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## Wavelet Analytical Study of Solar Wind Proton Density

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**Abstract:** *The solar wind is plasma, i.e., an ionized gas that fills the solar system. It results from the supersonic expansion of the solar corona. The solar wind consists primarily of electrons and protons with a smattering of alpha particles and other ionic species at low abundance levels. The structure of the heliosphere is strongly affected by the protons produced by the photo-ionization of the interstellar neutral hydrogen and by the charge exchange of the hydrogen atoms. The wavelet is a new analytical tool for turbulent or chaotic data to the physics community. It allows detection and characterization of short-lived structures in turbulence. Proton density fluctuations are studied using discrete wavelet transforms.*

**Keywords:** *Proton Density, Solar Wind, Turbulent Plasma, Wavelet, Wavelet Transforms*

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### I. INTRODUCTION

The Sun is an ideal laboratory since in-situ and remote sensing observations can give important details about our star, in particular, and some clues to understanding winds from other stars. The solar atmosphere is not in hydrostatic equilibrium but must expand into the interplanetary medium as the wind [1]. The density fluctuation spectrum evolves quickly during the transition from the outer corona to the interplanetary space, indicates that the Kolmogorov spectrum for density fluctuations is forming in a turbulent environment due to strong nonlinear interactions. Perhaps at small distances from the Sun density fluctuations might be strongly affected by transported structures, so that they can be considered as passively transported and in this case, spectral properties of fluctuations are subject to a Kolmogorov phenomenology [2].

Thus, the study of turbulence of solar wind plasma has attracted a growing interest. The Fourier and Fast Fourier Transforms (FFT) are a useful tool to study the power spectrum of stationary time series. Wavelet extracts both time evolution and frequency composition of a signal, while Fourier sine's extracts only frequency information from a time signal, thus losing time information ([3], [4], [5]). The wavelet is a new analytical tool for turbulent or chaotic data to the physics community. It allows detection and characterization of short-lived structures in turbulence. The main advantage of using the wavelet transform is that it preserves the information about local features (e. g. singularities) of the signal and allows reconstruction of the signal over a given range of scales. This property is of particular importance in studying turbulence, which often shows coherent structures apparently related to nonlinear processes ([6], [7]).

The wavelet analytical technique is able to disentangle coherent vortices from incoherent background flow of turbulence. Both components are multi-scale but present different statistics with different correlations. Wavelet techniques applied to histograms can be considered as a subset of density estimations in statistical analysis [8]. Skewness is a measurement of symmetry, or more precisely, the lack of symmetry. Kurtosis is a measurement of whether the data are peaked or flat relative to a normal distribution. That is, data sets with high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have a heavy tail. Skewness and kurtosis parameter for solar wind velocity are discussed.

### II. THEORETICAL BACKGROUND

The theory of turbulence is that at very large Reynolds numbers the injection scale  $L_0$  and the dissipative scale  $\ell_D$  are completely separated. In a stationary situation, the energy injection rate must be balanced by the energy dissipation rate and must also be the same as the energy transfer rate  $\varepsilon$  measured at any scale  $\ell$  within the inertial range  $\ell_D \ll \ell \ll L_0$ . Phenomenology for MHD turbulence does not differ from the fluid case, and hence the energy spectrum is going to follow a Kolmogorov  $k^{-5/3}$  law ([9], [10]). The fluctuations of the proton density in a Solar wind have an irregular, complicated and unpredictable evolution in time and space, as captured in a typical measurement of turbulence produced in the laboratory, where the different traces are registered at closely spaced points. The magnetosphere is holding but it is being seared by the very-dense solar winds emanating from the solar prominence that erupted on the Sun. The storm is only grazing the Earth's magnetosphere and the planet was spared a full frontal assault from the solar eruption but it is, nonetheless, still causing quite a stir ([11], [12]).

