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On Some Statistically Convergent Double Sequence Spaces Defined By Orlicz Functions

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Abstract: In this article we define different type of statistically convergent, statistically null and statistically bounded double sequence spaces on a semi-normed space by Orlicz functions. We study their different properties like solidness, denseness, symmetricity, completeness etc. We obtain some inclusion relations.

Keywords: Orlicz functions, Δ_2-condition, statistical convergence, solid space, complete space.

An Orlicz function M is said to satisfy Δ_2–condition if there exists a constant K > 0 such that M (2u) $\leq$ KM(u) for all values of u $\geq$ 0.

1. INTRODUCTION

The notion of statistical convergence was introduced by Fast [7] and Schoenberg [17] independently. It is also found in Zygmund [21]. Later on it was studied by Fridy and Orhan [8], Maddox [10], Salat [16], Rath and Tripathy [15], Tripathy [19, 20] and many others.

Throughout the article X_E denotes the characteristic function of E. The notion of statistical convergence depends on the density of subsets of N. A subset E of N is said to have density □(E) if

$$□(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{\infty} X_E(k) \text{ exists.}$$

A sequence (x_n) is said to be statistically convergent to L if for every □□> 0

$$□(\{k □ N : |x_k □ L| \geq □□□□\}) = 0,□$$

Throught a double sequence will be denoted by A = <a_{nk}> i.e. a double infinite array of elements a_{nk}, for all n,k □□N. The notion of statistical convergence for double sequences was introduced by Tripathy [20]. For this he introduced the notion of density of subsets of N $\times$ □N as follows:

A subset E of N $\times$ N is said to have density □(E) if

$$□(E) = \lim_{p,q\to\infty} \frac{1}{pq} \sum_{n\leq p} \sum_{k\leq q} X_E(n,k) \text{ exists.}$$

Throughout (X, q) will represent, a semi-normed space seminormed by q.

An Orlicz function M is a mapping M : $[0, \infty) \to [0, \infty)$ such that, it is continuous, non-decreasing and convex with M(0) – 0, M(x) > 0, for x > 0 and M(x) $\to \infty$, as x $\to \infty$.

2. Definitions and Preliminaries

A double sequence A = <a_{nk}> is said to converge in *Pringshcim's* to L if

$\lim_{n,k\to\infty} a_{nk}$ = L, where n and k tend to ∞, independent of each other.

A double sequence <a_{nk}> is said to converge regularly if it converges in Pringsheim's sense and in addition the following limits exist.

$$\lim_{n\to\infty} a_{nk} = L_k \ (k = 1, 2, 3, \ldots)$$

and

$$\lim_{k\to\infty} a_{nk} = J_k \ (k = 1, 2, 3, \ldots)$$

Throughout the article $w^2(q)$, $\ell^2_\infty(q)$, $c^2(q)$, $c^2_0(q)$, $_R c^2(q)$, $_R c^2_0(q)$ will denote the spaces of all, bounded, convergent in Pringsheim's sence, null in Pringsheim's sense, regularly convergent, regularly null X-valued double sequence spaces respectively.

An X-valued double sequence $<a_{nk}>$ is said to be statistically convergent to L if for every □ > 0, □({(n, k)∈ N × N : $q(a_{nk} - L) \geq$ □}) = 0.

An X-valued double sequence A is said to be statistically regularly convergent if it converges in Pringsheim's sense and the following statistical limits exist.

$$\text{stat} - \lim_{n\to\infty} a_{nk} = L_k \ (k = 1, 2, 3, \ldots)$$

and

$$\text{stat} - \lim_{k\to\infty} a_{nk} = J_n \ (n = 1, 2, 3, \ldots)$$

An X-valued double sequence A is said to be statistically bounded if there exists G > 0 such that □({(n, k) : $q(a_{nk}) > G$}) = 0.

