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Exponential H_∞ filtering design for discrete-time neural networks switched systems with time-varying delay

G. Mahendrakumar
Department of Mathematics,
Thiruvalluvar University,
Vellore - 632 115, Tamilnadu, India.
magimaths01@gmail.com

R. Manivannan
Department of Mathematics,
Thiruvalluvar University,
Vellore - 632 115, Tamilnadu, India.
manimath7@gmail.com

R. Samidurai
Department of Mathematics,
Thiruvalluvar University,
Vellore - 632 115, Tamilnadu, India.
samidurair@gmail.com

***Abstract:** This paper deals with the exponential H_∞ filtering problem for discrete-time neural networks switched singular systems with time-varying delays. The main purpose of this paper is to design a linear mode-dependent filter such that the resulting filtering error system is regular, causal, and exponentially stable with a prescribed H-infinity performance bound. In addition, the decay rate of the filtering error dynamics can also be tuned. By constructing an appropriate Lyapunov functional together with some zero inequalities and using the average dwell time scheme, a novel delay-dependent sufficient condition for the solvability of the H-infinity filtering problem is derived. Based on this condition, the desired filter gains can be obtained by solving a set of linear matrix inequalities (LMIs). A numerical example is presented to show the effectiveness of the proposed design method.*

***Keywords:** H_∞ filtering; Switched neural networks; Time-varying delays ; Lyapunov functional; linear matrix inequalities.*

1. Introduction

During the past few decades, neural networks (NNs) have received great attention due to their extensive applications in a variety of areas such as signal processing, pattern recognition, associative memories, parallel computation, optimization and other scientific areas. In hardware implementation of NNs, time delays are inevitably encountered and they are a source of oscillation and instability to a great degree. Therefore, it is important to investigate the stability issues of a delayed NNs (see [HCH]–[ZYW] and references therein). The deviations and perturbations in parameters have an effect on the performance of NNs. In addition, noise disturbance is a major source of instability and can lead to poor performances in NNs. It is noted that the synaptic transmission in a realistic NNs can be viewed as a noisy process introduced by random fluctuations. On the other hand, the study on time delay systems have become a hot topic of research in both theoretical and practical importance, because time delays are inherent features of many physical systems, such as chemical process, nuclear reactors and biological systems. Recently, the existence of time delay is an important phenomenon in many realistic neural network (NN) models see [RM1]–[RM3], and [MSA]).

In particular, the dynamic behavior of discrete-time has been widely studied in [ZY]–[D], since discrete-time systems can be witnessed in various fields, such as digital communication, secure communication, and etc. It is worth noting that, in implementing the discrete-time NNs for computer simulation, experimental or computational purposes, it is essential to formulate a discrete-time system which is an analogue of the discrete-time neural network. On the other hand, switched systems have attracted considerable attention in the control community within the last few decades (see [LYWC]) and references therein). Switched systems are a class of hybrid systems, which consist of a collection of continuous- or discrete-time subsystems and a switching rule that specifies the switching between them. Second, controller switching technique provides an effective mechanism to cope with highly complex systems and/or systems (see[WCA]). There are many issues have been investigated about switched systems, among which finding less conservative conditions to guarantee the stability under arbitrary switching is a basic problem. Besides switching properties, many engineering systems always involve time-delay phenomenon due to various reasons such as finite speed of information processing. Also, it has been well recognized that the presence of time delay often leads to instability and poor performance of a system. Therefore, the problems of stability analysis, control and filtering for switched time-delay systems are taken into consideration is very important in both theoretical and practical significance, which have received great interest in the past few years (see [RM1]–[RM3], and [MSA] and references therein).

