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Robust H_∞ Control of Discrete-Time Uncertain Recurrent Neural Networks with Discrete and Distributed Interval Time-Varying Delays

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Abstract— ***this paper is concerned with the problem of delay dependent H∞ control of discrete-time uncertain recurrent neural networks with time varying-delays. The neural network is subject to parameter uncertainty, and time-varying delay. For the robust H∞ stabilization problem, a state feedback controller is designed to ensure global robust stability of the closed-loop system about its equilibrium point for all admissible uncertainties. By using the Laypunov-Krasovskii functional, a linear matrix inequality (LMI) approach is developed to establish sufficient conditions. A simulation example is exploited to show the usefulness of the derived LMI-based stability conditions.***

Keywords— ***H∞ control; Uncertain; Discrete-time; Neural networks; Time-varying delay.***

I. INTRODUCTION

Recurrent neural networks (RNNs) have found successful applications in many areas such as image processing, signal processing, pattern recognition, and optimization problems. Recently, there has been a rapidly growing research interest on the dynamical properties of RNNs [1]. Since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing continuously distributed delays over a certain duration of time, such that the distant past has less influence compared to the recent behavior of the state. Therefore, stability analysis of neural networks has received much more attention over the past years [2]-[6]. Time delays are inevitable in the implementation of artificial neural networks as a result of the finite switching speed of amplifier. Therefore, various issues of neural networks with time delays have been addressed, and many results have been reported in the literature.

It is worth noting that, up to now, most recurrent neural networks have been analyzed by using a continuous-time model. However, when the continuous-time recurrent neural networks are implemented to simulate, experimentalize or compute based on computers, it is necessary to discretize the continuous-time networks to formulate a discrete-time system. But as pointed out in some previous articles, the discretization cannot preserve the dynamics of a continuous-time system even for a small sampling period. Therefore, a study on the dynamics of discrete-time neural networks is crucially needed [7]-[11]. However, the stability analysis for stochastic neural networks is difficult. Recently, although some results related to this issue have been reported in the literature.

In practice, uncertainties often exist in most engineering and communication systems and may cause undesirable dynamic network behaviours. More specifically, the connection weights of the neurons are inherently dependent on certain resistance and capacitance values that inevitably bring in uncertainties during the parameter identification process. The deviations and perturbations in parameters are the main sources of uncertainty. So, it is important to study the dynamical behaviours of neural networks by taking the uncertainty into account has been reported in [12]-[17]. During the last decade, the $H\infty$ control theory has attracted a lot of attention and made significant progress. The state-space approach for the $H\infty$ control of linear systems is developed based on the relation between the $H\infty$ norm and the algebraic Riccati equations which play an important role in the optimal control of linear systems [18]-[21]. Recently, $H\infty$ control approach has been extended to uncertain systems. The necessary and sufficient conditions for *H∞* control of uncertain linear systems are given [22, 23]. The robust stabilization problem has been addressed in [24] and LMI-based stability criteria and stabilization conditions have been proposed based on Lyapunov-Krasovskii functional method, while the *H∞* control problem has been studied in[25], and some sufficient conditions for the existence of an *H∞* controller have been presented by means of LMI format.

Motivated by the above discussions, we study the asymptotic stability problem for a new class of discrete-time neural networks with both discrete and distributed time-delays. We first deal with the deterministic neural network under mild conditions on the activation functions, where neither differentiability nor monotonicity is needed. By constructing a new Lyapnuov-Krasovskii functional, a linear matrix inequality (LMI) approach is developed to establish sufficient conditions for the discrete time neural networks to be globally asymptotically stable. A simulation example is presented to show the usefulness of the derived LMI-based stability condition.

2. Problem Formulation

Consider the following discrete-time recurrent neural network with interval time-varying discrete and distributed delay described by

$$x(k+1) = -(A+\Delta A)x(k) + (W+\Delta W)f(x(k)) + (W_1+\Delta W_1)f(x(k-\tau(k))) + (W_2+\Delta W_2)\sum_{i=-d(k)}^{-1} f(x(k+1)) + u(k) + v(k), \quad (1)$$

$$z(k) = Lx(k), \quad (2)$$

where $x(k) = (x_1(k), \quad x_2(k), ..., \quad x_n(k))^T$ is the state vector, $A = diag(a_1, a_2, ..., a_n)$ is real constant diagonal matrix $W = [w_{ij}]_{n\times n}$, and $W_1 = [(w_1)_{ij}]_{n\times n}$, $W_2 = [(w_2)_{ij}]_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively, $f(x(t)) = [f_1(x_1(t)), ..., f_n(x_n(t))]^T \in R^n$ is the neuron activation function with $f(0) = 0$.