For M an Orlicz function, we now introduce the following double sequence spaces :

$$\ell^2_\infty(M, q, □) = \{(a_{nk}, □_{nk}) \in w^2(q) : \sup_{n,k} M\left(q\frac{\lambda_{nk}a_{nk}}{r}\right) < \infty, \text{ for some } r > 0\},$$

$$c^2(M,q,□)=\{(a_{nk}) \in w^2(q): \text{stat} - \lim_{n,k\to\infty} M\left(q\left(\frac{\lambda_{nk}a_{nk} - L}{r}\right)\right) = 0, \text{ for some } r>0 \text{ and } L \in x\}$$

$$c^2_0(M, q, □) = \{(a_{nk}) \in w^2(q) : \text{stat} - \lim_{n,k\to\infty} M\left(q\left(\frac{\lambda_{nk}a_{nk}}{r}\right)\right) = 0, \text{ for some } r > 0\}.$$

A sequence $<a_{nk}> \in {}_R c^2(M, q, □)$ if $<a_{nk}> \in c^2(M, q, □)$ and the following statistical limits exist

$$\text{stat} - \lim_{k\to\infty} M\left(q\left(\frac{\lambda_{nk}a_{nk} - L_n}{r_1}\right)\right) = 0, \text{ for } n = 1, 2, 3, \ldots \qquad \ldots(1)$$

$$\text{stat} - \lim_{n\to\infty} M\left(q\left(\frac{\lambda_{nk}a_{nk} - J_k}{r_2}\right)\right) = 0, \text{ for } k = 1, 2, 3, \ldots \qquad \ldots(2)$$

A sequence $<a_{nk}> \in ({}_R\bar{c}^2_0)\ {}_R c^2_0(M, q, □)$, if $<a_{nk}> \in \bar{c}^2_0(M, q, □)$ and (1) and (2) hold with $L_n = J_k$ = □, the zero element of X, for all n,k ∈ N.

A double sequence space E is said to be solid if $<□_{nk}a_{nk}> \in$ E whenever $<a_{nk}> \in$ E for all sequences $<□_{nk}>$ of scalars with $|□_{nk}| \leq 1$ for all n,k ∈ N.

A double sequence space E is said to be symmetric if $<a_{nk}> \in$ E implies $\langle a_{\pi(n)\pi(k)} \rangle \in$ E, where □ is a permutations on N.

A double sequence space E is said to be sequence algebra if $<a_{nk}b_{nk}> \in$ E, whenever $<a_{nk}>$, $<b_{nk}> \in$ E.

A double sequence space E is said to be monotone if it contains the canonical preimages of all its step spaces.

A double sequence space E is said to be convergence free if $<b_{nk}> \in$ E, whenever $<a_{nk}> \in$ E and a_{nk} = □ implies b_{nk} = □,

Remark 1 : A sequence space E is said implies E is monotone.

The zero single sequence will be denoted by $\bar{\theta}$ = (□□□□□□□□□□□ …..) and the zero duble sequence will be denoted by $\begin{bmatrix} \theta & \theta & \ldots & \theta \\ \theta & \theta & \ldots & \theta \\ \theta & \theta & \ldots & \theta \end{bmatrix}$. Throughout e = (1, 1, 1, …..) and e_k = (0, 0, ….., 1, 0, 0, …..), where the only 1 appear at the place

Throughout the article $\bar{c}^2(M, q, □)$, $\bar{c}^2_0(M, q, □)$, $(\bar{c}^2)^B(M, q, □)$, $(\bar{c}^2_0)^B(M, q, □)$, $(\bar{c}^2)^R(M, q, □)$, $(\bar{c}^2_0)^R(M, q, □)$, $(\bar{c}^2)^{BR}(M, q, □)$, $(\bar{c}^2_0)^{BR}(M, q, □)$ denote the spaces of statistically convergent in Pringsheim sense, statistical null in Pringsheim's sense, bounded statistically convergent in Pringsheim sense, bounded statistically null, bounded regularly convergent, bounded regularly null X-valued double sequences defined by Orlicz function respectively.

