INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X
Impact factor: 4.295
(Volume3, Issue3)
Available online at www.ijariit.com

Application of Graph Theory in Matrix Re-Presentation

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Abstract: Graph is completely determined by specifying either its adjacency structure or its incidence structure. These specifications provide for more efficient ways of representing a large or complicated graph then a pictorial representation. As computers are more adept at manipulating numbers than at recognizing picture. It is standard practice to communicate the specification of a graph to a computer in matrix.

Keyword: Direct Graph, Simple Graph, Graph Node, Edge, Path, Cycle, Circuits.

INTRODUCTION

We have discussed the graph terminology in this paper. A diagrammatic representation of a graph may gave a limited usefulness. We can represent the graph by another important mathematical structure. Matrices which is very useful in computer processing. Here in this paper we represent the graph with the help of matrix having elements O and I, matrices can be used as input data by computers to study graph theory. This paper is actually a simplified representation of graph in matrix.

1. Incidence matrix: let G be a graph with n vertices m edges and without self-loops. The incidence matrix A of G is an n x m matrix A = [aij] whose n rows correspond to the n vertices and the m columns correspond to m edges such that.

$$a_{ij} = \begin{cases} I, & \text{if } j\text{th edge } m_j \text{ is incident on the } i^{th} \text{ vertex} \\ O & \text{Otherwise} \end{cases}$$

It is also called vertex – edge incidence matrix and is denoted by A (G)
Example:

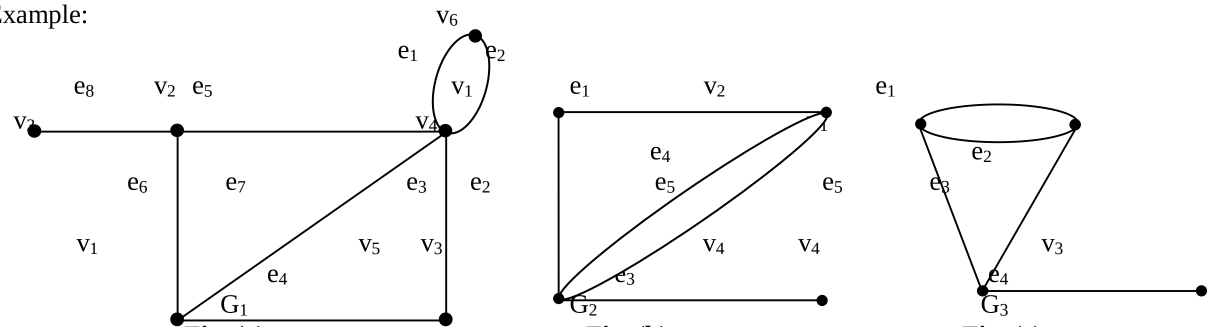

Fig. (a) Fig. (b) Fig. (c)

Consider the graph given in figure.
The incidence matrix G_1 is

$$A(G_1) = \begin{array}{cc} & \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} & \begin{pmatrix} O & O & O & I & O & I & O & O \\ O & O & O & O & I & I & I & I \\ O & O & O & O & O & O & O & I \\ I & I & I & O & I & O & O & O \\ O & O & I & I & O & O & I & O \\ I & I & O & O & O & O & O & O \end{pmatrix} \end{array}$$

The incidence matrix of G_2 is

$$A(G_2) = \begin{array}{cc} & \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} & \begin{pmatrix} I & I & O & O & O \\ I & O & O & I & I \\ O & I & I & I & I \\ O & O & I & O & I \end{pmatrix} \end{array}$$

The incidence matrix of G_3 is

$$A(G_3) = \begin{array}{cc} & \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{array}{c} V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} & \begin{pmatrix} I & I & O & O & I \\ I & I & I & O & O \\ O & O & O & I & O \\ O & O & I & I & I \end{pmatrix} \end{array}$$

The incidence matrix contains only two types of elements O and I this clearly is a binary matrix or a (O, I) matrix.
We have the following observations about the incidence matrix A.

i. Since every edges is incident on exactly two vertices, each column of A has exactly two one's.
ii. The number of one's in each row equals the degree of the corresponding vertex.
iii. A row with all zeros represents an isolated vertex.
iv. Parallel edges in a graph produce identical column in its incidence matrix.
v. If a graph is disconnected and consists G_1 and G_2. The incidence matrix A (G) of graph G can be written in a block diagonal form as.

$$A(G) = \begin{pmatrix} A(G_1) & O \\ O & A(G_2) \end{pmatrix}$$

When $A(G_1)$ and $A(G_2)$ are the incidence matrices of components G_1 and G_2.

vi. Permutation of any two rows or columns in an incidence matrix simply. Corresponds to relabeling the vertices and edges of the same graph.

2. **Path matrix :** Graph G (V, E) is a simple graph with no parallel edges and m vertices labeled $V_1, V_2, V_3 \ldots\ldots\ldots V_m$ then the path matrix for (V_P, V_J) vertices is denoted by $P(V_P, V_J)$ or $[P_{ij}]$, where

$$P_{ij} = \begin{cases} I & \text{IJ there is an edge between } V_l \text{ and } V_j \\ O & \text{Otherwise} \end{cases}$$

The row in by $P_{ij} = P(V_P, V_J)$ correspondence to different paths between vertices V_i and V_j and column corresponds to the edge in graph G.