The differential energy spectra of incident protons are derived from the solar proton fluxes measured on the board of the satellites and used to calculate the ionization rates at the altitudes 50- 100 km using an energy deposition model. Proton density counts in the solar wind stream have been changing dramatically over the year. It is important for the understanding of solar wind variability to determine if these fluctuations are generated during transit from the corona to 1 AU, or if they are signatures of a process occurring in the corona in the formation of the solar wind. Periodic density structures could be created through local compressive processes; given the observed plasma and magnetic field behavior, it is not obvious what that local process would be. An alternative explanation is that these structures form in the solar corona, are frozen into the solar wind, and connect out to Earth. This results in plasma expansion, magnetic reconnection at the tip of the helmet streamer and subsequent release of plasmoids into the solar wind. The frequency spectrum of fluctuations has been obtained after a linear trending and tapering of the time series, applied to remove high-frequency spurious components in the spectral analysis. The Fourier analysis of the UVCS and Helios 2 data reveals the existence of significant power in the spectra derived from the proton density fluctuations at the coronal and heliospheric levels, both in the low- and high-speed solar wind. Hence, the coronal region where the slow wind is generated is perhaps permeated by stochastic fluctuations which cannot be described by a simple Brownian motion, rather some long-range correlations must be present. These correlations should be the low-frequency driving for the occurrence of a turbulent energy cascade which characterizes the transition to the heliosphere, where the spectral slope becomes close to a Kolmogorov-like spectrum, as usually observed in the solar wind turbulence [13]. The characterization and understanding of strong turbulence are especially urgent in the field of plasma and astrophysics, where the so-called anomalous transport which deteriorates the energy confinement due to turbulence is an important phenomenon that is far from understood.

*A. Multiresolution analysis*

The discrete wavelet transform (DWT) provides a frequency band-wise decomposition of the signal, which is, called multiresolution analysis (MRA). A multiresolution analysis (MRA) for  $L^2(\mathbb{R})$  introduced by Mallat [14] and extended by other researchers ([15], [16]), consists of a Sequence  $V_j: j \in \mathbb{Z}$  of closed subspaces of  $L^2(\mathbb{R})$  satisfying following properties;

- (i)  $V_{j+1} \subset V_j: j \in \mathbb{Z}$ ,
- (ii)  $\bigcap_{j \in \mathbb{Z}} V_j = \{0\}$ ,  $\overline{\bigcup_{j \in \mathbb{Z}} V_j} = L^2(\mathbb{R})$
- (iii) For every  $L^2(\mathbb{R})$ ,  $f(x) \in V_j \Rightarrow f\left(\frac{x}{2}\right) \in V_{j+1}$ ,  $\forall j \in \mathbb{Z}$
- (iv) There exists a function  $\phi(x) \in V_0$  such that  $\{\phi(x - k) : k \in \mathbb{Z}\}$  is orthonormal basis of  $V_0$ .

The function  $\phi$  is called a scaling function of the given MRA. Since

$$V_{j+1} = V_j \oplus W_j$$

where

$$V_{j+1} = span \left( \overline{\phi_{j+1,k}(x)} \right)$$

$$V_j = span \left( \overline{\phi_{j,k}(x)} \right)$$

and

$$W_j = span \left( \overline{\psi_{j,k}(x)} \right), j, k \in \mathbb{Z}$$

Here  $\psi$  represents wavelet function. A signal  $f$  can be decomposed as in simplest form;

$$f = A + D$$

Where  $A$  and  $D$  are called Approximation and Details of the given signal  $f$ . Approximation is average of the signal and hence represents low frequency components, while Detail is the difference of the signal and hence represents high frequency components. Detail plays very important role and provides hidden information of any signal. Approximation and details coefficients are determines as;

Approximation coefficient  $c_{j,k} = \langle f, \phi_{j,k} \rangle$   
 $= \int f(x) \phi_{j,k}(x) dx \quad \forall j, k \in \mathbb{Z}$

and Details coefficient

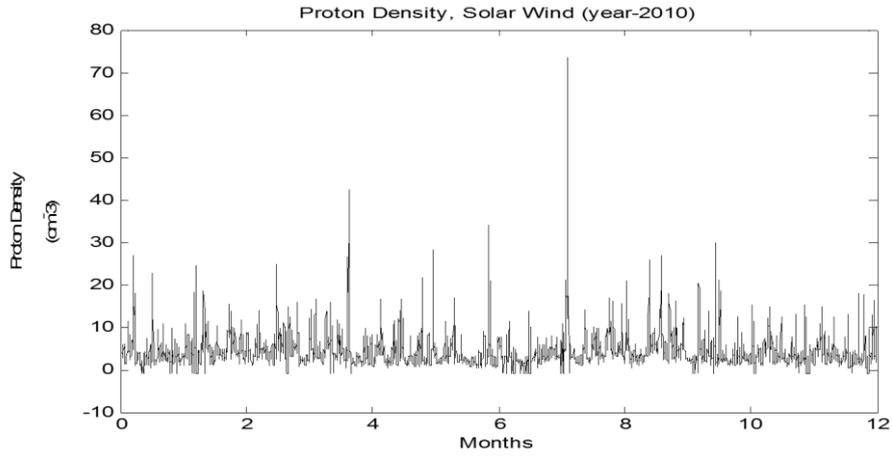
$$d_{j,k} = \langle f, \psi_{j,k} \rangle$$

$$= \int f(x) \psi_{j,k}(x) dx .$$

The wavelet is a new analytical tool for turbulent or chaotic data to the physics community. It allows detection and characterization of short-lived structures in turbulence.