3. Main Results

The proof of the following result is a routine verification in view of the existing technique.

Theorem 1. The classes Z(M, q, □), where $Z = \bar{c}^2$, $\bar{c}_0^2$, $(\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^R$, $(\bar{c}_0^2)^R$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ and $\bar{\ell}_\infty^2$ are linear spaces.

Theorem 2. The spaces Z(M, q, λ), where $Z = (\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ and $\bar{\ell}_\infty^2$ are seminormed spaces, seminormed by

$$f(\langle a_{nk}\rangle) = \inf\left\{\rho > 0 : \sup_{n,k} M\left(q\left(\frac{\lambda_{nk}a_{nk}}{\rho}\right)\right) \le 1\right\}.$$

Proof. Since q is a seminorm, so we have $f(A) \ge 0$ for all A : $f(\bar{\theta}^2) = 0$ and $f(□A) = |□|f(A)$ for all scalar □.

Let $\langle a_{nk}\rangle$ and $\langle b_{nk}\rangle$ $(\bar{c}^2)^B$ (m, q, λ). There exist ρ_1, $\rho_2 > 0$ such that

$$\sup_{n,k} M\left(q\left(\frac{\lambda_{nk}a_{nk}}{\rho_1}\right)\right) \le 1$$

and

$$\sup_{n,k} M\left(q\left(\frac{\lambda_{nk}b_{nk}}{\rho_2}\right)\right) \le 1$$

Let □ = □$_1$ + □$_2$. The we have,

$$\sup_{n,k} M\left(q\left(\frac{\lambda_{nk}a_{nk} + \lambda_{nk}b_{nk}}{\rho}\right)\right) \le \left(\frac{\rho_1}{\rho_1+\rho_2}\right) \sup_{n,k} M\left(q\left(\frac{\lambda_{nk}a_{nk}}{\rho_1}\right)\right) □□ \left(\frac{\rho_2}{\rho_1+\rho_2}\right) □ \sup_{n,k} M\left(q\left(\frac{\lambda_{nk}b_{nk}}{\rho_2}\right)\right)$$

Since □$_1$, □$_2$ > 0, so we have

$$f(\langle a_{nk}\rangle + \langle b_{nk}\rangle) = \inf\left\{\rho = \rho_1 + \rho_2 > 0 : \sup_{n,k} M\left(q\left(\frac{\lambda_{nk}a_{nk} + \lambda_{nk}b_{nk}}{\rho}\right)\right) \le 1\right\}$$

$$= \inf\left\{\rho_1 > 0 : \sup_{n,k} M\left(q\left(\frac{\lambda_{nk}a_{nk}}{\rho_1}\right)\right) \le 1\right\} □□□ \inf\left\{\rho_2 > 0 : \sup_{n,k} M\left(q\left(\frac{\lambda_{nk}b_{nk}}{\rho_2}\right)\right) \le 1\right\}$$

$= f(\langle a_{nk}\rangle + \langle b_{nk}\rangle)$.

Hence *f* is a seminorm.

Theorem 3. Let (X, q, □) be a complete semi normed space. Then the spaces Z(M, q, □), where $Z = (\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$, and $\bar{\ell}_\infty^2$ are complete semi-normed spaces semi normed by *f*.

Proof. We prove the theorem for space $(\bar{c}^2)^B$ (M, q, □) and the proof for the other cases can be established following similar technique. Let $A^i = (\lambda_{nk}^i a_{nk}^i)$ be a Cauchy sequence in $(\bar{c}^2)^B$ (M, q, □). We have to show the following:

(i) $\lambda_{nk}^i a_{nk}^i \to \lambda_{nk}a_{nk}$, as $i \to \infty$, for each (n, k) $\in N \times N$

(ii) $a_i \to a$, as $i \to \infty$, where stat-lim $a_{nk}^i = a_i$, for each $i \in N$.

