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On Almost Contra (b, μ)-Continuous Functions

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***Abstract:** The purpose of this paper is to introduce the notion of almost contra (b, μ)-continuous function. Also, the relationships between almost contra (b, μ)-continuous functions and the other forms are investigated. 2010 Mathematics Subject Classification: 57C10, 57C08, 57C05*

***Keywords:** Contra (b, μ)-Continuity, Almost Contra (b, μ)-Continuity, Strongly Contra λ-b-Closed, λ-b-Closed Compact.*

1. INTRODUCTION

In [2, 3, 4], Cs□sz□r founded the theory of generalized topological spaces and studied the extremely elementary character of these classes. Especially he introduced the notions of continuous functions on generalized topological spaces and investigated characterizations of generalized continuous functions (= (λ, μ)-continuous functions in [4]). In [7, 8, 9], Min introduced the notions of weak (λ, μ)-continuity, almost (λ, μ)-continuity, (α, μ)-continuity, (σ, μ)-continuity, (π, μ)-continuity and (β, μ)-continuity on generalized topological spaces. In [6], Jayanthi introduced the notion of contra continuous functions on generalized topological spaces.

The purpose of this paper is to introduce the notion of almost contra (b, μ)-continuous function. Also, the relationships between almost contra (b, μ)-continuous functions and the other forms are investigated.

2. PRELIMINARIES

Definition 2.1 [4]

Let X be a nonempty set and λ be a collection of subsets of X. Then λ is called a generalized topology (briefly GT) on X if $\phi \in \lambda$ and $G_i \in \lambda$ for $i \in I \neq \phi$ implies $G = \bigcup_{i \in I} G_i \in \lambda$. We say λ is strong if $X \in \lambda$, and we call the pair (X, λ) a generalized topological space (briefly GTS) on X.

The elements of λ are called λ-open sets and their complements are called λ-closed sets. The generalized closure of a subset S of X, denoted by $c_\lambda(S)$, is the intersection of λ-closed sets including S. And the interior of S, denoted by $i_\lambda(S)$, is the union of λ-open sets contained in S.

Definition 2.2

Let (X, λ) be a generalized topological space and $A \subset X$. Then A is said to be

(i) λ-semi-open [3] if $A \subset c_\lambda(i_\lambda(A))$,
(ii) λ-preopen [3] if $A \subset i_\lambda(c_\lambda(A))$,
(iii) λ-α-open [3] if $A \subset i_\lambda(c_\lambda(i_\lambda(A)))$,
(iv) λ-β-open [3] if $A \subset c_\lambda(i_\lambda(c_\lambda(A)))$,
(v) λ-b-open [14] if $A \subseteq c_\lambda(i_\lambda(A)) \cup i_\lambda(c_\lambda(A))$,
(vi) λ-regular open [10] if $A = i_\lambda(c_\lambda(A))$.

The complement of λ-semi-open (resp. λ-preopen, λ-α-open, λ-β-open, λ-b-open, λ-regular open) is said to be λ-semi-closed (resp. λ-preclosed, λ-α-closed, λ-β-closed, λ-b-closed, λ-regular closed).

Let us denote by $\sigma(\lambda_X)$ (briefly σ_X or σ) the class of all λ-semi-open sets on X, by $\pi(\lambda_X)$ (briefly π_X or π) the class of all λ-preopen sets on X, by $\alpha(\lambda_X)$ (briefly α_X or α) the class of all λ-α-open sets on X, by $\beta(\lambda_X)$ (briefly β_X or β) the class of all λ-β-open sets on X, by $b(\lambda_X)$ (briefly b_X or b) the class of all λ-b-open sets on X.

Remark 2.3 [101]

Let (X, λ) be a generalized topological space. Then we have the following implications.

$$\begin{tikzcd} & & & \lambda\text{-semi-open} \arrow[dr] & \\ \lambda\text{-regular open} \arrow[r] & \lambda\text{-open} \arrow[r] & \lambda\text{-}\alpha\text{-open} \arrow[ur] \arrow[dr] & & \lambda\text{-}\beta\text{-open} \\ & & & \lambda\text{-preopen} \arrow[ur] & \end{tikzcd}$$

Lemma 2.4 [3]

Let (X, λ) be a generalized topological space and $A \subset X$. Then A is λ-α-open if and only if it is λ-semi-open and λ-preopen.