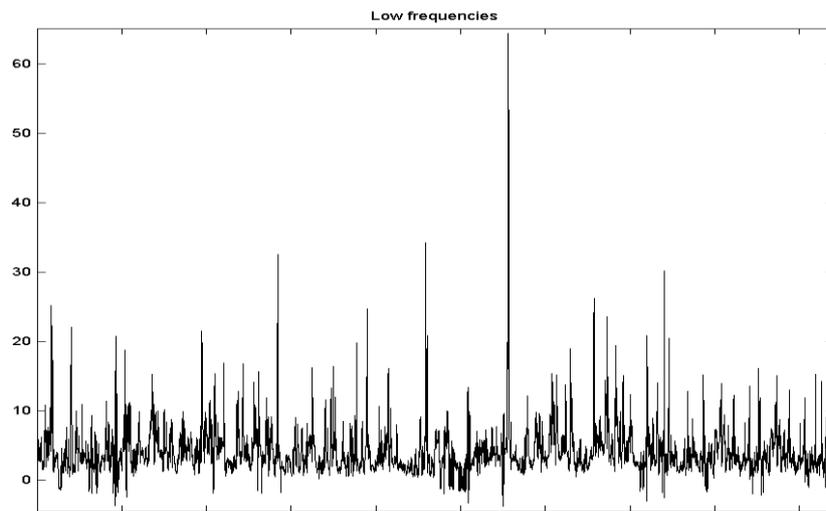
**III. METHODOLOGY**

We have used data of solar wind proton density from SOHO satellite obtained in the public internet address <http://umto.f.umd.edu/pm/crn/> [17] for the year 2010, with a sampling rate of one measurement by the hour. We draw this data and use it as an input signal.

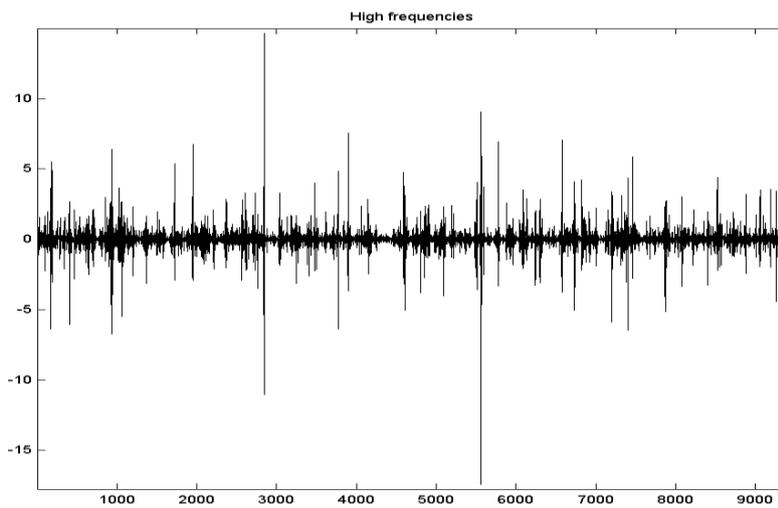


**Fig. 1 (Solar wind proton density)**

Using Daubechies4 wavelet, we decompose above signal and obtain the Approximation and Details coefficients.



**Fig. Fig. 2 (Approximation coefficients)**



**Fig. 3 (Details coefficients)**

Approximation coefficients represent average of signal and Details coefficients represent difference of the signal.

