



INTERNATIONAL JOURNAL OF ADVANCE RESEARCH, IDEAS AND INNOVATIONS IN TECHNOLOGY

ISSN: 2454-132X

Impact factor: 4.295

(Volume3, Issue1)

Available online at: www.ijariit.com

Propagation of Spherical Shock Wave in A Non-Uniform Medium Under the Monochromatic Radiation and Gravitation

Dr. Rohitashw Kumar, Arvind Kumar

Department of Physics, BRS-BSM (PG) College, Babrala, Sambhal (U.P.)

rohitashsharma0001@gmail.com

Abstract: A self-similar, theoretical model of propagation of spherical shock wave in a magneto-gas dynamics rotating non uniform atmosphere in the presence of monochromatic radiation and gravitation is considered. The result discussed depends upon the variation of flow variables behind the shock, which are represented by graphically. A special case is considered in present and absence of gravitation along with rotation to observe the influence of gravitation.

Keywords: Self-similar motion, Monochromatic radiation and Magneto-gas dynamic shock wave.

I. INTRODUCTION

The radiation flux moves through the gas with a constant intensity in the direction opposite to that of the propagation of shock wave. Further the rotating gas does not radiate itself and energy is absorbed only behind the shock wave. Also, it is assumed that the radiation pressure and radiation energy are very small in comparison to the material pressure and energy, hence neglected. The results discussed depend upon the variation of flow variables behind the shock, which are shown graphically. A special study is undertaken to consider the influence of gravitation on the variation of flow parameters in the presence and absence of rotation. Recently there has been considerable interest in the study of processes occurring under the action of radiation on gaseous substances in the stellar interior. **Sedov 1959** made it possible to analyze certain classes of self-similar solution in a number of problems with disturbed energy release with the help of methods of theory of dimensionality. **Onkar Nath 1989** presented a theoretical model of spherical magneto-hydrodynamic shock waves under the action of monochromatic radiation in non-uniform stellar atmosphere, following the work of Khudyakov on the self-similar problem of motion of a gas under the action of monochromatic radiation. **Nath 1998** further studied a model of spherical shock waves in a non-uniform rotating atmosphere under the action of monochromatic radiation, where the omitted magnetic field effect. In the present paper, we have studied the propagation of spherical shock wave in a magneto-gas dynamic rotating non-uniform atmosphere in the presence of monochromatic radiation and gravitation. Gas is assumed to be gray and opaque and shock to be transparent. The shock is assumed to be propagating in a conducting medium at rest with density varying as r^β ($-2 < \beta \leq 0$). The magnetic field distribution varies as r^α ($\alpha < 0$) and is directed tangential to the advancing shock front.

Sakurai 1956 has considered the problem of a shock wave arriving at the edge of a gas in a medium in which the density varies as power law. **Wang 1964, Koch 19654, Helliwell 1969, Ray and Banerjee 1980** and others have investigated the propagation of spherical shock waves. **Rays 1974, Verma and Vishwakarma 1976**, has already obtained a self-similar solution for spherical shock waves in magneto dynamic. **Singh and Srivastava 1982** have considered the problems of spherical shock waves in an exponentially increasing medium under the material pressure. The purpose of this study is therefore, to obtain self-similar solutions for the shock propagation in non-uniform gas with self-gravitational in the pressure of radiation. The propagation of spherical shock waves in a dusty gas with radiation heat-flux and exponentially varying density is investigated by **Singh 2007**. **Singh and Yadav 2009** have found solutions for spherical shock with heat flux.