Definition 2.5 [12]

A GTS (X, λ) is said to be λ-extremally disconnected if the λ-closure of every λ-open set of X is λ-open in X.

Definition 2.6 [5]

A set A in a GTS (X, λ) is called λ-clopen if it is both λ-open and λ-closed.

Definition 2.7 [5]

A subset A in a GTS (X, λ) is said to be λ-locally λ-closed set (briefly, λ-LC set) if $A = M \cap N$, where M is a λ-open set and N is a λ-closed set.

Definition 2.8 [13]

A GTS (X, λ) is said to be λ-connected if it cannot be expressed as the union of two nonempty, disjoint λ-open sets.

Definition 2.9 [9]

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be (λ, μ)-open if the image of each λ-open set in X is an μ-open set of Y.

Definition 2.10 [11]

Let $f : (X, \lambda) \to (Y, \mu)$ be a function from a GTS (X, λ) to a GTS (Y, μ). Then the function $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ is called the λ-graph function of f. Recall that for a function $f : (X, \lambda) \to (Y, \mu)$, the subset $\{(x, f(x)) : x \in X\} \subseteq X \times Y$ is called the λ-graph of f and is denoted by G(f).

Theorem 2.11 [6]

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : X \to Y$ is contra (α, μ)-continuous if and only if it is both contra (π, μ)-continuous and contra (σ, μ)-continuous.

Definition 2.12 [11]

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be

(i) (b, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-b-open in X,

(ii) (λ-LC, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-locally λ-closed in X.

Definition 2.13 [11]

A filter base Λ is said to be λ-b-convergent to a point x in X if for any λ-b-open set M in X containing x, there exists a set $N \in \Lambda$ such that $N \subseteq M$.

Definition 2.14 [11]

A GTS (X, λ) is said to be λ-b-connected if it cannot be expressed as the union of two nonempty, disjoint λ-b-open sets.

Definition 2.15 [11]

A GTS (X, λ) is said to be λ-b-T_1 if for each pair of distinct points x and y of X, there exist λ-b-open sets M and N containing x and y respectively such that $y \notin M$ and $x \notin N$.

Definition 2.16 [11]

A GTS (X, λ) is said to be λ-b-compact if every λ-b-open cover of X has a finite subcover.

Definition 2.17 [1]

A GTS (X, λ) is said to be λ-Urysohn if for each pair of distinct points x and y in X, there exist λ-open sets U and V such that $x \in U$, $y \in V$ and $c_\lambda(U) \cap c_\lambda(V) = \phi$.

Definition 2.18 [11]

A GTS (X, λ) is said to be λ-b-T_2 if for each pair of distinct points x and y of X, there exist λ-b-open sets M and N containing x and y respectively such that $M \cap N = \phi$.

3. Almost Contra (b, μ)-Continuous Functions

In this section, we introduce the notion of almost contra (b, μ)-continuous function.

Definition 3.1

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be

(i) contra (b, μ)-continuous if for each μ-open set U in Y, $f^{-1}(U)$ is λ-b-closed in X,

(ii) almost contra (b, μ)-continuous if the inverse image of each μ-regular open set in Y is λ-b-closed in X.

Theorem 3.2

Let $f : (X, \lambda) \to (Y, \mu)$ be a function and let $g : X \to X \times Y$ be the λ-graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is almost contra (b, μ)-continuous, then f is almost contra (b, μ)-continuous.

Proof

Let A be a μ-regular closed set in Y, then $X \times A$ is a λ-regular closed set in $X \times Y$. Since g is almost contra (b, μ)-continuous, then $f^{-1}(A) = g^{-1}(X \times A)$ is λ-b-open in X. Thus, f is almost contra (b, μ)-continuous.