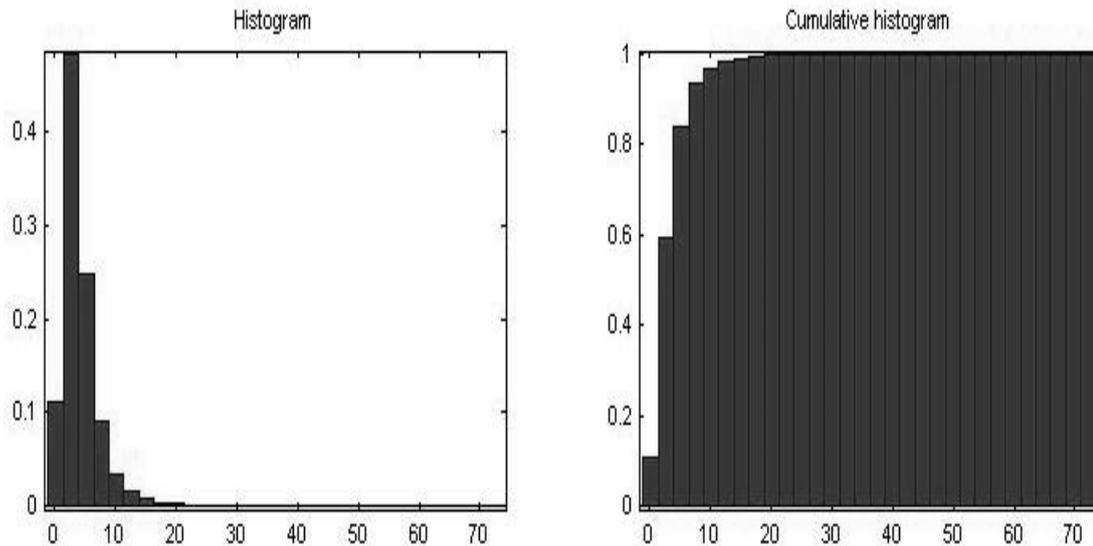


Fig. 4 (Histogram and cumulative histogram)

#### A. Skewness and Kurtosis parameter

A fundamental task in many statistical analyses is to characterize the location and variability of a data set. A further characterization of the data includes skewness and kurtosis [18, 19]. Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point. Kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. That is, data sets with high kurtosis tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. A uniform distribution would be the extreme case. For univariate data  $Y_1, Y_2, \dots, Y_N$ , the formula for skewness is:

$$\text{Skewness parameter } (S) = \frac{\sum_{i=1}^N (Y_i - \bar{Y})^3}{(N-1)s^3}$$

where  $\bar{Y}$  is the mean,  $s$  is the standard deviation, and  $N$  is the number of data points. The skewness for a normal distribution is zero, and any symmetric data should have a skewness near zero. Some measurements have a lower bound and are skewed right. For example, in reliability studies, failure times cannot be negative. For univariate data  $Y_1, Y_2, \dots, Y_N$ , the formula for kurtosis is:

$$\text{Kurtosis parameter } (K) = \frac{\sum_{i=1}^N (Y_i - \bar{Y})^4}{(N-1)s^4}$$

where  $\bar{Y}$  is the mean,  $s$  is the standard deviation, and  $N$  is the number of data points.

### RESULTS

The first time series (Fig. 2) presents low-frequency components only, while other time series (Fig. 3) presents high-frequency components. The histogram of solar wind proton density presents a strong skewness, centered at  $2 \text{ cm}^{-3}$ , as shown in Fig. 4 (a). The bulk density is usually between 0 and  $15 \text{ cm}^{-3}$  with an average of  $2 \text{ cm}^{-3}$ . The skewness parameter is 3.4228 and Kurtosis parameter is 35.6280 for solar wind proton density. From the analyses presented above, it is clear that the proton density time-series are more intermittent than the solar wind velocity. We did not get a negative value of the skewness for proton density.

### CONCLUSION

Approximation and details coefficients are drawn in Fig.2 and Fig. 3 using Daubechies wavelet transforms (db4). The total area of a histogram used for probability density is always normalized to 1. The fig. 4(a) represents a strong skewness centered at  $2 \text{ cm}^{-3}$ . These facts show the presence of gradients in time series. A cumulative histogram is a mapping that counts the cumulative number of observations in all of the bins up to the specified bin. According to the behavior studied above it is possible to conjecture that the difference between both time series can be directly associated with a large number of solar activity. Analyzing the time series it is clear that the energy of high pass filtered time series has a small contribution to the time series as compared with the low-pass filtered time series. The results show that the energy from large scale can be useful for the creation of the coherent structures at large scales (low frequencies).

The behavior of skewness parameter tells about the intermittency phenomenon. Skewness parameter for proton density in the solar wind is positive and Kurtosis parameter is high. The high value of Kurtosis for proton density indicates the strong intermittency in the flow. By Daubechies wavelet spectrum analyses, the disturbances increase the energy in lower periods for both the time-series, but being much efficient in the solar wind velocity time-series. Taking into account these results we have shown that the generalized wavelet analytical approach provides a simple and accurate framework for modeling the statistical behavior of plasma turbulence involved in the solar-terrestrial plasma dynamics. From the analyses presented above, we found that the proton density time-series are highly intermittent.

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