II. BASIC EQUATIONS AND BOUNDARY CONDITIONS

In accordance with the above assumptions the motion of an inviscid perfect gas in a magneto-gas dynamic rotating non-uniform medium in presence of monochromatic radiation and gravitation can be described by the following system of differential equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + \frac{\rho u}{r} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{h}{\rho r} \frac{\partial(hr)}{\partial r} + \frac{Gm}{r} - \frac{v^2}{r} = 0 \quad (2)$$

$$\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial r} + \frac{h}{r} \frac{\partial(ur)}{\partial r} = 0 \quad (3)$$

$$\frac{d}{dt}(vr) = 0 \quad (4)$$

$$\frac{\partial e}{\partial t} + u \frac{\partial e}{\partial r} - \frac{p}{\rho^2} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) + \frac{1}{\rho r} \frac{\partial(jr)}{\partial r} = 0 \quad (5)$$

$$\frac{\partial m}{\partial r} = 2\pi \rho r \quad (6)$$

$$\frac{\partial j}{\partial r} = Kj \quad (7)$$

where u, p, ρ, h, m, v, j and e are the radial component of velocity, pressure, density, magnetic field, mass per unit volume, azimuthal component of velocity, monochromatic radiation flux, energy per unit mass at a radial distance r and time t respectively. G represents the gravitational constant and K is the absorption coefficient.

Moreover, $e = p / \{\rho(\gamma - 1)\}$ (8)

Where γ is the ratio of specific heats.

The radial velocity u₀, azimuthal component of velocity v₀, density distribution ρ₀, magnetic field h₀ in the undisturbed medium are taken as

$$u_0 = 0; v_0 = 0; \rho_0 = \rho^* R^\beta, (-2 < \beta \leq 0); h_0 = h^* R^{-1} \quad (9)$$

Where ρ*, h*, β are constants, h₀ is assumed to be directed tangentially to advancing shock front and R is the radius given by (dR/dt)² = A² R^{-α}, A and α are constants.

The boundary conditions at the shock are given by [Zel'dovich and Raizer [4]:

$$\begin{aligned} U_1 &= 2(dR/dt)/(\gamma+1), & V_1 &= 2(dR/dt)/(\gamma+1) \\ \rho_1 &= \{\rho_0(\gamma+1)\}/(\gamma-1), & p_1 &= 2\rho_0(dR/dt)^2/(\gamma+1) \\ e_1 &= 2(dR/dt)^2/(\gamma+1)^2, & h_1 &= h_0(\gamma+1)/(\gamma-1), \\ m_1 &= m_0 = 2\pi\rho^* r^{2+\beta}/(2+\beta) \end{aligned} \quad (10)$$

Where the suffixes '1' and '0' refer to the conditions just behind and ahead of the shock respectively.

The Alfvén Mach number and the usual Mach number are defined as $M_A^2 = \frac{\rho_0 V^2}{h_0}$ and $M^2 = \frac{\rho_0 V^2}{\gamma p_0}$ respectively

where V=dR/dt is the speed of the shock.

The absorption co-efficient K is considered as:

$$K = K_0 \rho^n j^q r^s t^l \quad (11)$$

Where the dimension of the constant K₀ is given by

$$[K_0] = M^{-n-m-q} L^{3n+m-s} T^{2m+3q-1} \quad (12)$$

The dimension less constants J₀, p₀, ρ₀ are related as

$$j_0 = p_0^{3/2} \rho_0^{-1/2} \quad (13)$$

Under the equilibrium condition, we have from (2)

$$G = -[1/\gamma M^2 + 1(2M_A^2)](2 + \beta)(1 + \beta)(dR/dt)^2 / (\pi \rho^* r^{\beta+2}) \quad (14)$$