H_∞ Control has been one of the most important topics in control theory and has attracted much attention of numerous researchers in the past two decades. Recently, much appreciable work has been performed on the design of H_∞ filters for many kinds of switched system in various settings, including systems with or without time delay systems with switching, and discrete-time switched systems under arbitrary switching signals [SS], [LYC]. H_∞ Concept was proposed to reduce the effect of the disturbance input on the regulated output to within a prescribed level. Hence H_∞ finite time boundless for switched NNs with time varying delays takes considerable attention [WPN]. Due to the fact that the existence of switching may have great influence on the finite-time stability of a switched NNs, that is when all the subsystems are finite-time stable, the switched system may be not finite-time stable, which increases difficulties to discuss in this topic (see [SYZZ]). Based on the switched Lyapunov function approach, some attention has been focused on the filtering problem for discrete-time switched time-delay systems under arbitrary switching signals, and the delay-independent sufficient condition for the filter design was presented in [WT] for switched systems with constant time delay. Motivated by the above discussion, this paper is concerned with the event-triggered H_∞ filter state estimation problem for a class of discrete-time switching and time-varying delays. The main contributions of this paper are summarized as follows: 1) The NNs under consideration are comprehensive that include discrete-time, switching, time-varying delays, external disturbances; 2) We make the first attempt to introduce the event-triggered H_∞ filter mechanism into the state estimation problem for the nns addressed, which effectively reduce the information communication burden; 3) By using the Lyapunov functional theory and the linear matrix inequality (LMI) technique, an event-triggered H_∞ filter state estimator is designed that can well estimate the concentrations of the switching signal. Finally, a numerical example is presented to show the effectiveness of the proposed H_∞ filter estimation scheme.

Notations: The notations used in this paper are standard. R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $l_2[0,\infty)$ is the space of square sum able sequences. A^T represents the transpose of the matrix A. I stands for the identity matrix with appropriate dimensions. Matrices are assumed to have compatible dimensions if they are not explicitly specified.

2. Problem Formulation

We consider the following discrete-time neural network switched system with time-varying delay

$$\begin{aligned} x(k+1) =& A_{\rho(k)}x(k) + B_{\rho(k)}f(x(k)) + A_{d\rho(k)}f(x(k-d(k))) + B_{d\rho(k)}w(k), \\ y(k) =& C_{\rho(k)}f(x(k)) + C_{d\rho(k)}f(x(k-d(k))) + D_{\rho(k)}w(k), \\ z(k) =& L_{\rho(k)}x(k) + L_{d\rho(k)}x(k-d(k)) + L_{d\rho(k)}w(k), \end{aligned} \tag{1}$$

where $x_f(k)$ is the filter state vector, $z_f(k)\epsilon R^p$ is the estimated signal. $A_{fi,m}$, $B_{fi,m}$, $C_{fi,m}$ and $D_{fi,m}$ are filter gain to be determined. Define the augmented state vector $\tilde{x}(k) = [x^T(k) \quad x_f^T(k)]^T$, and the estimations error vector $\tilde{z}(k) = z(k) - z_f(k)$, the filtering error system is obtained as follows:

$$\bar{x}(k+1) = \bar{A}_i x(k) + \bar{B}_i f(x(k)) + \bar{A}_{di} f(x(k-d)) + \bar{D}_i w(k),$$

$$\tilde{z}(k) = \bar{L}_i x(k) + \bar{L}_{di} x(k-d) + \bar{G}_i w(k), \tag{4}$$

where $\bar{A_i} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fi} \end{bmatrix}$, $\bar{B_i} = \begin{bmatrix} B_i \\ B_{fi}C_i \end{bmatrix}$, $\bar{A_{di}} = \begin{bmatrix} A_i & 0 \\ B_{fi}C_{di} & 0 \end{bmatrix}$, $\bar{B_{di}} = \begin{bmatrix} B_{di} \\ B_{fi}D_i \end{bmatrix}$,

$\bar{L_i} = \begin{bmatrix} L_i - C_{fi} & 0 \end{bmatrix}$, $\bar{L_{di}} = \begin{bmatrix} L_{di} & 0 \end{bmatrix}$, $\bar{G_i} = \begin{bmatrix} G_i - D_{fi}D_i & 0 \end{bmatrix}$.

In order to more precisely describe the main objective, we introduce the following definitions and lemmas for the underlying system.

In this paper, without loss of generality, we always assume that f(0) = 0, f is the activation function. For vector -valued function f, we assume :

$$[f(x) - f(y) - G_1(x-y)]^T [f(x) - f(y) - G_2(x-y)] \le 0, \forall x, y\epsilon R^n, \tag{5}$$

where G_1, G_2 $\epsilon R^{n\times n}$ are known diagonal matrices.