The discrete time-varying delay $\tau(k)$ and distributed time-varying delay $d(k)$ are bounded, namely $\tau_1 \le \tau(k) \le \tau_2$, $0 < d_m \le d(k) \le d_M$. Where τ_1 and τ_2 are known integers. $u(k) \in R^m$ is the control input vector, $z(k) \in R^l$ is the controlled output, L is a known real constant matrix. $v(k) \in R^p$ is the disturbance input which is assumed to belong to $l_2[0, \infty]$. ΔA, ΔW, ΔW_1 and ΔW_2 are unknown matrices representing time-varying parameter uncertainties, which are assumed to be of the form

$$[\Delta A \quad \Delta W \quad \Delta W_1 \quad \Delta W_2] = MF(k)[N_1 \quad N_2 \quad N_3 \quad N_4], \quad (3)$$

where M, N_1, N_2, N_3 and N_4 are known real constant matrices, and F(.) is time-varying matrix function satisfying

$$F^T(k)F(k) \le I, \quad k \in N. \quad (4)$$

The uncertain matrices ΔA, ΔW, ΔW_1 and ΔW_2 are said to be admissible if both (3) and (4) hold. In order to obtain our main results, the following assumption are made throughout the paper.

Assumption 1: The activation function $f(x)$ is nondecreasing, bounded and globally Lipschitz; that is

$$0 \leq \frac{f_i(\varsigma_1) - f_i(\varsigma_2)}{\varsigma_1 - \varsigma_2} \leq \sigma_i., \forall \varsigma_1 \neq \varsigma_2 \quad i = 1, 2, ..., n. \tag{5}$$

Definition 2.1. *The discrete-time uncertain recurrent neural network(1) is said to be globally robustly stable about its equilibrium point with disturbance attenuation level γ if it is globally robustly stable and under zero initial conditions, such that*

$$\|Z\|_2 \leq \gamma \|v\|_2, \tag{6}$$

For all nonzero v and all admissible uncertainties, where $\gamma > 0$ is a given scalar.

1.Robust stabilization problem: Determine a state feedback controller

$$u(k) = Kx(k), \tag{7}$$

for the system (1) such that the resulting closed-loop system is globally robustly stable about its equilibrium point.

2.Robust H_∞ control problem: Given a constant scalar $\gamma > 0$, determine a state feedback in the form of (7) such that the resulting closed-loop system is globally robustly stable with disturbance attenuation level γ, thus (6) in fact gives an estimation of the deviation of the perturbed trajectory from the equilibrium point.

Lemma 2.2. [26] *Let A, D, S, F and P be real matrices of appropriate dimensions with $P > 0$ and F satisfying $F^T(k)F(k) \leq I$. Then the following statements hold.*
(a) For any $\epsilon > 0$ and vectors $x, y \in R^n$

$$2x^T DFSy \leq \epsilon^{-1} x^T DD^T x + \epsilon y^T S^T Sy,$$

(b) For vectors $x, y \in R^n$

$$2x^T DSy \leq x^T DPD^T x + y^T S^T P^{-1} Sy.$$

For any matrices E_i, S_i, T_i (i = 1, 2) and H_1 of appropriate dimensions, the equations are hold:

$$\phi_1 = 2[(x^T(k)E_1 + x^T(k - \tau(k))E_2]?[x(k) - x(k - \tau(k)) - \sum^{k} (x(j) - x(j-1))] = 0, \tag{8}$$

$$\phi_2 = 2[x^T(k - \tau(k))S_1 + x^T(k - \tau_2)S_2]?[x(k - \tau(k)) - x(k - \tau_2) - \sum_{j=k-\tau_2+1}^{k-\tau_k} (x(j) - x(j-1))] = 0, \tag{9}$$

$$\phi_3 = -2[x^T(k - \tau(k))T_1 + x^T(k - \tau_1)T_2][x(k - \tau_1) - x(k - \tau(k)) - \sum_{j=k-\tau(k)+1}^{k-\tau_1} (x(j) - x(j-1))] = 0, \tag{10}$$

$$\begin{aligned}\phi_4 = &-2[x^T(k+1)H_1][x(k+1) + (A + \Delta A)x(k) - (W + \Delta W)f(x(k)) - (W_1 + \Delta W_1)f(x(k - \tau(k))) \\ &- (W_2 + \Delta W_2) \sum_{i=-d(k)}^{-1} f(x(k+1)) - u(k) - v(k)] = 0,\end{aligned} \tag{11}$$

$$\phi_5 = 2f^T(x(k))R_1 f(x(k)) - 2f^T(x(k))R_1 f(x(k)) + 2f^T(x(k - \tau(k)))R_2 x(k - \tau(k))$$

$$-2f^T(x(k-\tau(k)))R_2x(k-\tau(k)) = 0. \tag{12}$$

Theorem 2.3. *Consider the system (1) with $v(k) = 0$, for given scalars τ_1, τ_2, d_m, d_M. The equilibrium point of model(1) is globally robustly stabilizable if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$ a nonsingular H_1 and matrix Y diagonal matrices $R_1 > 0$, $R_2 > 0$ and matrices E_i, S_i, H_i, Y_i, N_i and $T_i(i = 1, 2)$ of appropriate dimensions and positive scalar ϵ such that the following LMI holds*

$$\begin{bmatrix} \Pi & \tau_2 E & \tau_{21}S & \tau_{21}S & \tau_{21}T & HM & \epsilon N \\ * & -\tau_2 Z_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\tau_{21}(Z_1+Z_2) & 0 & 0 & 0 & 0 \\ * & * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & * & -\tau_{21}Z_2 & 0 & 0 \\ * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \tag{13}$$

where

$\Pi = \Pi(i,j)$, i, j $= 1, \ldots, 8$,

$\Pi_{11} = -P + (\tau_{21}+1)Q_1 + Q_2 + Q_3 + \tau_2 Z_1 + \tau_{21}Z_2 + E_1 + E_1^T, \Pi_{12} = E_2^T - E_1$, $\Pi_{13} = 0$, $\Pi_{14} = 0$,