Definition 3.3

A filter base Λ is said to be λ-rc-convergent to a point x in X if for any λ-regular closed set M in X containing x, there exists a set $N \in \Lambda$ such that $N \subseteq M$.

Theorem 3.4

If a function $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous, then for each point $x \in X$ and each filter base Λ in X λ-b-converging to x, the filter base f(Λ) is μ-rc-convergent to f(x).

Proof

Let $x \in X$ and Λ be any filter base in X λ-b-converging to x. Since f is almost contra (b, μ)-continuous, then for any μ-regular closed set M in Y containing f(x), there exists a λ-b-open set N in X containing x such that $f(N) \subseteq M$. Since Λ is λ-b-converging to x, there exists a $P \in \Lambda$ such that $P \subseteq N$. This means that $f(P) \subseteq M$ and therefore the filter base f(Λ) is μ-rc-convergent to f(x).

Theorem 3.5

If $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous surjective function and X is λ-b-connected space, then Y is μ-connected space.

Proof

Suppose that Y is not μ-connected space. Then there exists nonempty disjoint μ-open sets M and N such that $Y = M \cup N$. Therefore, M and N are μ-regular open sets in Y. Since f is almost contra (b, μ)-continuous, then $f^{-1}(M)$ and $f^{-1}(N)$ are λ-b-closed in X. Moreover, $f^{-1}(M)$ and $f^{-1}(N)$ are nonempty disjoint and $X = f^{-1}(M) \cup f^{-1}(N)$. This shows that X is not λ-b-connected. This is a contradiction. By contradiction, Y is λ-connected.

Definition 3.6

A GTS (X, λ) is said to be λ-b-normal if every pair of nonempty disjoint λ-closed sets can be separated by disjoint λ-b-open sets.

Definition 3.7

A GTS (X, λ) is said to be strongly λ-normal if for every pair of nonempty disjoint λ-closed sets M and N in X there exist disjoint λ-open sets P and Q such that $M \subseteq P$, $N \subseteq Q$ and $c_\lambda(P) \cap c_\lambda(Q) = \phi$.

Theorem 3.8

If (Y, μ) is strongly λ-normal and $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous λ-closed injective function, then (X, λ) is λ-b-normal.

Proof

Let M and N be disjoint nonempty λ-closed sets of X. Since f is injective and λ-closed, f(M) and f(N) are disjoint λ-closed sets. Since (Y, μ) is strongly λ-normal, there exists λ-open sets P and Q such that $f(M) \subseteq P$, $f(N) \subseteq Q$ and $c_\mu(P) \cap c_\mu(Q) = \phi$. Then, since $c_\mu(P)$ and $c_\mu(Q)$ are λ-regular closed and f is almost contra (b, μ)-continuous, $f^{-1}(c_\mu(P))$ and $f^{-1}(c_\mu(Q))$ are λ-b-open sets. Since $M \subseteq f^{-1}(c_\mu(P))$, $N \subseteq f^{-1}(c_\mu(Q))$, and $f^{-1}(c_\mu(P))$ and $f^{-1}(c_\mu(Q))$ are disjoint, (X, λ) is λ-b-normal.

Definition 3.9

A GTS (X, λ) is said to be weakly λ-T_2 if each element of X is an intersection of λ-regular closed sets.

Theorem 3.10

If $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous injective function and (Y, μ) is μ-Urysohn, then (X, λ) is λ-b-T_2.

Proof

Suppose that (Y, μ) is μ-Urysohn. By the injectivity of f, it follows that $f(x) \neq f(y)$, for any distinct singletons x and y in X. Since Y is μ-Urysohn, there exists λ-open sets M and N such that $f(x) \in M$, $f(y) \in N$ and $c_\mu(M) \cap c_\mu(N) = \phi$. Since f is almost contra (b, μ)-continuous, there exist λ-b-open sets P and Q in X containing x and y respectively, such that $f(P) \subseteq c_\mu(M)$ and $f(Q) \subseteq c_\mu(N)$. Hence $P \cap Q = \phi$. This shows that X is λ-b-T_2.