Definition 2.1. [27] *System (4) is said to be exponentially stable, if there exist some scalars $K > 0$ and $0 < \chi < 1$, such that the solution $\tilde{x}(k)$ of system (4) with w(k)=0 satisfies $\|\tilde{x}(k)\|^2 < K\chi^k\|\Phi\|_L^2$, $k \ge 0$,*
where

Lemma 2.3. [27] *For any constant matrices $W = W^T \ge 0$, two positive integers r_1 and r_0 satisfying $r_1 \ge r_0 \ge 1$, the followig inequality holds*

$$\sum_{i=r_0}^{r_1} x^T(i) W \sum_{i=r_0}^{r_1} x(i) \le \tilde{r} \sum_{i=r_0}^{r_1} x^T(i) W x(i) \tag{6}$$

where $\tilde{r} = r_1 - r_0 + 1$.

Lemma 2.4. [27] *For matrix A, $Q = Q^T$ and $P > 0$, the following matrix inequality*

$$A^T PA - Q < 0 \tag{7}$$

holds if and only if there exist a matrix G of appropriate dimensions, such that

$$\begin{bmatrix} -Q & A^T G \\ * & P - G - G^T \end{bmatrix} < 0 \tag{8}$$

Now, the H_∞ filtering design problem to be addressed can be stated as follows:

Exponential H_∞ filtering problem:

For the given scalar $\gamma > 0$, we design a mode-dependent filter (4) exponentially stable, and meets the performance of $\sum_{s=0}^{\infty} \tilde{z}^T(s)\tilde{z}(s) \le \sum_{s=0}^{\infty} \gamma^2 w^T(s) w(s)$ for all nonzero $w(k)\epsilon l_2[0,\infty)$ under zero initial condition.

3. Main results

Theorem 3.1. *For given scalars $0 < \alpha < 1$, $\mu > 1$, the filtering error system (4) is exponentially stable and achives a prescribed H_∞ performance γ, if the average dwell time for any switching signal, and the following inequlities (10)-(13) for all i=1,2,... admit a solution with positive-definite $P_i, Q_{1i}, Q_{2i}, Q_{3i}, Z_{1i}, Z_{2i}$, any matrices*

$$X_i = \begin{bmatrix} X_{11i} & X_{12i} \\ * & X_{22i} \end{bmatrix} \geq 0, M_i = \begin{bmatrix} M_{1i} \\ M_{2i} \end{bmatrix}, N_i = \begin{bmatrix} N_{1i} \\ N_{2i} \end{bmatrix},$$

$\tilde{Z}_i$ of appropriate dimensions, and any matrix $\mathcal{R} \epsilon R^{2n\times 2n}$ satisfying $\tilde{R} = 0$

$$\begin{bmatrix} \Phi_{1i} & d_1\Phi_{2i}^T Z_{1i} & d_{12}\Phi_{2i}^T Z_{1i} & \Phi_{3i}^T & \Phi_{4i}^T P_i \\ * & -Z_{1i} & 0 & 0 & 0 \\ * & * & -d_{12}Z_{2i} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -P_i \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} X_{11i} & X_{12i} & M_{1i} \\ * & X_{22i} & M_{2i} \\ * & * & \alpha^{d_2} Z_{2i} \end{bmatrix} \geq 0, \tag{10}$$

$$\begin{bmatrix} X_{11i} & X_{12i} & N_{1i} \\ * & X_{22i} & N_{2i} \\ * & * & \alpha^{d_2} Z_{2i} \end{bmatrix} \geq 0, \tag{11}$$

$$P_i \leq \mu P_j, \quad Q_{1i} \leq \mu Q_{1j}, \quad Q_{2i} \leq \mu Q_{2j}, \quad Q_{3i} \leq \mu Q_{3j}, \quad Z_{1i} \leq \mu Z_{1j}, \quad Z_{2i} \leq \mu Z_{2j}, \forall i, j\epsilon M, i \neq j \tag{12}$$

where

$$\Phi_{1i} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} & \Phi_{17} \\ * & -\lambda_1 I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\lambda_2 I & 0 & -\lambda_2 G_2^T & 0 & 0 \\ * & * & * & \Phi_{44} & 0 & N_{2i}^T & 0 \\ * & * & * & * & -\alpha^{d_2}Q_{2i} & -M_{2i}^T & 0 \\ * & * & * & * & * & \Phi_{66} & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}, \tag{13}$$