$\Pi_{15} = L^T R_1$, $\Pi_{16} = 0$, $\Pi_{17} = -\tau_2 Z_1 - \tau_{21}Z_2 - A^T H_1 + KH_1^T$, $\Pi_{18} = 0$,

$\Pi_{22} = -Q_1 - E_2 - E_2^T + S_1 + S_1^T + T_1 + T_1^T$, $\Pi_{23} = -S_1 + S_2^T$, $\Pi_{24} = -T_1 + T_2^T$, $\Pi_{25} = 0$,

$\Pi_{26} = R_2^T$, $\Pi_{27} = 0$, $\Pi_{28} = 0$, $\Pi_{33} = -Q_3 - S_2 - S_2^T$, $\Pi_{34} = 0$, $\Pi_{35} = 0$, $\Pi_{36} = 0$, $\Pi_{37} = 0$, $\Pi_{38} = 0$,

$\Pi_{44} = -T_2 - T_2^T - Q_2$, $\Pi_{45} = 0$, $\Pi_{46} = 0$, $\Pi_{47} = 0$, $\Pi_{48} = 0$,

$\Pi_{55} = d_0T_1 + \frac{1}{2}(d_0 - d_m)(d_0 + d_m - 1)T_1 - 2R_1$, $\Pi_{56} = 0$, $\Pi_{57} = W^T H_1^T$, $\Pi_{58} = 0$,

$\Pi_{66} = -R_2L^{-1} - R_2^T L^{-1}$, $\Pi_{67} = W_1^T H_1^T$, $\Pi_{68} = 0$,

$\Pi_{77} = -H_1 - H_1^T + P + \tau_2 Z_1 + \tau_{21}Z_2 + \tau_2 Z_1^T + \tau_{21}Z_2^T$, $\Pi_{78} = W_2^T H_1^T$, $\Pi_{88} = H_1^T W_2 + W_2^T H_1 - \frac{1}{d_0}T_1$,

$E = [E_1^T \quad E_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$, $S = [0 \quad S_1^T \quad S_2^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T$,

$T = [0 \quad T_1^T \quad 0 \quad T_2^T \quad 0 \quad 0 \quad 0 \quad 0]^T$, $H = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad H_1^T \quad 0]^T$,

$N = [N_1^T \quad 0 \quad 0 \quad 0 \quad N_2^T \quad N_3^T \quad N_4^T \quad 0]^T$, $\tau_{21} = \tau_2 - \tau_1$.

Proof:

Consider the following Lyapunov-Krasovskii functional for the discrete-time system in (1) with (7) and $v(k) = 0$

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k) + V_5(k)$$

$$V_1(k) = x^T(k)Px(k),$$

$$V_2(k) = \sum_{i=-\tau_2}^{-1} \sum_{j=k+i+1}^{k} (x(j) - x(j-1))^T Z_1 (x(j) - x(j-1)),$$

$$V_3(k) = \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+1}^{k} ((x(j) - x(j-1))^T Z_2 (x(j) - x(j-1)),$$

$$V_4(k) = \sum_{j=k-\tau(k)}^{k-1} x^T(j)Q_1x(j) + \sum_{j=k-\tau(k)}^{-1-\tau_1} x^T(j)Q_1x(j) + \sum_{j=k-\tau(1)}^{k-1} x^T(j)Q_2x(j)$$

$$+ \sum_{j=k-\tau(\tau_2)}^{k-1} x^T(j)Q_3x(j),$$

$$V_5(k) = \sum_{i=-d_k,1}^{-1} \sum_{j=k+i}^{k-1} f^T(x(j))T_1f(x(j)) + \sum_{i=-d_0}^{-d_m-1} \sum_{j=i+1}^{-1} \sum_{l=k+j}^{k-1} f^T(x(j))T_1f(x(j)).$$