Example:

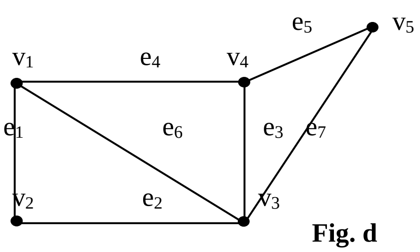

Fig. d

Consider the graph G in fig (d) with vertices V_1, V_2, V_3, V_4 and V_5 there are three different paths from V_1 to V_3.
$P_1 = \{ e_6 \}$, $P_2 = \{ e_4 , e_3 \}$, $P_3 = \{ e_1 , e_2 \}$ and seven edge in G as, e_1, e_2, e_3, e_4, e_5, e_6 an e_7 the path matrix P (V_1, V_3) is of order 3 x 7 is given us

$$P(V_1, V_3) = P_{13} = \begin{array}{c c} & \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \end{array} & \begin{pmatrix} O & O & O & O & O & I & O \\ O & O & I & I & O & O & O \\ I & I & O & O & O & O & O \end{pmatrix} \end{array}$$

We have the following observation about the path matrix.

i. All O's entry of a column corresponds to an edge that does not lie any path between V_i and V_j.
ii. All I's entry of a column correspond to and edge that line in every path between V_i and V_j.
iii. There is no information about the loop of a graph by a path matrix P (V_i, V_j).
iv. If two vertices lie in different components then we cannot define the path matrix. It implies that path matrix is possible only for connected graph.
v. In a path matrix there exist at least one I's in it.
vi. Number of I's in a raw is equal to the length of path matrix corresponding to the row.
vii. The ring sum of any two rows in P (V_i, V_j) corresponds to a circuit or an edge disjoint union or circuits.
viii. The path matrix of a tree is a row matrix.
ix. If two rows are identical there are multiple edges between the vertices which lie in the path corresponds to raw.

3. Circuit matrix: A graph G = (V , E) with n different circuits and t edges. Then the circuit matrix C = [c_{ij}] of graph G is a n x t binary matrix defined as follows :

$$C(G) = C_{ij} = \begin{cases} I & \text{If } I^{th} \text{ circuit consists of } j^{th} \text{ edge} \\ O & \text{Otherwise} \end{cases}$$

The circuit matrix of the graph given us

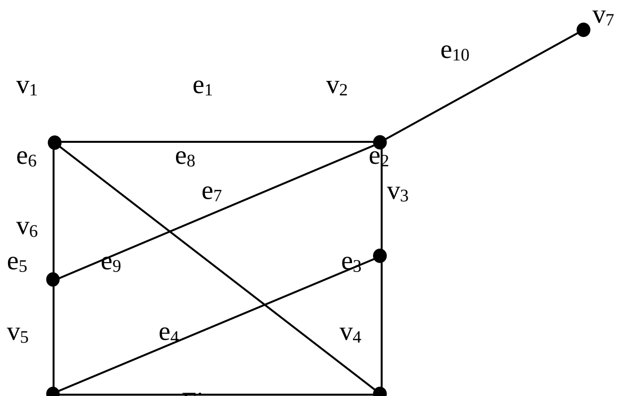

Fig. e

$$C = \begin{array}{c c} & \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array} \\ \begin{array}{c} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} & \begin{pmatrix} I & O & O & 0 & O & I & O & I & O & O \\ O & O & I & I & O & O & O & O & I & O \\ O & I & O & O & I & O & O & I & I & O \\ I & I & I & O & O & O & I & O & O & O \\ O & O & 0 & I & I & I & O & I & O & O \\ I & I & I & I & I & I & O & O & O & O \end{pmatrix} \end{array}$$

We have the following observation about the circuit matrix.

i. All O's entry of a column corresponds to an edge that does not belong to any circuit.
ii. The circuit matrix of a disconnected graph can be written in a block diagonal form as

$$C(G) = \begin{pmatrix} C(g_1) & I & O \\ -- & I & -- \\ O & I & C(g_2) \end{pmatrix}$$

where $C(g_1)$ and $C(g_2)$ are circuit matrixes of two components g_1 and g_2. It implies that there is no common edge between the circuit g_1 and g_2.

iii. The number of I's in a row is equal to the number of edges in the corresponding circuit.
iv. Two graph G_1 and G_2 are 2 – isomorphic If they have same circuit matrix.
v. A row with exactly one I's in corresponds to a self-loop.
vi. Permutation matrix of circuit matrix corresponds to relabeling the circuits and edges.

CONCLUSION

The main aim of this paper is to present the importance of graph theoretical idea in matrix.

Researcher may set some information related to graph theory and matrix and can get some ideas related to their field of research.

REFERENCE

1. Graph theory with application by NARSING DEV
2. Combinotrics and graph theory by S.B.SINGH
3. Graph theory by G. SURESH SINGH
4. Discrete mathematic and graph theory by BHAVANARI SATYA NARAYANA, KUNCHAM Shyam Prasad.
5. Discrete mathematics and its applications (with combinotrics and Graph theory) by KAMALA KRITHIVASAN.
6. A beginner's guide to Graph theory by W.D. WALLIS.
7. Graph theory by Neeraj Pant, SHAHAB FARUGI.
8. Graph theory modelery, applications and algorithms by GEIR AGNARSSON RAYMOND GREENLAW.
9. Harary Graph theory by V. Krishna Murti.
10. Introduction to Graph theory by DOUGLAS B. WEST.
11. Graph theory and combinotrics by RALPHP GRIMALDI.
12. Invitation to Graph theory by S. ARUMUGAM, S. RAMA CHANDRAN.