$\Phi_{11} = -\alpha P_i + Q_{1i} + Q_{2i} + (d_{12}+1)Q_{3i} - \alpha^{d_1} Z_{1i} + A_i^T R Z_i + A_i R^T Z_i + X_{11i} - \lambda_1 G_1^T$,
$\Phi_{12} = B_i R^T Z_i - \lambda_1 G_2^T$, $\Phi_{13} = A_{di} R^T Z_i$, $\Phi_{14} = \alpha^{d_1} Z_{1i} + N_{1i}$, $\Phi_{15} = -M_{1i}$,
$\Phi_{16} = M_{1i} - N_{1i} + X_{12i} d_{12}$, $\Phi_{17} = B_{di} R^T Z_i$, $\Phi_{44} = -\alpha^{d_1} Q_{1i} - z_{1i}\alpha^{d_1}$, $\Phi_{46} = N_{2i}$,
$\Phi_{56} = -M_{2i}$, $\Phi_{66} = -\lambda_1 G - \alpha^{d_2} Q_{3i} + M_{2i} - N_{2i} + d_{12} X_{22i} + M_{2i}^T - N_{2i}^T$.
$\Phi_{1i} = \Phi_{7\times 7}$, $\Phi_{2i} = [(A_i - I) \quad A_{di} \quad B_i \quad 0 \quad 0 \quad 0 \quad B_{di}]$,
$\Phi_{3i} = [L_i \quad 0 \quad 0 \quad 0 \quad 0 \quad L_{di} \quad G_i]$,

$$\Phi_{4i} = [A_i \quad A_{di} \quad B_i \quad 0 \quad 0 \quad 0 \quad B_{di}].$$

4. Proof

We now prove that the system (4) with w(k)=0 is exponential stable. Choose the following Lyapunov-Krasovskii functional:

$$
\begin{aligned}
V_{1i}(k) =& x^T(k)P_i x(k),\\
V_{2i}(k) =& \sum_{s=k-d_1}^{k-1} \alpha^{(k-s-1)} x^T(s)Q_{1i}x(s) + \sum_{s=k-d_2}^{k-1} \alpha^{(k-s-1)} x^T(s)Q_{2i}x(s),\\
V_{3i}(k) =& d_1 \sum_{=-d_1}^{-1} \sum_{m=k+s}^{k-1} \alpha^{k-m-1}\eta^T(m)Z_{1i}\eta(m) + \sum_{=-d_2}^{-d_1-1} \sum_{m=k+s}^{k-1} \alpha^{k-m-1}\eta^T(m)Z_{2i}\eta(m),
\end{aligned}
\tag{14}
$$

and $\eta(m) = x(m+1) - x(m)$.

Thus, we have

$$
\begin{aligned}
\Delta V_{1i} - \alpha V_{1i}(k) =& x^T(k+1)P_i x(k+1) - \alpha x^T(k)P_i x(k),\\
\Delta V_{2i} - \alpha V_{2i}(k) =& x^T(k)(Q_{1i} + Q_{2i} + (d_{12}+1)Q_{3i})x(k) - \alpha^{d_1} x^T(k-d_1)Q_{1i}x(k-d_1)\\
& - \alpha^{d_2} x^T(k-d_2)Q_{2i}x(k-d_2) - \alpha^{d_2} x^T(k-d(k))Q_{3i}x(k-d(k)),
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
\Delta V_{3i} - \alpha V_{3i}(k) =& d_1^2 \eta^T(k)Z_{1i}\eta(k) + d_{12}\eta^T(k)Z_{2i}\eta(k)\\
& - d_1\alpha^{d_1} \sum_{m=k-d_1}^{k-1} \eta^T(k)Z_{1i}\eta(k) - \alpha^{d_2} \sum_{mk-d_2}^{k-d_1-1} \eta^T(k)Z_{1i}\eta(k).
\end{aligned}
\tag{16}
$$

It follows readily from (5) that

$$
-\lambda_1 \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} G_1 & G_2 \\ * & I \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \le 0, \tag{17}
$$

$$
-\lambda_2 \begin{bmatrix} x(k-d(k)) \\ f(x(k-d(k)) \end{bmatrix}^T \begin{bmatrix} G_1 & G_2 \\ * & I \end{bmatrix} \begin{bmatrix} x(k-d(k)) \\ f(x(k-d(k))) \end{bmatrix} \le 0. \tag{18}
$$