Theorem 3.11

Let (X_i, λ_i) be a GTS for all $i \in I$ and I be finite. Suppose that $(\prod_{i \in I} X_i, \sigma)$ is a product space and $f : (X, \lambda) \to (\prod_{i \in I} X_i, \sigma)$ is any function. If f is almost contra (b, μ)-continuous, then $pr_i \circ f$ is almost contra (b, μ)-continuous where pr_i is projection function for each $i \in I$.

Proof

Let $x \in X$ and $(pr_i \circ f)(x) \in M_i$ and M_i be a λ-regular closed set in (X_i, λ_i). then $f(x) \in pr_i^{-1}(M_i) = M_i \times \prod_{i \neq j} X_j$ is a λ-regular closed set in $(\prod_{i \in I} X_i, \sigma)$. Since f is almost contra (b, μ)-continuous, there exists a λ-b-open set N containing x such that $f(N) \subseteq M_i \times \prod_{i \neq j} X_j = pr_i^{-1}(M_i)$ and hence $N \subseteq (pr_i \circ f)^{-1}(M_i)$ and we obtain that $pr_i \circ f$ is almost contra (b, μ)-continuous for each $i \in I$.

Definition 3.12

A λ-graph G(f) of a function $f : (X, \lambda) \to (Y, \mu)$ is said to be strongly contra λ-b-closed if for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a λ-b-open set M in X containing x and a λ-regular closed set N in Y containing y such that $(M \times N) \cap G(f) = \phi$.

Lemma 3.13

The following properties are equivalent for the λ-graph G(f) of a function f:

(i) G(f) is strongly contra λ-b-closed;

(ii) for each $(x, y) \in (X \times Y) \setminus G(f)$, there exist a λ-b-open set M in X containing x and a λ-regular closed set N in Y containing y such that $f(M) \cap N = \phi$.

Theorem 3.14

If $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous and (Y, μ) is μ-Urysohn, then G(f) is strongly contra λ-b-closed in $X \times Y$.

Proof

Suppose that (Y, μ) is μ-Urysohn. Let $(x, y) \in (X \times Y) \setminus G(f)$. It follows that $f(x) \neq y$. Since Y is μ-Urysohn, there exist λ-open sets M and N in Y with $f(x) \in M$ and $y \in N$ such that $c_\mu(M) \cap c_\mu(N) = \phi$. Since f is almost contra (b, μ)-continuous, there exists a λ-b-open set P in X containing x such that $f(P) \subseteq c_\mu(M)$. Therefore, we obtain $f(P) \cap c_\mu(N) = \phi$ and G(f) is strongly contra λ-b-closed in $X \times Y$.

Theorem 3.15

Let $f : (X, \lambda) \to (Y, \mu)$ have a strongly contra λ-b-closed graph G(f). If f is injective, then X is λ-b-T_1.

Proof

Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. By Lemma 3.13, there exist a λ-b-open set M in X and a λ-regular closed set N in Y such that $x \in M$, $f(y) \in N$ and $f(M) \cap N = \phi$; hence $M \cap f^{-1}(N) = \phi$. Therefore, we have $y \notin M$. This implies that X is λ-b-T_1.

4. THE RELATIONSHIPS

In this section, the relationships between almost contra (b, μ)-continuous functions and other forms are investigated.

Definition 4.1

Let (X, λ) and (Y, μ) be generalized topological spaces. A function $f : (X, \lambda) \to (Y, \mu)$ is said to be weakly almost contra (b, μ)-continuous if for each point $x \in X$ and each λ-regular closed set M in Y containing $f(x)$, there exists a λ-b-open set N in X containing x such that $f(N) \subseteq M$.

Definition 4.2

Let (X, λ) and (Y, μ) be generalized topological spaces. A function $f : (X, \lambda) \to (Y, \mu)$ is said to be λ-(b, σ)-open if the image of each λ-b-open set is μ-semi-open.

Theorem 4.3

If a function $f : (X, \lambda) \to (Y, \mu)$ is weakly almost contra (b, μ)-continuous and λ-(b, σ)-open, then f is almost contra (b, μ)-continuous.