Taking the difference of $V(k)$,

$$\Delta V_1(k) = x^T(k+1)Px(k+1) - x^T(k)Px(k), \tag{14}$$

$$\begin{aligned}\Delta V_2(k) = &\sum_{i=-\tau_2}^{-1} \sum_{j=k+i+2}^{k} (x(j)-x(j-1))^T Z_1(x(j)-x(j-1))\\ &- \sum_{i=-\tau_2}^{-1} \sum_{j=k+i+1}^{k} (x(j)-x(j-1))^T Z_1(x(j)-x(j-1)),\\ &\leq \tau_2(x(k+1)-x(k))^T Z_1(x(k+1)-x(k)) - \sum_{j=k-\tau_2+1}^{k-\tau(k)} (x(j)-x(j-1))^T Z_1(x(j)-x(j-1))\\ &- \sum_{j=k-\tau(k)+1}^{k} (x(j)-x(j-1))^T Z_1(x(j)-x(j-1)),\end{aligned} \tag{15}$$

$$\begin{aligned}\Delta V_3(k) = &\sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+2}^{k+1} (x(j)-x(j-1))^T Z_2(x(j)-x(j-1))\\ &- \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+1}^{k} (x(j)-x(j-1))^T Z_2(x(j)-x(j-1)),\\ &\leq (\tau_2-\tau_1)(x(k+1)-x(k))^T Z_2(x(j)-x(j-1))\\ &- \sum_{j=k-\tau(k)+1}^{k-\tau_1} (x(j)-x(j-1))^T Z_2(x(j)-x(j-1)),\end{aligned} \tag{16}$$

$$\begin{aligned}\Delta V_4(k) = &\sum_{j=k+1-\tau(k+1)}^{k+1-1} x^T(j)Q_1x(j) + \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k+i+2}^{k+1} x^T(j)Q_2x(j) + \sum_{j=k-\tau_1+1}^{k+1-1} x^T(j)Q_2x(j)\\ &+ \sum_{j=k+1-\tau_2}^{k+1-1} x^T(j)Q_3x(j) - \sum_{k-\tau(k)}^{k-1} x^T(j)Q_1x(j) - \sum_{i=-\tau_2}^{-1-\tau_1} \sum_{j=k-\tau_2+1}^{k} x^T(j)Q_2x(j)\\ &- \sum_{j=k-\tau_1}^{k-1} x^T(j)Q_2x(j) - \sum_{j=k-\tau_2}^{k-1} x^T(j)Q_3x(j),\\ &\leq x^T(k)[(\tau_2-\tau_1+1)Q_1+Q_2+Q_3]x(k) - x^T(k-\tau(k))Q_1x(k-\tau(k))\\ &- x^T(k-\tau_1)Q_2x(k-\tau_1) - x^T(k-\tau_2)Q_3x(k-\tau_2),\end{aligned} \tag{17}$$

$$\begin{aligned}\Delta V_5(k) = &\sum_{i=-d_{k+1},1}^{-1} \sum_{j=k+i+1}^{k} f^T(x(j))T_1f(x(j)) - \sum_{i=-d_k,1}^{-1} \sum_{j=k+i}^{k-1} f^T(x(j))T_1f(x(j))\\ &+ \sum_{i=-d_0}^{-d_m-1} \sum_{j=i+1}^{-1} \Big[\sum_{i=k+j+1}^{k} - \sum_{i=k+j}^{k-1} \Big] f^T(x(j))T_1f(x(j)),\end{aligned}$$

$$= \sum_{i=-d_{k+1},1}^{-1} \sum_{j=k+i+1}^{k-1} f^T(x(j))T_1 f(x(j)) + \sum_{i=-d_{k+1},1}^{-1} f^T(x(j))T_1 f(x(j))$$

$$+ \sum_{i=-d_k,1}^{-1} \sum_{j=k+i+1}^{k-1} f^T(x(j))T_1 f(x(j)) - \sum_{i=k-d_k,1}^{k-1} f^T(x(j))T_1 f(x(j))$$

$$+ \sum_{i=-d_0}^{-d_m-1} \sum_{j=i+1}^{-1} [f^T(x(j))T_1 f(x(j)) - f^T(x(k+1))T_1 f(x(k+1))],$$

$$\leq \sum_{i=-d_m}^{-1} \sum_{j=k+i+1}^{k-1} f^T(x(j))T_1 f(x(j)) + d_0 f^T(x(j))T_1 f(x(j))$$

$$- \sum_{i=-d_m}^{-1} \sum_{j=k+i+1}^{k-1} f^T(x(j))T_1 f(x(j)) - \sum_{i=-d_k,1}^{-1} f^T(x(k+1))T_1 f(x_{k+1})$$

$$+ \frac{1}{2}(d_0 - d_m)(d_0 + d_m - 1) f^T(x(j))T_1 f(x(j))$$

$$- \sum_{i=-d_0}^{-d_m-1} \sum_{j=k+i+1}^{k-1} f^T(x(j))T_1 f(x(j)),$$

$$\leq d_0 f^T(x(k))T_1 f(x(k)) + \frac{1}{2}(d_0 - d_m)(d_0 + d_m - 1) f^T(x(k))T_1 f(x(k))$$

$$- \frac{1}{d_0}\left(\sum_{i=-d_k}^{-1} f(x(k+1))^T T_1 \left(\sum_{i=-d_k}^{-1} f(x(k+1)).\right.\right. \qquad (18)$$

Defining the following new variables

$$\eta(k) = [x^T(k) \quad x^T(k-\tau(k)) \quad x^T(k-\tau_2) \quad x^T(k-\tau_1) \quad f^T(x(k)) \quad f^T(x(k-\tau(k))) \quad x^T(k+1) \quad \sum_{i=-d_k}^{-1} f^T(x(k+1))]^T,$$