(iii) $\lambda_{nk}a_{nk} \overset{stat}{\to} a$ (relative to M).

Let □ > 0 be given. For a fixed $x_0 > 0$, choose r > 0 such that $M\left(\frac{rx_0}{3}\right) \ge 1$ and $M_0 \in N$ be such that

$$f\left(\left\langle \lambda_{nk}a^i_{nk} - \lambda_{nk}a^j_{nk} \right\rangle\right) \square\square \frac{\varepsilon}{rx_0}, \text{ for all } i, j \geq m_0.$$

By the definition of f we have,

$$M\left(q\left(\frac{\lambda_{nk}a^i_{nk} - \lambda_{nk}a^j_{nk}}{f\left(\left\langle a^i_{nk} - a^j_{nk}\right\rangle\right)}\right)\right) \leq 1 \leq M\left(\frac{rx_0}{3}\right), \quad \text{for all } i, j \geq m_0.$$

$$\Rightarrow q\left(\lambda_{nk}a^i_{nk} - \lambda_{nk}a^j_{nk}\right) < \frac{rx_0}{3} \cdot \frac{\varepsilon}{rx_0} = \frac{\varepsilon}{3}, \text{ for all } i, j \geq m_0. \qquad \text{.....(3)}$$

Hence $\left\langle \lambda_{nk}a^i_{nk} \right\rangle$ is a Cauchy sequence in X for all (n, k) $\in N \times N$. Since X is complete, so there exists $a_{nk} \in X$, such that $a^i_{nk} \to a_{nk}$, as $i \to \infty$, for each (n, k) $\in N \times N$.

(ii) We have stat-lim $a^i_{nk} = a_i$ for each $i \in N$. Thus there exists a subset $E_i \subset N \times N$ such that $\square(E_i) = 1$, for each $i \in N$ and for a givrn $\square > 0$,

$$M\left(q\left(\frac{a^i_{nk} - a_i}{r}\right)\right) \leq M\left(\frac{\varepsilon}{3r}\right), \text{ for all } (n, k) \in E_i, \text{ for each } i \in N \text{ and some } r > 0.$$

$$\Rightarrow \left(a^i_{nk} - a_i\right) < \frac{\varepsilon}{3}, \text{ for all } (n, k) \in E_i, \text{ for each } i \in N \text{ and by continuity of } M. \qquad \text{....(4)}$$

Let $i, j \geq m_0$ and (n, k) $\in E_i \bigcap E_j$. Then we have

$$q(a_i \square a_j) \leq q\left(a^i_{nk} - a_i\right) + q\left(a^i_{nk} - a^j_{nk}\right) + q\left(a^i_{nk} - a_j\right)$$

$$< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \square, \text{ by (3) and (4).}$$

Hence $\langle a_i \rangle$ is a Cauchy sequence in X, which is complete. Thus $\langle a_i \rangle$ converges in X and let $\lim_{i\to\infty} a_i = a$.

(iii)For $\varepsilon_1 > 0$ given, let $i \geq m_0$ and $r > 0$ be so chosen that $M\left(\frac{\varepsilon}{r}\right) < \varepsilon_1$ and the following hold. From (ii) we have a subset $E \subset N \times N$ such that

$$q\left(a^i_{nk} - a_i\right) < \frac{\varepsilon}{3}.$$

By (i) we have $q\left(a_{nk} - a^i_{nk}\right) < \frac{\varepsilon}{3}$, for all $i \geq m_0$.

By (ii) we have $q(a^i \square a) < \frac{\varepsilon}{3}$, for all $i \geq m_0$.