It follows from Lemma 2.3 that

$$
-d_1 \sum_{m=k-d_1}^{k-1} \eta^T(k)Z_{1i}\eta(k) \le -(x(k) - x(k-d_1))^T Z_{1i}(x(k) - x(k-d_1)) \tag{19}
$$

For any matrices $M_{1i}, M_{2i}, N_{1i}, N_{2i}$ and

$$
X_i = \begin{bmatrix} X_{11i}^T & X_{12i}^T \\ * & X_{22i}^T \end{bmatrix}, \tag{20}
$$

the following equations hold:

$$
0 = 2[x^T(k)M_{1i} + x^T(k-d(k))M_{2i}] \times [x(k-d(k)) - x(k-d_2) - \sum_{m=k-d_2}^{k-d(k)-1} \eta(m)], \tag{21}
$$

$$0 = 2[x^T(k)N_{1i} + x^T(k-d(k))N_{2i}] \times [x(k-d_1) - x(k-d(k)) - \sum_{m=k-d(k)}^{k-d_1-1} \eta(m)], \qquad (22)$$

$$\begin{aligned} 0 =& \sum_{m=k-d_2}^{k-d_1-1} \xi_1^T(k)X_i\xi_1(k) - \sum_{m=k-d_2}^{k-d_1-1} \xi_1^T(k)X_i\xi_1(k), \\ =& d_{12}\xi_1^T(k)X_i\xi_1(k) - \sum_{m=k-d_2}^{k-d(k)-1} \xi_1^T(k)X_i\xi_1(k) - \sum_{m=k-d(k)}^{k-d_1-1} \xi_1^T(k)X_i\xi_1^T(k), \end{aligned} \qquad (23)$$

where $\xi_1(k) = [x^T(k) \quad x^T(k-d(k))]^T$.
On other hand, we have for any matrix Z_i that

$$\begin{aligned} 2x^T(k+1)RZ_i^Tx(k) =& \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}^T \begin{bmatrix} X_i & M_i \\ * & \alpha^{d_2}Z_{2i} \end{bmatrix}^T \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix} \\ & - \sum_{m=k-d(k)}^{k-d_1-1} \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}^T \begin{bmatrix} X_i & M_i \\ * & \alpha^{d_2}Z_{2i} \end{bmatrix}^T \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}, \end{aligned} \qquad (24)$$

where
$\xi(k) = [x^T(k) \quad f^T(x(k)) \quad f^T(x(k-d(k))) \quad x^T(k-d_1) \quad x^T(k-d_2) \quad x^T(k-d(k))]^T$,

$$\Phi_{1i} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ * & -\lambda_1 I & 0 & 0 & 0 & 0 \\ * & * & -\lambda_2 I & 0 & -\lambda_2 G_2^T & 0 \\ * & * & * & \Phi_{44} & 0 & N_{2i}^T \\ * & * & * & * & -\alpha^{d_2}Q_{2i} & -M_{2i}^T \\ * & * & * & * & * & \Phi_{66} \end{bmatrix}, \qquad (25)$$

$\Phi_{2i} = [(A_i - I) \quad A_{di} \quad B_i \quad 0 \quad 0 \quad 0]$,

$\Phi_{3i} = [L_i \quad 0 \quad 0 \quad 0 \quad 0 \quad L_{di}]$,

$\Phi_{4i} = [A_i \quad A_{di} \quad B_i \quad 0 \quad 0 \quad 0]$.

By using Schur complement, it is clear that (10) guarantees $V_i(k+1) - \alpha V_i(k) < 0$.