III. SOLUTION OF THE PROBLEM

Let us assume the solution of the fundamental equations (1)-(8) in the similarity form as –

$$u = (dR/dt)U(x), \tag{15}$$

$$v = (dR/dt)V(x), \tag{16}$$

$$\rho = \rho^0 F(x) \tag{17}$$

$$p = \rho^0 (dR/dt)^2 P(x) \tag{18}$$

$$m = m_1 S(x) \tag{19}$$

$$h = \rho_0^{1/2} (dR/dt)H(x) \tag{20}$$

$$j = J_0 J(x) \tag{21}$$

$$e = (dR/dt)^2 E(x) \tag{22}$$

$$\text{where } X = r/R(t) \tag{23}$$

Substituting these in the fundamental equations (1) – (8), we get a set of differential equations as given below:

$$\beta F - X (dF/dX) + U (dF/dX) + F(dU/dX) + FU/X = 0 \tag{24}$$

$$(U-X)(dU/dX) - (\alpha U)/2 + [(dP/dX)/F] + H/(XF)[H+X(dH/dX)] - \{1/(\gamma M^2) + 1/(2M_A^2)\}2(1+\beta)(S/X) - V^2/X = 0 \tag{25}$$

$$(\beta - \alpha)/2 + (u-x)(dH/dX) / H + U/X + (dU/dX) = 0 \tag{26}$$

$$U[V+X(dV/dX)] - X[(\alpha V/2) + X(dV/dX)] = 0 \tag{27}$$

$$-[\alpha E + X(dE/dX)] + U(dE/dX) - (P/F^2)[\beta F + (U-X)(dF/dX) + 1/(XF\gamma^{3/2}M^3)[J+X(dJ/dX)]] = 0 \tag{28}$$

$$\frac{dJ}{dX} = \alpha_1 X^s F^n P^m J^{q+1} \tag{29}$$

$$\frac{dP}{dX} = (\gamma - 1)[E(dF/dX) + F(dE/dX)] \tag{30}$$

$$\frac{dS}{dX} = [(2 + \beta)F/X] - [(2 + \beta)S/X] \tag{31}$$

$$\alpha_1 = K_0 \rho_0^{n+m-1} j_0^{q+1} (dR/dt)^{2m-2+s} \tag{32}$$

The above set of differential equations (24) – (32), after some simplifications can be put in the following form :

$$\frac{dP}{dX} = [(\alpha FUX(U - X))/2 + \gamma PU + (\beta - \alpha)PX]$$

$$+ (\gamma - 1)(1/\gamma^{3/2}M^3)(\alpha_1 X^{s+1} F^n P^m J^{q+1} + J) + (XH^2/2)(\beta - \alpha + 2) \tag{33}$$

$$+ (1/\gamma M^2 + 1/2M_A^2)2(1 + \beta)S(U - X)F + V^2 F(U - X)]$$

$$\div (FX(U - X)^2 - \gamma PX - XH^2)$$

$$\frac{dF}{dX} = -(F/(U - X))[U/X] + (dU/dX) + \beta \tag{34}$$

$$\frac{dH}{dX} = -(H/(U - X))[U/X] + (dU/dX) + (\beta - \alpha)/2 \tag{35}$$

$$\frac{dE}{dX} = -(E/(U - X))[\gamma - 1][(U/X) + (dU/dX)] \tag{36}$$

$$+ (\gamma - 1)/PX(1/\gamma^{3/2}M^3)(\alpha_1 X^{s+1} F^n P^3 J^{q+1} + J) - \alpha]$$

$$\frac{dP}{dX} = (\gamma - 1)[E(dF/dX) + F(dE/dX)] \tag{37}$$

$$\frac{dV}{dX} = (-V/(U - x))[U/X] - \alpha/2 \tag{38}$$

$$\frac{dJ}{dX} = \alpha_1 X^s F^n P^m J^{q+1} \tag{39}$$

$$\frac{dS}{dX} = [(2 + \beta)F/X] - [(2 + \beta)S/X] \tag{40}$$

The boundary conditions (10) changes to-

$$U = 2/(\gamma + 1), V = 2/(\gamma + 1), F = 1, P = 1, E = 1, J = 1, H = 1$$

and $S = 1$ for $X = 1$ (41)