Combining (8)-(12) and (14)-(18), we have

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(K) + \Delta V_4(K) + \Delta V_5(K) + \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5$$

$$\leq x^T(k+1)Px(k+1) - x^T(k)Px(k) + \tau_2(x(k+1) - x(k))^T Z_1(x(k+1) - x(k))$$

$$- \sum_{j=k-\tau_2+1}^{k-\tau(k)} (x(j) - x(j-1))^T Z_1(x(j) - x(j-1)) - \sum_{j=k-\tau(k)+1}^{k} (x(j) - x(j-1))^T Z_1(x(j) - x(j-1))$$

$$+ (\tau_2 - \tau_1)(x(k+1) - x(k))^T Z_2(x(j) - x(j-1)) - \sum_{j=k-\tau(k)+1}^{k-\tau_1} (x(j) - x(j-1))^T Z_2(x(j) - x(j-1))$$

$$+ x^T(k)[(\tau_2 - \tau_1 + 1)Q_1 + Q_2 + Q_3]x(k) - x^T(k-\tau(k))Q_1 x(k-\tau(k)) - x^T(k-\tau_1)Q_2 x(k-\tau_1)$$

$$- x^T(k-\tau_2)Q_3 x(k-\tau_2) + 2\eta^T(k)E[x(k) - x(k-\tau(k)) - \sum_{j=k-\tau(k)+1}^{k} (x(j) - x(j-1))]$$

$$+ 2\eta^T(k)S[x(k-\tau(k)) - x(k-\tau_2) - \sum_{j=k-\tau_2+1}^{k-\tau_k} (x(j) - x(j-1))] - 2\eta^T(k)T[x(k-t_1)$$

$$- x(k-\tau(k)) - \sum_{j=k-\tau(k)+1}^{k-\tau_1}(x(j)-x(j-1))] - 2\eta^T(k)H_1[x(k+1)+(A+\Delta A)x(k)$$

$$-(W+\Delta W)f(x(k)) - (W_1+\Delta W_1)f(x(k-\tau(k))) - (W_2+\Delta W_2)\sum_{i=-d(k)}^{-1} f(x(k+1)) - Kx(k)]$$

$$+ x^T(k+1)H_1W_2\sum_{i=-d(k)}^{-1} f(x(k+1)) + 2x^T(k+1)H_1Kx(k) - 2f^T(x(k))R_1f(x(k))$$

$$+ 2f^T(x(k))R_1f(x(k)) + 2f^T(x(k-\tau(k)))R_2x(k-\tau(k)) - 2f^T(x(k-\tau(k)))R_2x(k-\tau(k)). \quad (19)$$

By Lemma 2.2,

$$-2\eta^T(k)E\sum_{j=k-\tau(k)+1}^{k}(x(j)-x(j-1)) \le \tau_2\eta^T(k)EZ_1^{-1}E^T\eta(k)$$

$$+ \sum_{j=k-\tau(k)+1}^{k}(x(j)-x(j-1))^TZ_1(x(j)-x(j-1)), \quad (20)$$

$$-2\eta^T(k)S\sum_{j=k-\tau_2+1}^{k-\tau(k)}(x(j)-x(j-1)) \le (\tau_2-\tau_1)\eta^T(k)S(Z_1+Z_2)^{-1}S^T\eta(k)$$

$$-2\eta^T(k)S\sum_{j=k-\tau_2+1}^{k-\tau(k)}(x(j)-x(j-1)) \le (\tau_2-\tau_1)\eta^T(k)S(Z_1+Z_2)^{-1}S^T\eta(k)$$

$$+ \sum_{j=k-\tau_2+1}^{k-\tau(k)}(x(j)-x(j-1))^T(Z_1+Z_2)(x(j)-x(j-1)), \quad (21)$$

$$2\eta^T(k)T\sum_{j=k-\tau(k)+1}^{k-\tau_1}(x(j)-x(j-1)) \le (\tau_2-\tau_1)\eta^T(k)TZ_2^{-1}T^T\eta(k)$$

$$+ \sum_{j=k-\tau(k)+1}^{k-\tau_1}(x(j)-x(j-1))^TZ_2(x(j)-x(j-1)). \quad (22)$$

Noting that for any diagonal matrices $R_1 > 0$ and $R_2 > 0$ we have,

$$2f^T(x(k))R_1f(x(k)) \le 2f^T(x(k))R_1Lx(k), \quad (23)$$

$$2f^T(x(k-\tau(k)))R_2x(k-\tau(k)) \le 2f^T(x(k-\tau(k)))R_2L^{-1}f(x(k-\tau(k))). \quad (24)$$

where $L = diag(\sigma_1, \sigma_2, ..., \sigma_n)$.