We now choose three non-singular matrices G, H, and W such that
$G = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $G^{-1}RW = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$, $GA_i = \begin{bmatrix} A_{1i} & 0 \\ 0 & I \end{bmatrix}$, $G^{-T}P_iG^{-1} = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{3i} & P_{4i} \end{bmatrix}$,
$GA_{di} = \begin{bmatrix} A_{d1i} & A_{d2i} \\ A_{d3i} & A_{d4i} \end{bmatrix}$, $Z_iW^{-T} = \begin{bmatrix} Z_{1i} & Z_{2i} \\ Z_{3i} & Z_{4i} \end{bmatrix}$.
Let $\eta(k) = \begin{bmatrix} \eta_1(k) \\ \eta_2(k) \end{bmatrix} = H^{-1}x(k) = x(k)$, the filtering error system (4) with w(k)=0 can be decomposed as

$$\begin{aligned} \eta_1(k+1) =& A_{1i}\eta_1(k) + A_{d1i}\eta_1(k-d(k)) + A_{d2i}\eta_2(k-d(k)), \\ 0 =& \eta_2(k) + A_{d3i}\eta_1(k-d(k)) + A_{d4i}\eta_2(k-d(k)). \end{aligned} \qquad (26)$$

The stability of the filtering error system(4) with w(k)=0 is equivalent to that of system (26). It follows from (10) that

$$\begin{bmatrix} \Phi_{11}^i & \Phi_{12}^i \\ * & \Phi_{22}^i \end{bmatrix} < 0, \qquad (27)$$

which implies

$$\begin{bmatrix} \bar{\Phi}_{11}^i & \Phi_{12}^i \\ & \Phi_{22}^i \end{bmatrix} < 0, \tag{28}$$

where

$$\bar{\Phi}_{11} = -\alpha P_i + [Q_{3i} - \alpha^{d_1} Z_{1i}] + A_i^T R Z_i^T + Z_i R^T A_i^T. \tag{29}$$

Following [26], there exist a small enough scalar $\vartheta > 0$, such that

$$\begin{bmatrix} \hat{\Phi}_{11}^i & \Phi_{12}^i & 0 \\ * & \Phi_{22}^i & 0 \\ * & * & -\alpha^{d_2}\vartheta I \end{bmatrix} < 0, \tag{30}$$

where

$\hat{\Phi}_{11}^i = -\alpha P_i + \begin{bmatrix} Q_{3i} & 0 \\ * & \vartheta I \end{bmatrix} \text{-} \alpha^{d_1} Z_{1i} + A_i^T R Z_i^T + Z_i R^T A_i^T.$

Denoting

$$\begin{bmatrix} Q_{3i} & 0 \\ 0 & \vartheta I \end{bmatrix} = \begin{bmatrix} Q_{31i} & Q_{32i} \\ * & Q_{33i} \end{bmatrix}, \tag{31}$$

we get

$$\begin{bmatrix} Z_{4i}^T + Z_{4i} + Q_{33i} & Z_{4i} A_{d4i} \\ * & -\alpha^{d_2} Q_{33i} \end{bmatrix} < 0. \tag{32}$$

Then, there exist a positive constant $\vartheta > 0$ such that

$$\begin{bmatrix} Z_{4i}^T + Z_{4i} + Q_{33i} & Z_{4i} A_{d4i} \\ * & -\alpha^{d_2} Q_{33i} \end{bmatrix} \leq - \begin{bmatrix} -\nu_2 I & 0 \\ 0 & 0 \end{bmatrix}. \tag{33}$$

We now construct the following function:

$$L(k) = \eta_2^T(k) Q_{33i} \eta_2(k) - \eta^T(k - d(k)) \alpha^{d_2} Q_{33i} \eta_2(k - d(k)). \tag{34}$$

By pre-multiplying the second equation of (26) with $\eta_2^T(k) Z_{4i} e(k)$.

$$0 = \eta_2^T Z_{4i} \eta_2(k) + \eta_2^T(k) Z_{4i} A_{d4i} \eta_2(k - d(k)) + \eta_2^T(k) Z_{4i} e(k). \tag{35}$$

Adding (30) to (31) yields

$$\begin{aligned} L(k) =& \eta_2^T(k)(Z_{4i}^T + Z_{4i} + Q_{33i})\eta_2(k) + 2\eta_2^T(k) Z_{4i} A_{d4i} \eta_2(k - d(k)) \\ & - \eta_2^T(k - d(k)) \alpha^{d_2} Q_{33i} \eta_2(k - d(k)) + 2\eta_2^T(k) Z_{4i} e(k), \\ \leq & \begin{bmatrix} \eta_2(k) \\ \eta_2(k - d(k)) \end{bmatrix}^T \begin{bmatrix} Z_{4i}^T + Z_{4i} + Q_{33i} & Z_{4i} A_{d4i} \\ & -\alpha^{d_2} Q_{33i} \end{bmatrix} \begin{bmatrix} \eta_2(k) \\ \eta_2(k - d(k)) \end{bmatrix} \\ & + \nu_1 \eta_2^T(k) \eta_2(k) + \nu_1^{-1} e^T(k) Z_{4i}^T Z_{4i} e(k), \end{aligned} \tag{36}$$

where ν_1 is any positive constant.