Hence for all $i \geq m_0$ and for all $(n, k) \in E$ with $\square(E) = 1$, we have

$$q(a_{nk} \square a) \leq q\left(a_{nk} - a^i_{nk}\right) + q\left(a^i_{nk} - a_i\right) + q(a^i\square\square\square a) < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \square.$$

$$\Rightarrow M\left(q\left(\frac{a_{nk} - a}{r}\right)\right) \leq M\left(\frac{\varepsilon}{r}\right) = \square_1, \text{ for some } r > 0 \text{ and all } (n, k) \in E \text{ with } \square(E) = 1.$$

$\Rightarrow$ stat-lim $a_{nk} = a$.

Hence $<a_{nk}>$ $(\bar{c}^2)^B$ (M, q, $\square$).

Thus $(\bar{c}^2)^B$ (M, q, $\square$) is a complete semi normed space.

Proposition 4. The spaces $\bar{c}_0^2$ (M, q, $\square$), $(\bar{c}_0^2)^{BR}$ (M, q, $\square$), $(\bar{c}_0^2)^B$ (M, q, $\square$), $(\bar{c}_0^2)^R$ (M, q, $\square$), ℓ^2_∞(M, q, $\square$) and $\bar{\ell}^2_\infty$ (M, q, l) are solid and hence are monotone.

Proof. Let $<\square_{nk}>$ be a double sequence of scalars such that $|\square a_{nk}| \leq 1$, for all $n, k \in N$.

Then the proof for $\bar{c}_0^2$ (M, q, λ), ($\bar{c}_0^2$)^{BR} (M, q, λ), ($\bar{c}_0^2$)^{B} (M, q, λ), ($\bar{c}_0^2$)^{R} (M, q, λ), ℓ_∞^2(M, q, λ) and $\bar{\ell}_\infty^2$(M, q, λ) are obvious in view of the following inequality.

$$M\left(q\left(\frac{\alpha_{nk}\lambda_{nk}a_{nk}}{\rho}\right)\right) \le M\left(q\left(\frac{\lambda_{nk}a_{nk}}{\rho}\right)\right), \text{ for all } (n, k) \in K.$$

The rest of the proof of first part follows from Remark 1.

Proposition 5. The spaces Z(M, q, λ) where Z = $\bar{c}^2$, $\bar{c}_0^2$, ($\bar{c}^2$)^B, ($\bar{c}_0^2$)^B, ($\bar{c}^2$)^R, ($\bar{c}_0^2$)^R, ($\bar{c}^2$)^{BR}, ($\bar{c}_0^2$)^{BR}, $\bar{\ell}_\infty^2$ are not symmetric.

The above result follows from the following examples.

Example 1. Let X = c, the class of convergent single sequences and M(x) = x^p, $p \ge 1$ and q(x) = $\sup_i|x^i|$, for x = (x^i) $\in$ c. Define <a_{nk}> by

$$a_{nk} = \begin{cases} \varepsilon, \text{ for all } n = k \in N \\ \theta \qquad \text{Otherwise.} \end{cases}$$

Let <b_{nk}> be a rearrangement of <a_{nk}>, defined as

$$b_{nk} = \begin{cases} \varepsilon, \text{ for all } k \text{ even and all } n \in N, \\ \theta, \qquad \text{Otherwise.} \end{cases}$$

Let <b_{nk}> be a rearrangement of <a_{nk}>, defined as follows

$$b_{nk} = \begin{cases} \frac{k+1}{2}\varepsilon, \text{ if } k \text{ is odd, and for all } n \in N \\ \theta \qquad \text{Otherwise.} \end{cases}$$

Then a_{nk} > $\in$ $\bar{\ell}_\infty^2$(M, q, λ), but <b_{nk}> $\notin$ $\bar{\ell}_\infty^2$(M, q, λ).

Hence, $\bar{\ell}_\infty^2$(M, q, λ) is not symmetric.