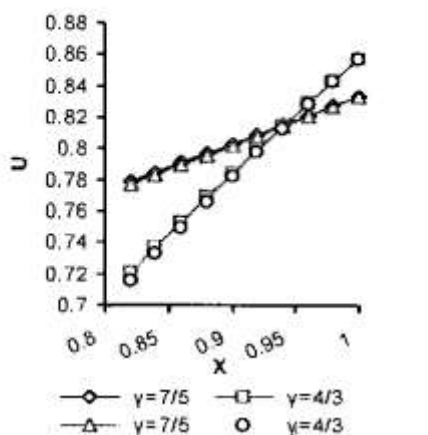


Fig 1. Variation of Radial velocity with Distance series 1&2 shows with Gravitation($\gamma=7/5,4/3$) & series 3&4 without Gravitation($\gamma=7/5,4/3$)

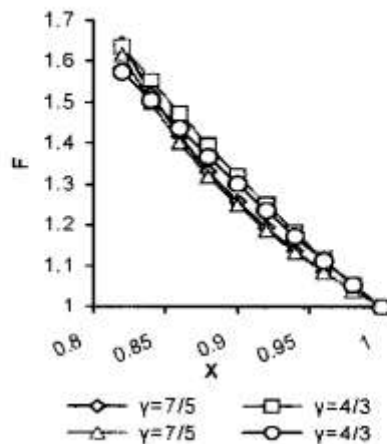


Fig 2. Variation of Density with Distance series 1&2 shows with Gravitation ($\gamma=7/5,4/3$) & series 3&4 without Gravitation($\gamma=7/5,4/3$)

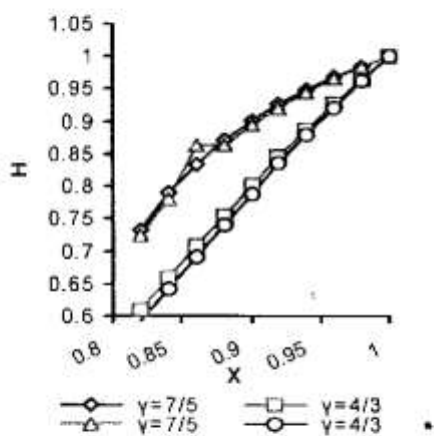


Fig 3. Variation of Magnetic Field with Distance series 1&2 shows with Gravitation($\gamma=7/5,4/3$) & series 3&4 without Gravitation($\gamma=7/5,4/3$)

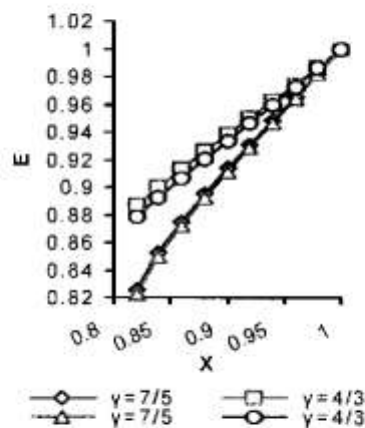


Fig 4. Variation of Energy with Distance series 1&2 shows with Gravitation ($\gamma=7/5,4/3$) & series 3&4 without Gravitation($\gamma=7/5,4/3$)

