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Bitopological Connected Graph

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***Abstract:** A graph G = (V, E) is called a bitopological graph if there exist a set X and a set-indexer f on G such that both f(V) and $f^{\oplus}(E) \cup \phi$ are topologies on X. The corresponding set-indexer is called a bitopological set-indexer of G. We prove the existence of bitopological set-indexer. We give a characterization of bitopological complete graphs. We define equi-bitopological graphs and establish certain results on equi-bitopological graphs. We identify certain classes of graphs, which are bitopological and define bitopological index β_τ (G) of a finite graph G as the minimum cardinality of the underlying set X. We discuss about embedding and NP-completeness problems of some classes of non-bitopological graphs.*

***Keywords:** Topology, Set- indexer, set-Graceful, Embedding and Completeness.*

INTRODUCTION

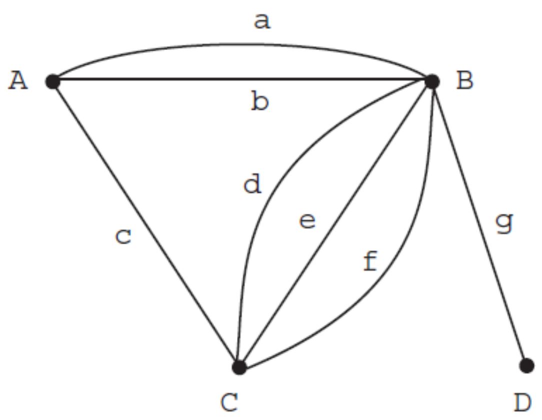
In bitopological graph, let V be a finite set and T be a topology on V . The transitive digraph corresponding to this topology is got by drawing a line from u to v, iff u is in every open set containing v. Conversely , let D be a transitive family digraph on V the family B = {Q(a) : a ∈ V } forms a base for a topology on V, where bases Q(a) = {a}∪ {b ∈ V : (b, a) ∈ E(D)}.

In 1968 T.N. Bhargav and T.J. Ahlborn analysed the topological spaces associated with digraphs. According to them a subset A of V (D) is open if and only if for every pair of points i, j ∈ V with j in A and i not in A, (i, j) is not a line in D. Sampath kumar extended this results to the case in which the point set is infinite. Sampath kumar also investigated the topological spaces associated with signed graphs and semigraphics. Let S = (V, E, σ) be a signed graph. A subset A of V is an open set in the positive E-topology on S denoted by $\tau^+(S)$ iff u ∈ A, uv ∈ $E^+(S)$ implies that v ∈ A. Similarly he defined negative E-topology $\tau^-(S)$. He defined the topology τ_V on the vertex set V (D) of a disemigraph D = (V, E) as follows: A subset S of V (D) is open whenever u ∈ S and v ∈ V (D) such that vu is a partial arc, then v ∈ S.

In 1983, Acharya established another link between graph theories and point-set topology. He proved that for every graph G, there exists a set X and a set-indexer f : V (G) → 2^X such that the family f(V) is a topology on X.

In 2005 Antoine Vella tried to express combinatorial concepts in topological language. As a part of his investigation, he defined the classical topology. Given a hyper-graph H, the classical topology on $V_H \cup E_H$ is the collection of all sets U such that, if U contains a vertex v, then it also contains all hyper-edges incident with v. It is interesting to note that all these topologies are either defined on the vertex set, or on the union of vertex set and edge set. Here we study a special case of set-indexed graphs in which both f(V) and $f^{\oplus}(E) \cup \phi$ are topologies on the ground set X of the set-indexer f. We prove that a uniform binary tree with one pendant edge added at the root vertex is set-graceful and establish a necessary and sufficient condition for the graph H + K_m to be set-graceful, where H is a set-graceful graph with n edges and n + 1 vertices. We characterize complete set-graceful tripartite graph $K_{1,m,n}$. If G_f is the full augmentation of the graph G, a set-graceful graph, then we prove that the corona of G_f and K_1 is set-graceful. We also give an embedding of a graph into a set-graceful graph and a cycle into an Eulerian set-graceful graph and also we discuss bi set-graceful graphs and establish some results on bi set-graceful graphs with characterization of the r-regular connected bi set-graceful graph. It deals with set-sequential graphs. We characterize complete set-sequential bipartite graphs and identify some classes of graphs which are not set-sequential. We also prove that G is set-sequential if and only if G + K_1 is set-graceful. If G is a set-graceful graph then we prove that G ∪ K_t is set-sequential for some positive integer t and also prove that every graph can be embedded into a set sequential graph.

