (sulphate) that cause haze and reduce visibility due to their high light-scattering ability. These sulphate particles have a modifying effect on enhanced greenhouse warming because they reflect incoming heat from the sun. Sulphur dioxide gas can be carried long distances, but its concentration reduces with time due to its reactivity

with other atmospheric components. It reacts with water, nitrates, oxygen, ozone and OH radicals to produce sulphuric acid, sulphurous acid (H<sub>2</sub>SO<sub>3</sub>) and various sulphates, some of which fall out as particulate matter.

In the perspective of human health and environment protection, the monitoring and analytical study of sulphur dioxide have become of great importance. Therefore, this field has attracted the intense interest of Physicists and Environmentalists. The Fourier and Fast Fourier Transform (FFT) is a useful tool to study the spectrum of stationary time series. These theories extract only frequency information from a time signal, thus losing time information. Thus for non-stationary signals, they become insufficient and wavelet theory comes into play. With help of this theory, time evolution and frequency composition of a signal are extracted. The wavelet is a new analytical tool for turbulent or chaotic data to the Science community [4]. It allows detection and characterization of short-lived structures in the non-stationary signal.

## II. WAVELET THEORY

Wavelet is a special kind of the functions which exhibit oscillatory behaviour for a short time interval and then dies out. In wavelet we use a single function and its dilation and translation to generate a set of orthonormal basis functions to represent a signal ([5], [6]). A number of such functions are infinite and we choose one that suits to our application. The range of interval over which scaling function and wavelet function are defined is known as support of wavelet. Beyond this interval (support) the functions should be identically zero. There is an interesting relation between the length of support and number of coefficients in the refinement relation. For orthogonal wavelet system, the length of support is always less than no. of coefficients in the refinement relation. It is also very helpful to require that the mother function has a certain number of zero moments, according to:

$$\int_{-\infty}^{\infty} \psi(t) dt = 0$$

The mother function can be used to generate a whole family of wavelets by translating and scaling the mother wavelet.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) = T_b D_a \psi$$

Here  $b$  is the translation parameter and  $a$  is the dilation or scaling parameter. Provided that  $\psi(t)$  is real-valued, this collection of wavelets can be used as an orthonormal basis. A critical sampling of the continuous wavelet transform is

$$W_{a,b} = \int f(t) \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) dt$$

is obtained via  $a = 2^{-j}$ , where  $j$  and  $k$  are integers representing the set of discrete translations and discrete dilations. Upon this substitution, we can write discrete wavelet transform as;

$$W_{j,k} = \int f(t) 2^{j/2} \psi(2^j t - k) dt$$

Wavelet coefficients for every (a, b) combination whereas in discrete wavelet transform, we find wavelet coefficients only at very few points by the dots and the wavelets that follow these values are given by;

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k)$$

This wavelet coefficient for all  $j$  and  $k$  produce an orthonormal basis. We call  $\psi_{0,0}(t) = \psi(t)$  as mother wavelet. Other wavelets are produced by translation and dilation of mother wavelet. The wavelet transform of a signal captures the localized time frequency information of the signal. Suppose we are given a signal or sequences of data  $S = \{S_n\}_{n \in \mathbb{Z}}$  sampled at regular time interval  $\Delta t$ .  $S$  is split into a “blurred” version  $a_1$  at the coarser interval  $2\Delta t$  and “detail”  $d_1$  at scale  $\Delta t$ . This process is repeated and gives a sequence  $S_n, a_1, a_2, a_3, a_4, \dots$  of more and more blurred versions together with the details  $d_1, d_2, d_3, d_4, \dots$  removed at every scale ( $\Delta t = 2^m \tau$  in  $a_m$  and  $d_m$ ). Here  $a_m$ s and  $d_m$ s are approximation and details of original signal. After  $N$  iteration the original signal  $S$  can be reconstructed as