By using Lemma 2.2 (a) results in

$$2\eta^T(k)H_1(-\Delta Ax(k) + \Delta Wf(x(k)) + \Delta W_1f(x(k-\tau(k)) + \Delta W_2\sum_{i=-d(k)}^{-1} f(x(k+1)))$$

$$= 2\eta^T(k)H_1MF(k)N^T\eta(k),$$

$$\le \varepsilon^{-1}\eta^T(k)H_1MM^TH_1^T\eta(k) + \varepsilon\eta^T(k)NN^T\eta(k). \quad (25)$$

$$= 2\eta^T(k)H_1MF(k)N^T\eta(k),$$
$$\leq \varepsilon^{-1}\eta^T(k)H_1MM^TH_1^T\eta(k) + \varepsilon\eta^T(k)NN^T\eta(k). \tag{25}$$

Substituting (19)-(25) we can obtain,

$$\begin{aligned}\Delta V(k) \leq & x^T(k+1)Px(k+1) - x^T(k)Px(k) + \tau_2(x(k+1)-x(k))^TZ_1(x(k+1)-x(k)) \\ & + (\tau_2-\tau_1)(x(k+1)-x(k))^TZ_2(x(k+1)-x(k)) + x^T(k)[(\tau_2-\tau_1+1)Q_1+Q_2+Q_3]x(k) \\ & - x^T(k-\tau(k))Q_1x(k-\tau(k)) - x^T(k-\tau_1)Q_2x(k-\tau_1) - x^T(k-\tau_2)Q_3x(k-\tau_2) \\ & + 2x^T(k)E_1x(k) + 2x^T(k)E_2^Tx^T(k-\tau(k)) - 2x^T(k)E_1x(k-\tau(k)) \\ & - 2x^T(k-\tau(k))E_2^Tx(k-\tau(k)) + \tau_2\eta^T(k)EZ_1^{-1}E^T\eta(k) + \tau_{21}\eta^T(k)S(Z_1+Z_2)^{-1}S^T\eta(k) \\ & + (\tau_2-\tau_1)\eta^T(k)TZ_2^{-1}T^T\eta(k) + 2x^T(k-\tau(k))S_1x(k-\tau(k)) + 2x^T(k-\tau(k))S_2^T(k-\tau_2) \\ & - x^T(k-\tau_2)S_1x(k-\tau(k)) - 2x^T(k-\tau_2)S_2^Tx(k-\tau_2) - 2x^T(k-\tau(k))T_1x(k-\tau_1) \\ & - 2x^T(k-\tau_1)T_2^Tx(k-\tau_1) + 2x^T(k-\tau(k))T_1x(k-\tau(k)) + 2x^T(k-\tau(k))T_2x(k-\tau_1) \\ & - 2x^T(k+1)H_1x(k+1) + \tau_2\eta^T(k)EZ_1^{-1}E^T\eta(k) + (\tau_2-\tau_1)\eta^T(k)S(Z_1+Z_2)^{-1}S^T\eta(k) \\ & + (\tau_2-\tau_1)\eta^T(k)TZ_2^{-1}T^T\eta(k) + 2x^T(k)A^TH_1^Tx(k+1) + 2f^T(x(k))W^TH_1^Tx(k+1) \\ & + 2x^T(k+1)(H_1W_1)f(x(k-\tau(k)) + 2f^T(x(k-\tau(k))W_1^TH_1^Tx(k+1) \\ & + \varepsilon^{-1}\eta^T(k)H_1MM^TH_1^T\eta^T(k) + \varepsilon\eta^T(k)NN^T\eta(k) + 2\sum_{i=-d(k)}^{-1} f^T(x(k+1))(H_1^TW_2) \\ & \sum_{i=-d(k)}^{-1} f(x(k+1)) + 2x^T(k)L^TR_1^Tf(x(k)) - 2f^T(x(k-\tau(k)))R_2Lf(x(k-\tau(k))) \\ & + d_0f^T(x(k))T_1f(x(k)) + \frac{1}{2}(d_0-d_m)(d_0+d_m-1)f^T(x(k))T_1f(x(k)) \\ & - \frac{1}{d_0}(\sum_{i=-d_k}^{-1} f(x(k+1)))^TT_1(\sum_{i=-d_k}^{-1} f(x(k+1))) + 2x^T(k+1)H_1W_2\sum_{i=-d(k)}^{-1} f(x(k+1)).\end{aligned} \tag{26}$$

$$\begin{aligned}\Delta V(k) \leq & \eta^T(k)[\Pi + \tau_2EZ_1^{-1}E^T + \tau_{21}S(Z_1+Z_2)^{-1}S^T \\ & + \tau_{21}SZ_2^{-1}S^T + \tau_{21}TZ_2^{-1}T^T + \varepsilon^{-1}H_1MM^TH^T + \varepsilon NN^T]^T\eta(k),\end{aligned} \tag{27}$$

$$\Pi = \begin{bmatrix} \Pi_{11} & E_2^T - E_1 & 0 & 0 & L^TR_1 & 0 & \Pi_{17} & 0 \\ * & \Pi_{22} & S_2^T - S_1 & -T_1 + T_2^T & 0 & R_2^T & 0 & 0 \\ * & * & -Q_3 - S_2 - S_2^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - T_2 - T_2^T & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & W^TH_1^T & 0 \\ * & * & * & * & * & \Pi_{66} & W_1^TH_1^T & 0 \\ * & * & * & * & * & * & \Pi_{77} & H_1^TW_2^T \\ * & * & * & * & * & * & * & \Pi_{88} \end{bmatrix}$$

$$\begin{aligned}\eta(k) = [& x^T(k) \quad x^T(k-\tau(k)) \quad x^T(k-\tau_2) \quad x^T(k-\tau_1) \quad f^T(x(k)) \\ & f^T(x(k-\tau(k))) \quad x^T(k+1) \quad \sum^{-1} f^T(x(k+1))]^T.\end{aligned}$$

By using the Schur complement, then $\Delta V(k) < 0$ holds if (16) is satisfied.
By Lyapunov-Krasovskii stability, the discrete-time uncertain recurrent neural network with interval time-varying delay in (1) is globally asymptotically robust stable for all admissible uncertainties. This completes the proof of Theorem .