A graph is a pair G = (V, E) of sets satisfying E∈ V × V. The elements of V are the vertices of the graph G, the elements of E are its edges. Graphs are represented graphically by drawing a dot or circle for every vertex, and drawing an arc between two vertices if they are connected by an edge. If the graph is directed, the direction is indicated by drawing an arrow.



Pic-1

TOPOLOGY

A **Topology** on a set *X* is a collection τ of subsets of X , with the following properties:

1. ϕ, X ∈τ .
2. If $u_\alpha \in \tau$,α∈A , then ∪ u_α∈A.
3. If $u_i \in \tau$,i=1,…., n , then ∩ u_i∈τ .

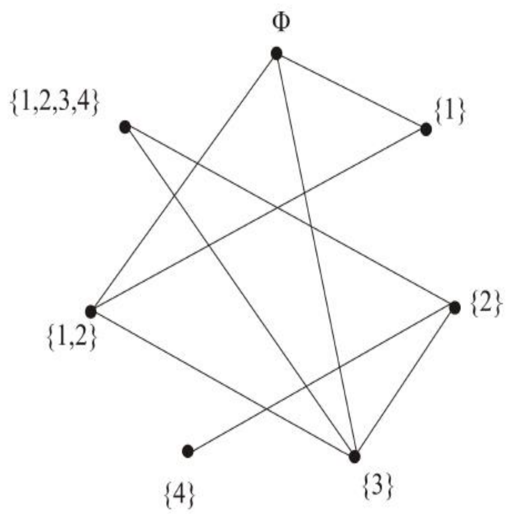
The elements of τ are called open sets.

A graph is one whose vertex set can be partitioned into two subsets X and Y so that each edge has one end in X and one end in Y.Such a partition (X,Y) is called a **bipartition of the graph**.

A **Complete bipartite graph** is a simple bipartite graph with bipartition (X,Y) in which each vertex of X is joined to each vertex of Y. If |X|=m and |Y|=n,such a graph is denoted by $K_{m,n}$ $K_{3,3}$ is an example of a complete bipartite graph.

SET-INDEXER

Let G = (V, E) be a graph, X be a nonempty set and 2^x denote the set of all subsets of X.A **Set- indexer** of G is an injective set-valued function f: V (G) → 2^x such that the function f^Δ: E(G)→ 2^x −ϕ; defined by $f^\Delta(uv)$ = f(u)Δf(v) for every uv ∈E(G) is also injective, where Δ denotes the symmetric difference of sets.

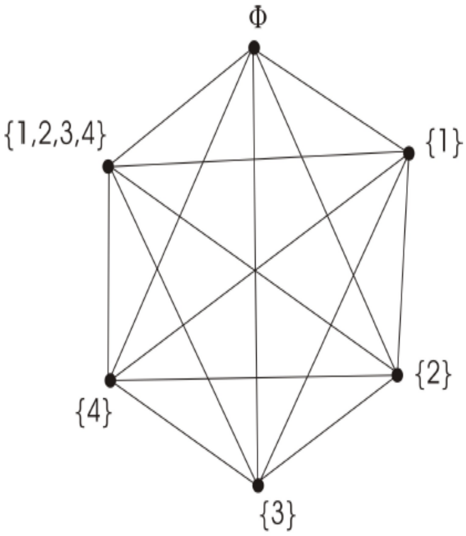


Pic-2

The **K-cube** is the graph whose vertices are the ordered K-tuples of 0's and 1's; Two vertices being joined if and only if they differ exactly in one coordinate. The graph $K_{1,m}$ is called a star for m≥1.