Proof

Let $x \in X$ and M be a μ-regular closed set containing $f(x)$. Since f is weakly almost contra (b, μ)-continuous, there exists a λ-b-open set N in X containing x such that $i_\lambda(f(N)) \subseteq M$. Since f is λ-(b, σ)-open, $f(N)$ is a μ-semi-open set in Y and $f(N) \subseteq c_\lambda(i_\lambda(f(N))) \subseteq M$. This shows that f is almost contra (b, μ)-continuous.

Definition 4.4

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be

(i) almost contra (λ, μ)-continuous if for each μ-regular open set U in Y, $f^{-1}(U)$ is λ-closed in X,
(ii) almost contra (α, μ)-continuous if for each μ-regular open set U in Y, $f^{-1}(U)$ is λ-α-closed in X,
(iii) almost contra (σ, μ)-continuous if for each μ-regular open set U in Y, $f^{-1}(U)$ is λ-semi-closed in X,
(iv) almost contra (π, μ)-continuous if for each μ-regular open set U in Y, $f^{-1}(U)$ is λ-preclosed in X,
(v) almost contra (β, μ)-continuous if for each μ-regular open set U in Y, $f^{-1}(U)$ is λ-β-closed in X.

Remark 4.5

Let $f : (X, \lambda) \to (Y, \mu)$ be a function between GTS's (X, λ) and (Y, μ). Then we have the following implications.

$$\begin{tikzcd}
 & 2 \arrow[r] \arrow[d] & 3 \arrow[d] \arrow[dr] & \\
1 \arrow[ur] \arrow[dr] & & & 6 \\
 & 4 \arrow[r] & 5 \arrow[ur] &
\end{tikzcd}$$

where 1. almost contra (λ, μ)-continuous

2. almost contra (α, μ)-continuous

3. almost contra (π, μ)-continuous

4. almost contra (σ, μ)-continuous

5. almost contra (b, μ)-continuous

6. almost contra (β, μ)-continuous

Remark 4.6

The reverse implications may not be true in general and this can be clearly seen from the following examples.

Example 4.7

Let $X = Y = \{a, b, c\}$. Consider two generalized topologies $\lambda = \{\phi, \{b\}, \{c\}, \{b, c\}\}$ and $\mu = \{\phi, \{c\}\}$ on X. Define $f : (X, \lambda) \to (Y, \mu)$ as follows: $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then $f^{-1}(\{c\}) = \{b\}$. We have f is almost contra (α, μ)-continuous and almost contra (σ, μ)-continuous but not almost contra (λ, μ)-continuous.

Example 4.8

Let $X = Y = \{a, b, c\}$. Consider two generalized topologies $\lambda = \{\phi, \{b\}, \{c\}, \{b, c\}\}$ and $\mu = \{\phi, \{c\}\}$ on X. Define $f : (X, \lambda) \to (Y, \mu)$ as follows: $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then $f^{-1}(\{c\}) = \{a\}$. We have f is almost contra (β, μ)-continuous, almost contra (b, μ)-continuous and almost contra (σ, μ)-continuous but not almost contra (π, μ)-continuous. Moreover f is not almost contra (α, μ)-continuous.

Example 4.9

Let $X = Y = \{a, b, c\}$. Consider two generalized topologies $\lambda = \{\phi, \{a, c\}\}$ and $\mu = \{\phi, \{a\}\}$ on X. Define $f : (X, \lambda) \to (Y, \mu)$ as follows: $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then $f^{-1}(\{a\}) = \{c\}$. We have f is almost contra (π, μ)-continuous and almost contra (b, μ)-continuous but not almost contra (σ, μ)-continuous. Moreover f is not almost contra (α, μ)-continuous.

Definition 4.10

A GTS (X, λ) is said to be λ-P_Σ if for any λ-open set M of X and each $x \in X$, there exists a λ-regular closed set N- containing x such that $x \in N \subseteq M$.

Theorem 4.11

If $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous and Y is μ-P_Σ, then f is (b, μ)-continuous.