We conclude that system (26) with $w(k)$=0 is exponentially stable with decay , which infers the filtering error system (4) is exponentially stable with decay.

We now consider the H_∞ performance of system (4). Following the similar line as before, we have

$$V_i(k+1) - \alpha V_i(k) + \Gamma(k) \leq \xi^T(k)[\Phi_{1i} + \Phi_{2i}^T(d_1^2 Z_{1i} + d_{12} Z_{2i})\Phi_{2i} + \Phi_{3i}^T P_i \Phi_{3i} + \Phi_{4i}^T \Phi_{4i}]\xi(k)$$
$$- \sum_{m=k-d_2}^{k-d(k)-1} \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}^T \begin{bmatrix} X_i & M_i \\ * & \alpha^{d_2} Z_{2i} \end{bmatrix} \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}$$
$$- \sum_{m=k-d(k)}^{k-d_1-1} \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}^T \begin{bmatrix} X_i & N_i \\ * & \alpha^{d_2} Z_{2i} \end{bmatrix} \begin{bmatrix} \xi_1(k) \\ \eta(m) \end{bmatrix}, \tag{37}$$

where
$\xi(k) = [x^T(k) \quad f^T(x(k)) \quad f^T(x(k-d(k))) \quad x^T(k-d_1) \quad x^T(k-d_2) \quad x^T(k-d(k)) \quad w^T(k)],$

$$\Gamma(k) = z^T(k)z(k) - \gamma^2 w^T(k)w(k). \tag{38}$$

. It is seen that (10) guarantees that

$$V_i(k+1) - \alpha V_i(k) + \Gamma(k) < 0.$$

This completes the proof.

5. Conclusion

The H_∞ filtering for discrete-time switched neural network system with time-varying delay has been investigated in this paper. The LMI optimization approach has been used to improve the conservativeness of the proposed results. Finally, a numerical example is given to illustrate the effectiveness of the proposed method. Furthermore, we are interested to extend this method to the discrete-time switched delayed NNs with Markov jump parameters. The results will appear in the near future.

6. Numerical Examples

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}, A_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.06 \end{bmatrix}, B_1 = \begin{bmatrix} -0.1 & 0 \\ -0.3 & 0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0 \\ -0.2 & -0.2 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} -0.2 & 0 \\ -0.3 & 0.2 \end{bmatrix}, C_{d1} = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.1 \end{bmatrix}, L_1 = \begin{bmatrix} 0.01 & -0.2 \\ 0.1 & 0.4 \end{bmatrix}, L_2 = \begin{bmatrix} 0.1 & -0.2 \\ 0.1 & 0.4 \end{bmatrix},$$
$$L_{d1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, L_{d2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix}, G_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, G_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.6 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0.2 & -0.3 \\ 0 & -0.1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.2 & -0.3 \\ 0 & -0.1 \end{bmatrix}, B_{d1} = \begin{bmatrix} -0.1 & -0.1 \\ 0.5 & 0.1 \end{bmatrix}, R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$d_1 = 2, \quad d_2 = 4, \quad t = 0.4.$$

The feasible solutions for LMI (33) can be found as follows

$$P_1 = \begin{bmatrix} 32.0219 & -0.0044 \\ -0.0044 & 32.0404 \end{bmatrix}, P_2 = \begin{bmatrix} 32.0190 & 0 \\ 0 & 32.0190 \end{bmatrix}, Q_{11} = \begin{bmatrix} 34.9032 & 0.0675 \\ 0.0675 & 34.8511 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 106.7299 & 0 \\ 0 & 106.7299 \end{bmatrix}, Q_{31} = 10^4 \begin{bmatrix} 9.8532 & -0.0001 \\ -0.0001 & -4.2150 \end{bmatrix}.$$

Therefore the concerned discrete time neural network switched system is asymptotically stable.

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