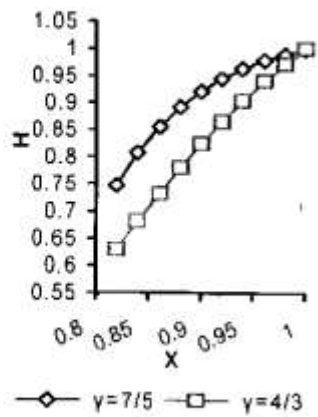


Fig 11. Variation of Magnetic Field with Distance

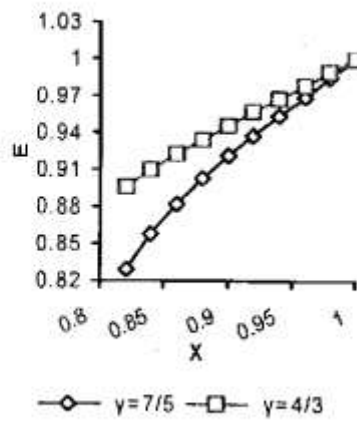


Fig 12. Variation of Energy with Distance

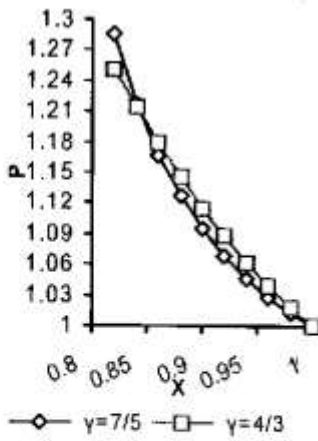


Fig 13. Variation of Pressure with Distance



Fig 14. Variation of Radiation Flux with Distance

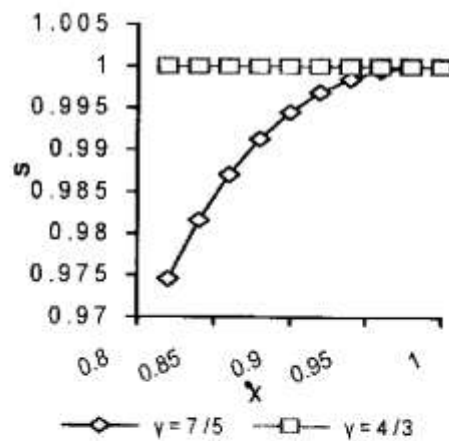


Fig 15. Variation of Mass with Distance

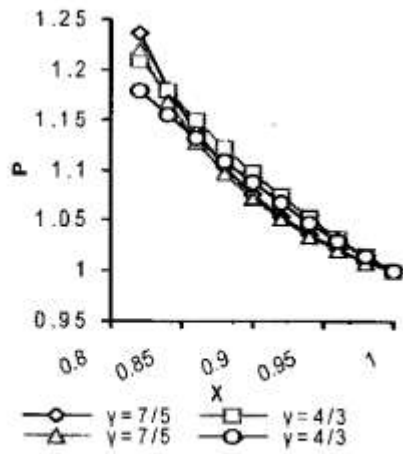


Fig 5. Variation of Pressure with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation($\gamma=7/5, 4/3$)

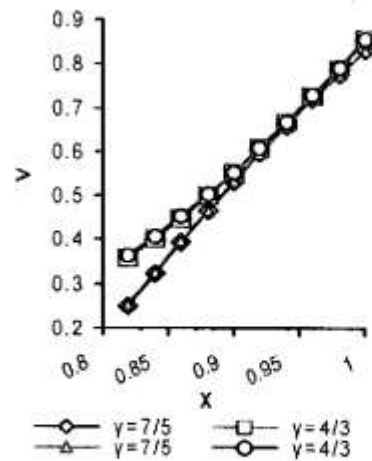


Fig 6. Variation of Rotational Velocity with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation($\gamma=7/5, 4/3$)

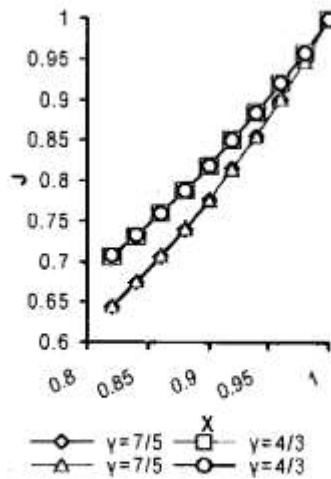


Fig 7. Variation of Radiation Flux with Distance series 1&2 shows with Gravitation($\gamma=7/5, 4/3$) & series 3&4 without Gravitation($\gamma=7/5, 4/3$)