$$S = a_N + d_1 + d_2 + d_3 + \dots \dots d_N$$

## III. METHODOLOGY

The primary and most important work in the spectral analysis of any signal using wavelet transforms is the selection of suitable wavelet according to the signal. Suitable wavelet is selected on the basis of compatibility with signal characteristics. Accurate wavelet selection retains the original signal and also enhances the frequency spectrum of the reconstructed signal. Wavelet extracts both time evolution and frequency composition of a signal ([7], [8]). A multiresolution analysis for  $L^2(\mathbb{R})$  introduced by Mallat [9] consists of a Sequence  $V_j, j \in \mathbb{Z}$  of closed subspaces of  $L^2(\mathbb{R})$ . Let  $f(x)$  be a function in  $L^2(\mathbb{R})$ . We can write  $f(x)$  in  $V_{j+1}$  space,

$$f(x) = \sum c_{j+1,k} \phi_{j+1,k}(x)$$

Since

$$V_{j+1} = V_j \oplus W_j$$

where

$$V_{j+1} = \text{span} \left( \overline{\phi_{j+1,k}(x)} \right)$$

$$V_j = \text{span} \left( \overline{\phi_{j,k}(x)} \right)$$

$$W_j = \text{span} \left( \overline{\psi_{j,k}(x)} \right)$$

Therefore

$$f(x) = \sum_k c_{j,k} \phi_{j,k}(x) + \sum_{j_0}^j \sum_k d_{j,k} \psi_{j,k}(x)$$

where

$$c_{j,k} = \langle f, \phi_{j,k} \rangle$$

$$= \int f(x) \phi_{j,k}(x) dx, \quad \forall k \in \mathbb{Z}$$

and

$$d_{j,k} = \langle f, \psi_{j,k} \rangle$$

$$= \int f(x) \psi_{j,k}(x) dx$$

are collectively known as an approximation and detailed coefficients. Thus given signal takes place a new version such as [10]

$$\begin{aligned} f(1) &= a_1 + d_1 \\ f(2) &= a_2 + d_2 + d_1 \\ f(3) &= a_3 + d_3 + d_2 + d_1 \\ f(4) &= a_4 + d_4 + d_3 + d_2 + d_1 \\ f(5) &= a_5 + d_5 + d_4 + d_3 + d_2 + d_1 \\ f(6) &= a_6 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1 \\ f(7) &= a_7 + d_7 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1 \\ f(8) &= a_8 + d_8 + d_7 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1 \end{aligned}$$

Here  $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$  are approximations of signal and  $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$  are details of signal at various scale or time frames. A signal  $f$  can be decomposed as in simplest form;

$$f = A + D$$

Where  $A$  and  $D$  are called Approximation and Details of the given signal  $f$ . Approximation is average of the signal and hence represents low frequency components, while Detail is the difference of the signal and hence represents high frequency components. Detail plays very important role and provides hidden information of any signal. The wavelet is a new analytical tool for chaotic data to the physics community. It allows detection and characterization of short-lived structures in data.

#### IV. RESULTS AND DISCUSSION

Our study area is Anand Vihar, Delhi and we have used data (online) observed at DPCC, Anand Vihar. We have taken an average quantity of  $\text{SO}_2$  for a day from 01-07-2016 to 13-03-2017.