Theorem 6. Let M and M_1 be two Orlicz functions, then Z(M, q, λ) $\subset$ Z(M_0M_1, q, λ) for Z = $\bar{c}^2$, $\bar{c}_0^2$, ($\bar{c}^2$)^B, ($\bar{c}_0^2$)^B, ($\bar{c}^2$)^R, ($\bar{c}_0^2$)^R, ($\bar{c}^2$)^{BR}, ($\bar{c}_0^2$)^{BR} and $\bar{\ell}_\infty^2$(M, q, λ).

Proof. We prove it for the case Z = $\bar{c}_0^2$, the other cases can be proved following similar technique. Let □ > 0 be given. Since M is continuous, so there exists □ > 0 such that M(□) = □. Let ($\lambda_{nk}a_{nk}$) $\in$ $\bar{c}_0^2$ (M_1, λ). Then there exists a subset K $\subset$ N $\times$ N with □(K) = 1 such that

$$M_1\left(q\left(\frac{\lambda_{nk}a_{nk}}{r}\right)\right) < □, \text{ for all } (n, k) \in K.$$

$$□□ M_0 M_1\left(q\left(\frac{\lambda_{nk}a_{nk}}{r}\right)\right) < □.$$

Hence, ($\lambda_{nk}a_{nk}$) > $\in$ $\bar{c}_0^2$ (M_0M_1, λ)

Thus $\bar{c}_0^2$ (M_1, q, λ) $\subset$ $\bar{c}_0^2$ (M_0M_1, q, λ).

Theorem 7. If M and M_1 are two functions then

$$Z(M, q, \lambda) \cap Z(M_2, q, \lambda) \subseteq Z(M_1 + M_2, q, \lambda)$$

For Z = $\bar{c}^2$, $\bar{c}_0^2$, ($\bar{c}^2$)^B, ($\bar{c}_0^2$)^B, ($\bar{c}^2$)^R, ($\bar{c}_0^2$)^R, ($\bar{c}^2$)^{BR}, ($\bar{c}_0^2$)^{BR}, $\bar{\ell}_\infty^2$.

Proof. We prove the result for $\bar{c}^2$ (M, q, λ). The other cases can be established following similar technique.

Let ($\lambda_{nk}a_{nk}$) $\in$ $\bar{c}^2$ (M_1, q, λ) $\cap$ $\bar{c}^2$ (M_2, q, λ). Let □ > 0 be given.

Then there exist subsets K and D of N $\times$ N such that □(K) = □(K) – □(D) = 1

$$M_1\left(q\left(\frac{\lambda_{nk}a_{nk} - L}{r_1}\right)\right) < \frac{\varepsilon}{2}, \text{ for all } (n, k) \in K, \text{ for some } r_1 > 0,$$

and

$$M_2\left(q\left(\frac{\lambda_{nk}a_{nk} - L}{r_2}\right)\right) < \frac{\varepsilon}{2}, \text{ for all } (n, k) \in D, \text{ for some } r_2 > 0.$$

Let $r = \max\{r_1, r_2\}$. Then for all $(n, k)\ (K \bigcap D)$ we have

$$(M_1 + M_2)\left(q\left(\frac{\lambda_{nk}a_{nk} - L}{r}\right)\right) \leq M_1\left(q\left(\frac{\lambda_{nk}a_{nk} - L}{r_1}\right)\right) + M_2\left(q\left(\frac{\lambda_{nk}a_{nk} - L}{r_2}\right)\right) < \square.$$

Hence $(\lambda_{nk}a_{nk})(M_1 + M_2, q, \lambda)$.

This completes the proof.

The proof of the following result is a consequence of Theorem 6.

Corollary 8. Let M be an Orlicz function then we have $Z(q) \subset Z(M, q, \lambda)$ for $Z = \bar{c}^2$, $\bar{c}_0^2$, $(\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^R$, $(\bar{c}_0^2)^R$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ and $\bar{\ell}_\infty^2$.

The proof of the following result is a routine work.