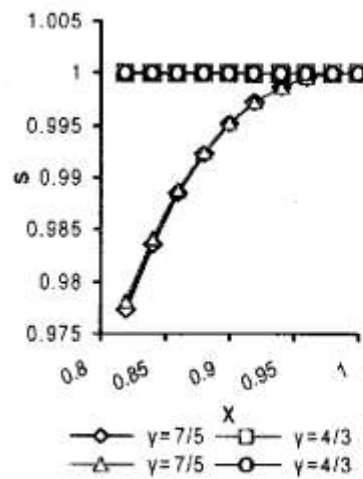


Fig 8. Variation of Mass with Distance series 1&2 shows with Gravitation ($\gamma=7/5, 4/3$) & series 3&4 without Gravitation($\gamma=7/5, 4/3$)

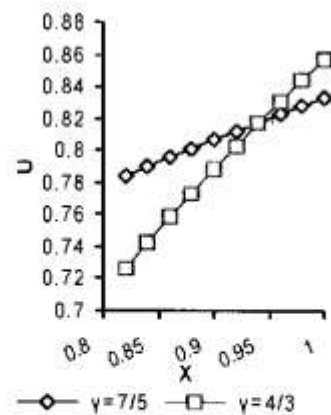


Fig 9. Variation of Radial velocity with Distance

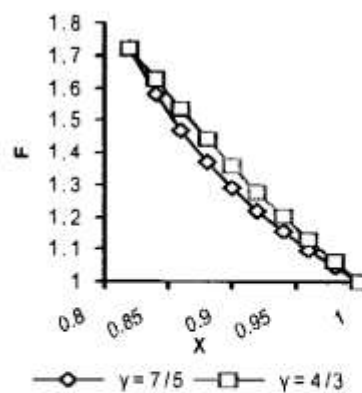


Fig 10. Variation of Density with Distance

RESULT AND DISCUSSIONS

The set of differential equations (33)- (40) have been integrated numerically with the help of boundary conditions (41) by the well known Runge-Kutta method $\alpha=2$ and 2.5 , ($\beta=-1.5$ and -2 , $\gamma=4/3, 7/5$; $M^2=5$, $M=10$, $\alpha =2(2-\gamma)/(\gamma+1)$ and $n=-1/2$, $m=3/2$, $q=0, s=1$). We have plotted the graphs showing the variations of various flow parameters with distance for different values of γ , β and α_1 in presence and absence of gravitational field and in presence and absence of rotation. This helped us to study the importance of gravitation and rotation respectively on flow parameters.

From Fig. 1 to Fig.8, we observe that the radial component of velocity, magnetic field, energy, rotational velocity, radiation flux decrease as we go towards the center of the explosion, while density and pressure increase as we go towards the center of the shock. TN fig.8, it is surprising to note that the mass decreases for the case of $\gamma=7/5$ for both presence and absence of gravitational field, it remains uniform for the case of $\gamma=4/3$. In absence of gravitational field decrease in the radial velocity, magnetic field and energy is more prominent. In the presence of gravitational field we see that decrease in rotational velocity, radiation flux is more prominent while the increase in the value of density and pressure is more prominent.

To draw a comparison between gravitational effects vis-a-vis rotational effect, we have also observed the variations of flow parameters in the absence of rotation in fig.9 to fig. 15 and compared it with the earlier drawn graphs for the absence of gravitation. We find that radial velocity, magnetic field, energy decrease more in presence of rotational velocity while radiation flux decreases more rapidly in the absence of rotational velocity. As expected, in the absence of rotation, the value of pressure and density increase more as we go towards the center of the shock. However, the peculiar phenomenon to be observed is that the variation of mass remains uniform in case of $\gamma=4/3$, $\alpha_1=2$ and $\beta=-2$ irrespective of the presence or absence of the rotational effect. We further note that the gravitational effect is important for the propagation of shock waves in the present problem.

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