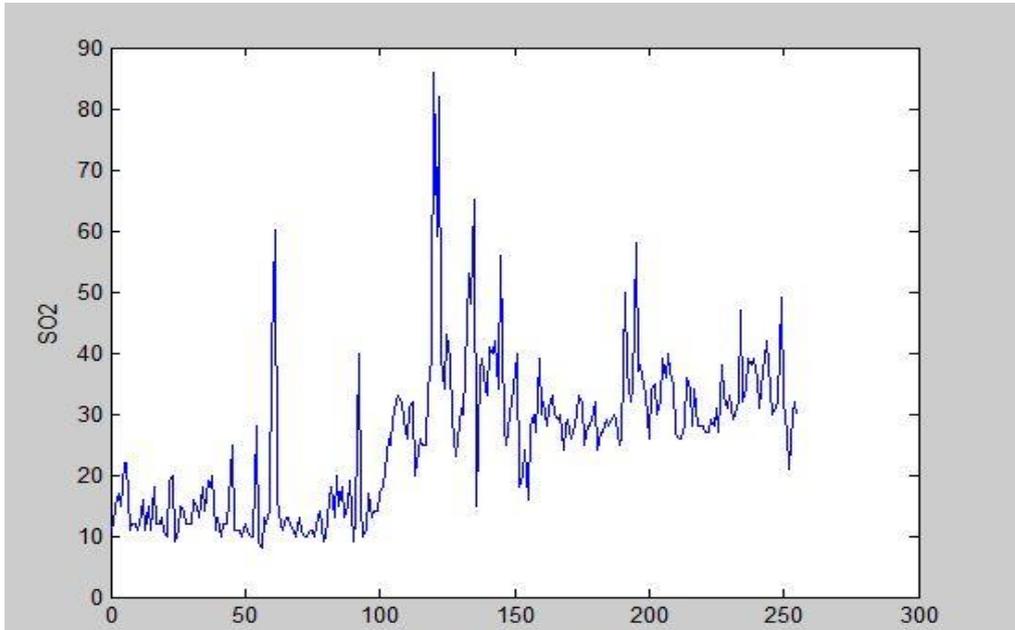


Fig. 1. Average SO<sub>2</sub> (in ppb)

In above figure, the quantitative behaviour of SO<sub>2</sub> for given time period is shown. For the more detailed investigation, we decompose the time series of SO<sub>2</sub> at different scale by using discrete wavelet transform (Daubechies 4 level 8 [11]). A full decomposition of SO<sub>2</sub> time series is shown in fig. 2. In decomposition figures  $a_8$  is approximation at level 8 while  $d_1, d_2, d_3, d_4, d_5, d_6, d_7,$  and  $d_8$  are details of the of SO<sub>2</sub> time series at different time modes. Original time series is the average value of SO<sub>2</sub>, taken as one day interval. Approximation  $a_8$  represents the behaviour of the signal  $S$  at level 8. While details  $d_1$  exhibits per day variation,  $d_2$  shows 2-days variation and  $d_3$  is 4-days variation in the value of SO<sub>2</sub> also  $d_4, d_5, d_6, d_7$  and  $d_8$  are variation in the value of SO<sub>2</sub> in 8, 16, 32, 64, and 128 days mode, respectively. The peak in a detail shows rapid fall or rise in the value of SO<sub>2</sub> in that time mode. The measurement of SO<sub>2</sub> depends on anthropogenic driving forces.