SET-GRACEFUL

A graph G = (V, E) to be **Set-graceful** if there exists a non-empty set X then a set-indexer f : V (G) $\rightarrow 2^x$ such that $f^{\Delta}(E(G)) = 2^x - \{\phi\}$, such an indexer being called a set-graceful labelling of G. A graph G, there exist a set-indexer f : V (G) $\rightarrow 2^x$ such that the family f(V) is a topology on X.



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Two graphs G and H are said to be isomorphic (written $G \cong H$) if there are bijections $\theta:V(G)\rightarrow V(H)$ and $\varphi:E(G)\rightarrow E(H)$ such that $\psi G(e)=uv$ if and only if $\psi H(\varphi(e)) = \theta(u)\theta(v)$;Such a pair (θ,φ) of mappings is called an **Isomorphism** between G and H.

An isomorphism of a graph G onto itself is called an **Automorphism** of G. Let Γ(G) denote the set of all automorphisms of G. Clearly the identity map $i:V\rightarrow V$ defined by i(v)=v is an automorphism of G so that i ϵ Γ(G). Further if α and β are automorphisms of G then α・β and α-1 are also automorphisms of G.

The **Complement** G^C of a simple graph G is the simple graph with vertex set V, two vertices being adjacent in G^C if and only if they are not adjacent in G.

A simple graph G is **Self-complementary** if G is isomorphic to G^c.C_5 is the only cycle which is self-complementary. All other cycles are not self-complementary.

A **Walk** in a graph G is a finite sequence $W=v_0\, e_1\, v_1 e\ldots\ldots v_{k-1}\, e_k\, v_k$ whose terms are alternatively vertices and edges such that each edge e_i is incident with vi_{-1} and vi.

A **Directed graph** D is transitive if whenever the line uv and vw are in D then uw is also in D**.** The **Distance between u and v** in G, denoted by $d_G(u,v)$,is the length of a shortest (u ,v) path in G;If there is no path connecting u and v we define $d_G(u,v)$ to be infinite.

Let mapping ζ of the vertices of G into the set of vertices of a graph H such that the subgraph induced by the set {ζ(u) : u ∈ V (G)} is isomorphic to G.A graph G is said to be embedded in a graph H, written G∠H, if there exists asubgraphG^0 of H such that G is isomorphic to G^0 .

A graph G is said to be Strongly bitopological if G and its line graph L(G) are bitopological. It is observed that every finite path P_n is strongly bitopological, since line graph of a path P_n is path P_{n-1} this implies it is bitopological.

Both the bounds for the inequality $3 \le p + q \le 2^{|X|+1} - 1$ are attainable, as the lower bound is attained by K_2 and the upper bound is attained by $K_{1,2}|X|_{-1}$.

It is to be noted that given a positive integer n, there exist a bitopological set-indexer f of $K_{1,n}$ such that both f(V) and $f^{\oplus}(E) \cup \phi$ give the same topology on the ground set X. Line graph of a bitopological graph may or may not be a bitopological graph. There exist graphs where G and L(G) are isomorphic. We call such graphs strongly bitopological.

A closed trail whose origin and internal vertices are distinct is called a **Cycle.**A cycle of length k is called a k-cycle.A cycle with n vertices is denoted by C_n.

The **Diameter of G** denoted by diam(G),is the maximum distance between two vertices of G. The **Circumference of a graph** G is defined to be the length of its shortest cycle.

Let X be a non empty set, τ_1 and τ_2 be a two non-empty set then (X,τ_1,τ_2) or simply X denote a **Bitopological space**, for any subset $A \subseteq X$, τ_i-int(A) and τ_i-cl(A) denote the interior and closure of a set A with respect to the topology τ_i.

CONCLUSIONS

There is no chance to work the electric circuit board without matrix and graph. By working this way the board has been taking more time to work. So I have chosen the Bitopological connected graph to work this electric circuit board to reduce the time. Hence I conclude that here after we can use the Bitopological connected graph in the circuit board in future.

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