3. Robust H_∞ control

This section will focus on the design of a state feedback controller such that the resulting closed-loop system is globally robustly stable with disturbance attenuation level $\gamma > 0$ for estimation of the deviation of the perturbed trajectory from the equilibrium point. The following delay-dependent global robust H_∞ performance analysis result is explored.

Theorem 3.1. *Under the Assumption 1, consider the system (1) for given scalars τ_k, d_k is globally robustly stabilizable with disturbance attenuation level $\gamma > 0$ if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Z_1 > 0$, $Z_2 > 0$, $Z_3 > 0$, diagonal matrices $R_1 > 0$, $R_2 > 0$ and matrices E_i, S_i, N_i, and $T_i (i = 1, 2)$ and H_1 of appropriate dimensions and positive scalar ϵ such that the following LMI holds*

$$\begin{bmatrix} \Pi & \tau_2 E & \tau_{21} S & \tau_{21} S & \tau_{21} T & HM & \epsilon N \\ * & -\tau_2 Z_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\tau_{21}(Z_1 + Z_2) & 0 & 0 & 0 & 0 \\ * & * & * & -Z_2 & 0 & 0 & 0 \\ * & * & * & * & -\tau_{21} Z_2 & 0 & 0 \\ * & * & * & * & * & -\epsilon I & 0 \\ * & * & * & * & * & * & -\epsilon I \end{bmatrix} < 0, \qquad (28)$$

$\Pi = \Pi(i, j)$, (i, j = 1, . . . , 9),

$\Pi_{11} = -P + (\tau_{21} + 1)Q_1 + Q_2 + Q_3 + \tau_2 Z_1 + \tau_{21} Z_2 + E_1 + E_1^T + LL^T$, $\Pi_{12} = E_2^T - E_1$, $\Pi_{13} = 0$,
$\Pi_{14} = 0$, $\Pi_{15} = L^T R_1$, $\Pi_{16} = 0$, $\Pi_{17} = -\tau_2 Z_1 - \tau_{21} Z_2 - A^T H_1 - Y^T$, $\Pi_{18} = 0$, $\Pi_{19} = 0$,
$\Pi_{22} = -Q_1 - E_2 - E_2^T + S_1 + S_1^T + T_1 + T_1^T$, $\Pi_{23} = -S_1^T + S_2^T$, $\Pi_{24} = -T_1 + T_2^T$, $\Pi_{25} = 0$, $\Pi_{26} = R_2^T$,
$\Pi_{27} = 0$, $\Pi_{28} = 0$, $\Pi_{29} = 0$, $\Pi_{33} = -Q_3 - S_2 - S_2^T$, $\Pi_{34} = 0$, $\Pi_{35} = 0$, $\Pi_{36} = 0$, $\Pi_{37} = 0$, $\Pi_{38} = 0$,
$\Pi_{39} = 0$, $\Pi_{44} = -T_2 - T_2^T - Q_2$, $\Pi_{45} = 0$, $\Pi_{46} = 0$, $\Pi_{47} = 0$, $\Pi_{48} = 0$, $\Pi_{49} = 0$,
$\Pi_{55} = d_0 T_1 + \frac{1}{2}(d_0 - d_m)(d_0 + d_m - 1)T_1 - 2R_1$, $\Pi_{56} = 0$, $\Pi_{57} = W^T H_1^T$, $\Pi_{58} = 0$, $\Pi_{59} = 0$,
$\Pi_{66} = -R_2 L^{-1} - R_2^T L^{-1}$, $\Pi_{67} = W_1^T H_1^T$, $\Pi_{68} = 0$,$\Pi_{69} = 0$, $\Pi_{77} = -H_1 - H_1^T + P + \tau_2 Z_1 + \tau_{21} Z_2$
$+\tau_2 Z_1^T + \tau_{21} Z_2^T$, $\Pi_{78} = H_1 W_2$, $\Pi_{79} = 0$, $\Pi_{88} = H_1^T W_2 + W_2^T H_1 - \frac{1}{d_0} T_1$, $\Pi_{89} = 0$, $\Pi_{99} = -\gamma^2 I$.