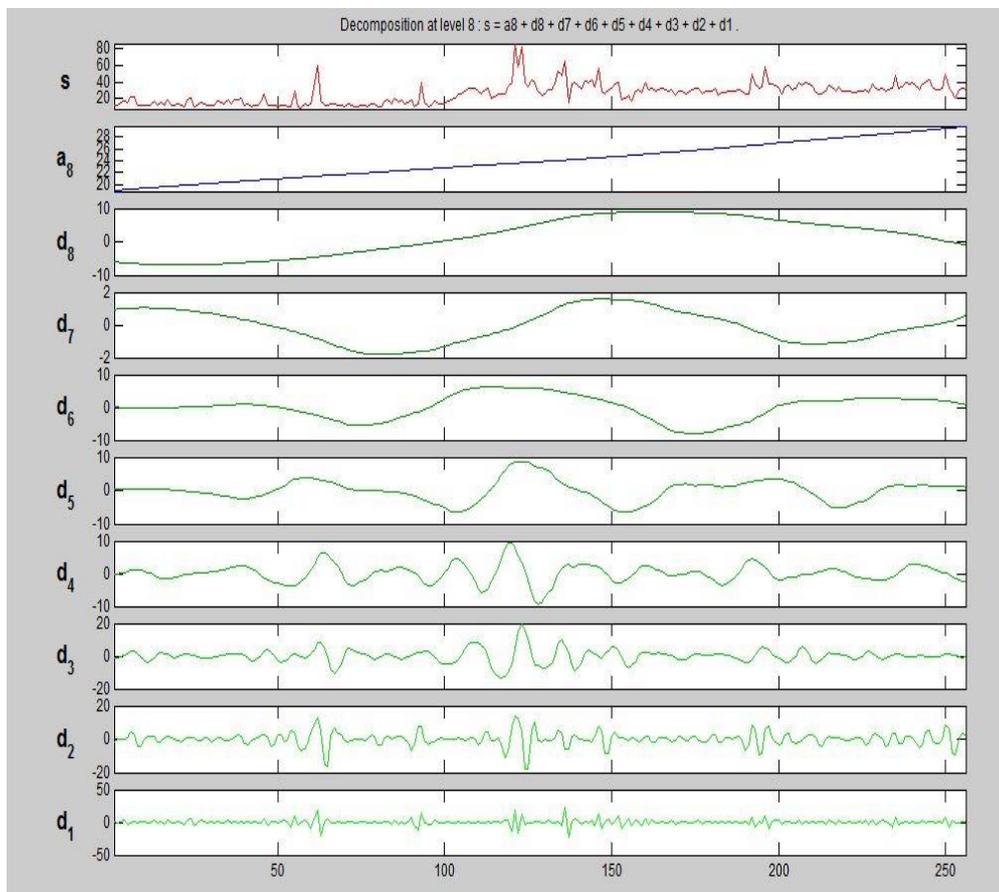
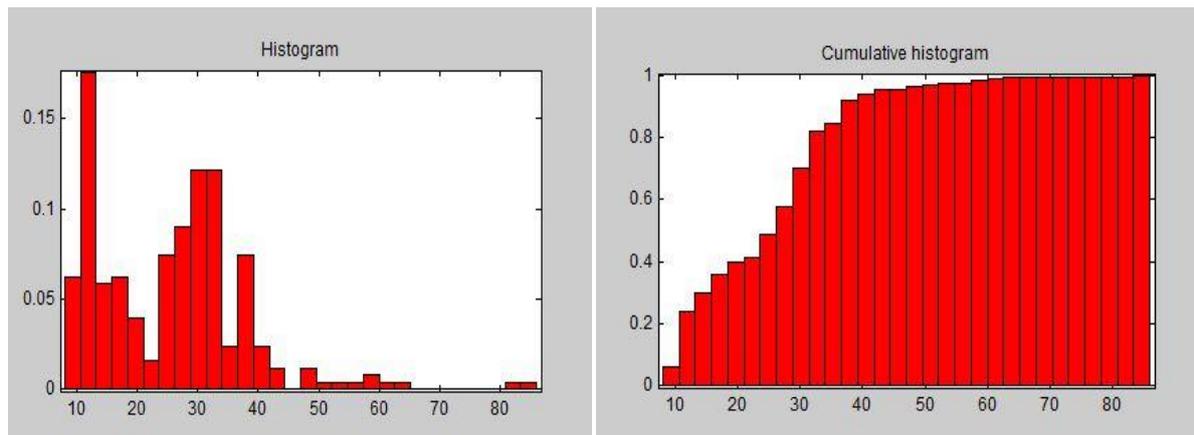


Fig. 2. Wavelet decomposition of time series of SO<sub>2</sub>

Fluctuation in quantity of SO<sub>2</sub> over time may be as a result of the effluent load. In data analysis, the low-frequency content of the signal is an important part, because it gives the identity of the signal. The trend is the slowest part of the signal means lowest frequency part of the signal. In wavelet analysis terms, this corresponds to the greatest scale value. As the scale increases, the resolution decreases, producing a better estimate of the unknown trend. A trend of the SO<sub>2</sub> signal exhibits in fig. 3.



**Fig. 3. Histogram and Cumulative Histogram**

Histogram depicts the trends of the average value of SO<sub>2</sub> for Approximation. A cumulative histogram is a mapping that counts the cumulative number of observations in all of the bins up to the specified bin. The histogram provides important information about the shape of a distribution. A variance in the value of SO<sub>2</sub> during a time interval (01-07-2016 to 13-03-2017) in individual sampling spot is clearly depicted in an approximation of the decomposition.

### CONCLUSION

The wavelet method allows the decomposition of the signal according to different frequency levels which characterize the intensity of natural and man-made disturbances. A histogram is a graphical representation showing a visual impression of the distribution of data. According to the behaviour studied, it is possible to conjecture that the difference between all-time series can be directly associated with a large number of anthropogenic activity. Taking into account these results we have shown, the wavelet analytical approach provides a simple and accurate framework for modelling the statistical behaviour of SO<sub>2</sub> variation.

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