Proof

Let M be a μ-open set in Y. Since Y is λ-P_Σ, there exists a family Ψ whose members are μ-regular closed sets of Y such that $M = \cup \{N: N \in \Psi\}$. Since f is almost contra (b, μ)-continuous, $f^{-1}(N)$ is λ-b-open in X for each $N \in \Psi$ and hence $f^{-1}(M)$ is λ-b-open in X. Therefore f is (b, μ)-continuous.

Definition 4.12

A GTS is said to be weakly λ-P_Σ if for any λ-regular open set M of X and each $x \in X$, there exists a λ-regular closed set N-containing x such that $x \in N \subseteq M$.

Definition 4.13

A function $f : (X, \lambda) \to (Y, \mu)$ is said to be almost (b, μ)-continuous at $x \in X$ if for each λ-open set M containing f(x), there exists a λ-b-open set N containing x such that $f(N) \subseteq i_\lambda(c_\lambda(M))$.

Theorem 4.14

Let $f : (X, \lambda) \to (Y, \mu)$ be a almost contra (b, μ)-continuous function. If Y is weakly μ-P_Σ, then f is almost (b, μ)-continuous.

Proof

Let M be a μ-regular open set in Y. Since Y is weakly μ-P_Σ, there exists a family Ψ whose members are μ-regular closed sets of Y such that $M = \cup\{N: N \in \Psi\}$. Since f is almost contra (b, μ)-continuous, $f^{-1}(N)$ is λ-b-open in X for each $N \in \Psi$ and hence $f^{-1}(M)$ is λ-b-open in X. Therefore f is almost (b, μ)-continuous.

Definition 4.15

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be λ-b-irresolute if the inverse image of each μ-b-open setin Y is λ-b-open in X.

Theorem 4.16

Let (X, λ), (Y, μ), (Z, γ) be GTS's and let $f : (X, \lambda) \to (Y, \mu)$ and $g : (Y, \mu) \to (Z, \gamma)$ be functions. If f is λ-b-irresolute and g is almost contra (b, μ)-continuous, then $g \circ f : (X, \lambda) \to (Z, \gamma)$ is almost contra (b, μ)-continuous function.

Proof

Let $M \subseteq Z$ be any γ-regular closed set and let $(g \circ f)(x) \in M$. Then $g(f(x)) \in M$. Since g is almost contra (b, μ)-continuous function, it follows that there exists a μ-b-open set N containing f(x) such that $g(N) \subseteq M$. Since f is λ-b-irresolute function, it follows that there exists a λ-b-open set P containing x such that $f(P) \subseteq N$. From here we obtain that $(g \circ f)(P) = g(f(P)) \subseteq g(N) \subseteq M$. Thus we show that $g \circ f$ is almost contra (b, μ)-continuous function.

Definition 4.17

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be λ-b-open if the image of each λ-b-open set is μ-open.

Definition 4.18

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be always λ-b-open if the image of each λ-b-open set is μ -b-open.

Theorem 4.19

Let (X, λ), (Y, μ), (Z, γ) be GTS's. If $f : (X, \lambda) \to (Y, \mu)$ is a surjective λ-b-open function and $g : (Y, \mu) \to (Z, \gamma)$ is a function such that $g \circ f : (X, \lambda) \to (Z, \gamma)$ is almost contra (b, μ)-continuous, then g is almost contra (b, μ)-continuous function.

Proof

Suppose that x is a singleton in X. Let M be a γ-regular closed set in Z containing $(g \circ f)(x)$. Then there exists a λ-b-open set N in X containing x such that $g(f(N)) \subseteq M$. Since f is λ-b-open, f(N) is μ-b-open in Y containing f(x) such that $g(f(N)) \subseteq M$. This implies that g is almost contra (b, μ)-continuous function.

Corollary 4.20

Let (X, λ), (Y, μ), (Z, γ) be GTS's. If $f : (X, \lambda) \to (Y, \mu)$ is a surjective λ-b- irresolute and λ-b-open function and let $g : (Y, \mu) \to (Z, \gamma)$ be a function. Then, $g \circ f : (X, \lambda) \to (Z, \gamma)$ is almost contra (b, μ)-continuous if and only if g is almost contra (b, μ)-continuous function.