In this case, an appropriate delay-dependent global robust stabilizing state feedback controller can be chosen as

$$u(k) = H_1^{-1} Y x(k).$$

Proof:
Define

$$J_N = \sum_{k=0}^{N} \Big[\mid Z(k) \mid^2 - \gamma^2 \mid v(k) \mid^2 \Big],$$

where the scalar $N > 0$ is an integer. Noting the zero initial condition.

$$J_N = \sum_{k=0}^{N} \Big[\mid Z(k) \mid^2 - \gamma^2 \mid v(k) \mid^2 + \Delta V(k) \Big] \le \sum_{k=0}^{N} \eta^T(k) \theta \eta(k),$$

Where

$$\Pi = \begin{bmatrix} \Pi_{11} & E_2^T - E_1 & 0 & 0 & L^T R_1 & 0 & \Pi_{17} & 0 & 0 \\ * & \Pi_{22} & S_2^T - S_1 & -T_1 + T_2^T & 0 & R_2^T & 0 & 0 & 0 \\ * & * & -Q_3 - S_2 - S_2^T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q_2 - T_2 - T_2^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & W^T H_1^T & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & W_1^T H_1^T & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & H_1^T W_2^T & H_1 \\ * & * & * & * & * & * & * & \Pi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Pi_{99} \end{bmatrix}$$

$$\eta(k) = [x^T(k) \quad x^T(k-\tau(k)) \quad x^T(k-\tau_2) \quad x^T(k-\tau_1) \quad f^T(x(k))$$
$$f^T(x(k-\tau(k))) \quad x^T(k+1) \quad \sum_{i=-d_k}^{-1} f^T(x(k+1)) \quad v^T(k)]^T$$

$$\theta = \Pi + \tau_2 E Z_1^{-1} E^T + \tau_{21} S(Z_1+Z_2)^{-1} S^T + \tau_{21} S Z_2^{-1} S^T + \tau_{21} T Z_2^{-1} T^T + \varepsilon^{-1} H M M^T H^T + \varepsilon N N^T.$$

Where Now, using Schur complent Lemma , it follows from (28) that $\theta < 0$, which together with (30) ensures that $\|z\|_2 = \gamma \|v\|_2$ holds under the zero initial condition. This completes the proof.

4. Numerical Examples

Example 4.1. Consider the following discrete-time uncertain recurrent neural network with discrete and distributed delays.

$$x(k+1) = -(A+\Delta A)x(k) + (W+\Delta W)f(x(k)) + (W_1+\Delta W_1)f(x(k-\tau(k)))$$
$$+ (W_2+\Delta W_2)\sum_{i=-d(k)}^{-1} f(x(k+1)) + u(k) + v(k),$$

with the following parameters:

$$A = \begin{bmatrix} -1.15427 & 0 \\ 0 & 0.8427 \end{bmatrix}, W = \begin{bmatrix} 0 & -0.125 \\ -0.125 & 0 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, W_2 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix}, M = \begin{bmatrix} 0 & 0.1 \\ -0.1 & -0.2 \end{bmatrix}, N_1 = \begin{bmatrix} 0.2 & 0.1 \\ -0.2 & 0.1 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -0.1 & -0.3 \\ 0 & -0.1 \end{bmatrix}, N_3 = \begin{bmatrix} -0.1 & -0.1 \\ 0.1 & -0.2 \end{bmatrix}, N_4 = \begin{bmatrix} -0.1 & -0.1 \\ 0.1 & -0.3 \end{bmatrix}, L = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$\tau_1 = 2$, $\tau_2 = 10$, $dm = 3$, $d_0 = 1$, The noise attenuation level $\gamma = 1.8$.

The feasible solutions for LMI (32) can be found as follows

$$P = 10^4 \begin{bmatrix} 3.6520 & -0.0007 \\ -0.0007 & 3.6274 \end{bmatrix}, Q_1 = 10^3 \begin{bmatrix} 5.8816 & -0.0139 \\ -0.0139 & 5.8811 \end{bmatrix}, Q_2 = 10^3 \begin{bmatrix} 6.2882 & 0.0295 \\ 0.0295 & 6.2683 \end{bmatrix},$$

$$Q_3 = 10^3 \begin{bmatrix} 6.5887 & 0.0159 \\ 0.0159 & 6.6675 \end{bmatrix}, Z_1 = \begin{bmatrix} 503.7141 & -2.2053 \\ -2.2053 & 485.8337 \end{bmatrix}, Z_2 = 10^5 \begin{bmatrix} 1.3259 & -0.0096 \\ -0.0096 & 1.3073 \end{bmatrix},$$

$$R_1 = 10^3 \begin{bmatrix} 4.1587 & 0.0053 \\ 0.0053 & 4.2575 \end{bmatrix}, R_2 = \begin{bmatrix} 831.7596 & 0.3352 \\ 0.3352 & 439.0143 \end{bmatrix}.$$

Therefore the concerned discrete time neural network is asymptotically stable.

5. CONCLUSION

In this paper, we have investigated the stability analysis problem of delay dependent H_∞ control for class of discrete-time recurrent neural networks with mixed delay and parametric uncertainty. LMI approach has been developed to derive sufficient conditions under which the controlled system is mean square asymptotically stable, where the conditions are dependent on the length of the time delays. A numerical example is given to illustrate the effectiveness of the results obtained.

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