Theorem 9. Let M be an Orlicz function, q_1, and q_2 be seminorms. Then

(i) $Z(M, q_1, \lambda) \bigcap Z(M, q_2, \lambda) \subseteq Z(M, q_1 + q_2, \lambda)$

(ii) If q_1 is stronger than q_2, then $Z(M, q_1, \lambda) \subset Z(M, q_2, \lambda)$ for $Z = \bar{c}^2$, $\bar{c}_0^2$, $(\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^R$, $(\bar{c}_0^2)^R$, $(\bar{c}^2)^{BR}$ and $\bar{\ell}_\infty^2$.

The following result can be proved by using the standard technique.

Theorem 10. Let M be an Orlicz function. Then $Z(M, q, \lambda) \subset \bar{\ell}_\infty^2(M, q, \lambda)$ for $Z = (\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ and the inclusions are strict.

The following result is a consequence of the above Theorem and Theorem 3.

Corollary 11. The spaces $Z(M, q, \lambda)$ for $Z = (\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ are a nowhere dense subset of $\bar{\ell}_\infty^2$.

Theorem 12. The spaces $Z(M, q, \lambda)$ where $Z = \bar{c}^2$, $\bar{c}_0^2$, $(\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^R$, $(\bar{c}_0^2)^R$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ and $\bar{\ell}_\infty^2$ are not convergence free.

The above result is clear from the following example.

Example 3. Let $M(x) = x^p$, for some $1 \leq p \leq \infty$, $X = \ell_\infty$, $q(x) = \sup_i|x^i|$, for $x = (x_i) \in \ell_\infty$. Then the double sequence $< a_{nk} >$ defined as $a_{nk} = (k^{\square 1}, k^{\square 1}, k^{\square 1}, \ldots)$, for all $n, k \in N$ belongs to all the spaces. Consider the sequence $< b_{nk} >$ defined as $b_{nk} = (k, k, k, \ldots)$, for all $n, k \in N$. Then $<b_{nk}>$ does not belong to any of these spaces. Hence none of the spaces is convergence free.

Proposition 13. The spaces $Z(M, q, \lambda)$, for $Z = \bar{c}^2$, $\bar{c}_0^2$, $(\bar{c}^2)^B$, $(\bar{c}^2)^R$, $(\bar{c}^2)^{BR}$ are not monotone and hence are not solid.

Proof. The first part follows from the following example. The second part follows from Remark 1.

Example 4. Let $M(x) = x$, $X = C$ and $q(x) = |x|$. Then the double sequence $(\lambda_{nk}a_{nk})$, defined by $a_{nk} = 1$, for all $n, k \in N$ belong to $Z(M, q, \lambda)$, for $Z = \bar{c}^2$, $(\bar{c}^2)^B$, $(\bar{c}^2)^R$, $(\bar{c}^2)^{BR}$.

Consider its pre-image $(\lambda_{nk}b_{nk})$ defined as

$$\lambda_{nk}b_{nk} = \begin{cases} 0, & \text{for } k \text{ even, for all } n \in N, \\ 1, & \text{for } k \text{ odd, for all } n \in N. \end{cases}$$

Then $(\lambda_{nk}b_{nk})$ does not belong to any of these spaces.

Remark 2. Let (X, ||.||) be a normed linear space. Then the spaces Z(M, ||.||), for Z = $(\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$, and ℓ_∞^2 will be normed linear spaces normed by

$$f(\lambda_{nk}a_{nk}) = \inf\{\rho > 0 : \sup_{n,k} M\left(\left\|\left(\frac{\lambda_{nk}a_{nk}}{\rho}\right)\right\|\right) \leq 1\}.$$

Remark 3. If X is a Banach space then it is clear that the spaces Z(M, ||.||), for Z = $(\bar{c}^2)^B$, $(\bar{c}_0^2)^B$, $(\bar{c}^2)^{BR}$, $(\bar{c}_0^2)^{BR}$ and ℓ_∞^2 are Banach spaces under the norm.

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