Proof

It can be obtained from Theorem 4.16 and Theorem 4.19.

Definition 4.21

A GTS (X, λ) is said to be λ-R-closed if every λ-regular closed cover of X has a finite subcover.

Theorem 4.22

The almost contra (b, μ)-continuous images of λ-b-compact spaces are μ-R-closed.

Proof

Suppose that $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous surjection. Let $\{M_i: i \in I\}$ be any μ-regular closed cover of Y. Since f is almost contra (b, μ)-continuous, then $\{f^{-1}(M_i): i \in I\}$ is a λ-b-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(M_i): i \in I_0\}$. Therefore, we have $Y = \cup\{M_i: i \in I_0\}$ and Y is μ-R-closed.

Definition 4.23

A GTS (X, λ) is said to be

(i) λ-b-closed compact if every λ-b-closed cover of X has a finite subcover.

(ii) nearly λ-compact if every λ-regular open cover of X has a finite subcover.

Theorem 4.24

The almost contra (b, μ)-continuous images of λ-b-closed compact spaces are nearly μ- compact.

Proof

Suppose that $f : (X, \lambda) \to (Y, \mu)$ is almost contra (b, μ)-continuous surjection. Let $\{M_i : i \in I\}$ be any μ-regular open cover of Y. Since f is almost contra (b, μ)-continuous, then $\{f^{-1}(M_i) : i \in I\}$ is a λ-b-closed cover of X. Since X is λ-b-closed compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(M_i) : i \in I_0\}$. Therefore, we have $Y = \cup\{M_i : i \in I_0\}$ and Y is nearly μ-compact.

Definition 4.25

Let $f : (X, \lambda) \to (Y, \mu)$ be a function. Then f is said to be

(i) (σ, μ)-open if the image of every λ-open set of X is μ-semi-open in Y,
(ii) (π, μ)-open if the image of every λ-open set of X is μ-preopen in Y,
(iii) (α, μ)-open if the image of every λ-open set of X is μ-α-open in Y,
(iv) (β, μ)-open if the image of every λ-open set of X is μ-β-open in Y,
(v) (b, μ)-open if the image of every λ-open set of X is μ-b-open in Y,
(vi) (λ-LC, μ) open if the image of every λ-open set of X is μ-locally μ-closed set in Y.

Theorem 4.26

Let $f : (X, \lambda) \to (Y, \mu)$, where Y is a λ-extremally disconnected space be a function. Then the following properties are equivalent.

(i) f is (λ, μ)-open.
(ii) f is (α, μ)-open and (λ-LC, μ) open.
(iii) f is (π, μ)-open and (λ-LC, μ) open.
(iv) f is (σ, μ)-open and (λ-LC, μ) open.
(v) f is (b, μ)-open and (λ-LC, μ) open.

Proof

It is obvious.

Definition 4.27

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be

(i) almost (λ, μ)-continuous if the inverse image of every μ-regular open set in Y is λ-open in X.
(ii) almost (α, μ)-continuous if the inverse image of every μ-regular open set in Y is λ-α-open in X.
(iii) almost (σ, μ)-continuous if the inverse image of every μ-regular open set in Y is λ-semi-open in X.
(iv) almost (π, μ)-continuous if the inverse image of every μ-regular open set in Y is λ-preopen in X
(v) almost (b, μ)-continuous if the inverse image of each μ-regular open set in Y is λ-b-open in X.
(vi) almost (λ-LC, μ)-continuous if the inverse image of each μ-regular open set in Y is λ-locally λ-closed in X.

Theorem 4.28

Let $f : (X, \lambda) \to (Y, \mu)$, where Y is a λ-extremally disconnected space be a function. Then the following properties are equivalent.

(i) f is a almost (λ, μ)-continuous.

(ii) f is almost (α, μ)-continuous and almost $(\lambda\text{-LC}, \mu)$-continuous.
(iii) f is almost (π, μ)-continuous and almost $(\lambda\text{-LC}, \mu)$-continuous.
(iv) f is almost (σ, μ)-continuous and almost $(\lambda\text{-LC}, \mu)$-continuous.
(v) f is almost (b, μ)-continuous and almost $(\lambda\text{-LC}, \mu)$-continuous.

Definition 4.29

Let (X, λ) and (Y, μ) be GTS's. Then a function $f : (X, \lambda) \to (Y, \mu)$ is said to be almost contra $(\lambda\text{-LC}, \mu)$-continuous if the inverse image of each μ-regular closed set in Y is λ-locally λ-closed in X.

Theorem 4.30

Let $f : (X, \lambda) \to (Y, \mu)$, where Y is a λ-extremally disconnected space be a function. Then the following properties are equivalent.

(i) f is a almost contra (λ, μ)-continuous.
(ii) f is almost contra (α, μ)-continuous and almost contra $(\lambda\text{-LC}, \mu)$-continuous.
(iii) f is almost contra (π, μ)-continuous and almost contra $(\lambda\text{-LC}, \mu)$-continuous.
(iv) f is almost contra (σ, μ)-continuous and almost contra $(\lambda\text{-LC}, \mu)$-continuous.
(v) f is almost contra (b, μ)-continuous and almost contra $(\lambda\text{-LC}, \mu)$-continuous.

Proof

It is obvious.

REFERENCES

[1] A. Al-omari and T. Noiri, A unified theory of contra-(μ, λ)-continuous functions in generalized topological spaces, Acta Math. Hungar., 135 (1-2) (2011), 31-41, doi: 10.1007/s10474-011-0143-x.
[2] A. Cs□sz□r, Generalized open sets, Acta Math. Hungar. 75(1-2) (1997), 65-87, doi: 10.1023/A: 1006582718102.
[3] A. Cs□sz□r, Generalized open sets in generalized topologies, Acta Math. Hungar., 106 (1-2) (2005), 53-66, doi: 10.1007/s10474-005-0005-5.
[4] A. Cs□sz□r, Generalized topology, generalized continuity, Acta Math. Hungar. 96 (4) (2002), 351-357, doi: 10.1023/A:1019713018007.
[5] R. Jamunarani and P. Jeyanthi, Regular sets in generalized topological spaces, Acta Math. Hungar., 135 (4) (2012), 342-349, , doi: 10.1007/s10474-011-0179-y.
[6] D. Jayanthi, Contra continuity on generalized topological spaces, Act Math. Hungar. (2012), 1-9, doi:: 10.1007/s10474-012-0211-x.
[7] W. K. Min, Almost continuity on generalized topological spaces, Acta Math. Hungar. 125 (1-2) (2009), 121-125, doi: 10.1007/s10474-009-8230-y.
[8] W. K. Min, Generalized continuous functions defined by generalized open sets on generalized topological spaces, Acta Math. Hungar. 128 (4) (2009), 299-306, doi: 10.1007/s10474-009-9037-6.
[9] W. K. Min, Some results on generalized topological spaces and generalized systems, Acta Math. Hungar. 108 (1-2) (2005), 171-181, doi: 10.1007/s10474-005-0218-7.
[10] W. K. Min, Weak continuity on generalized topological spaces, Acta Math. Hungar. 124 (1-2) 2008, 73-81, doi: 10.1007/s10474-008-8152-0.
[11] P. Santhi, R. Poovazhaki and O. Ravi, On Slightly (b, μ)-Continuous Functions, submitted.
[12] R. D. Sarma, on extremally disconnected generalized topologies, Acta Math. Hungar. 134 (4) (2012), 583-588, doi: 10.1007/s10474-011-0153-8.
[13] R. X. Shen, A note on generalized connectedness, Acta Math. Hungar. 122 (3) (2008), 231-235, doi: 10.1007/s10474-008-8009-6.
[14] P. Sivagami, Remarks on γ-interior, Acta Math. Hungar. 119 (1-2) (2008), 81-94, doi: 10.1007/s1 047-007-7007-4.